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Problem Solving: Main Steps

1. Problem definition
2. Algorithm design / Algorithm Specification
3. Algorithm analysis / Performance
4. Implementation
5. Testing
6. Maintenance

I. Problem Definition

- What is the task to be accomplished?
 - Calculate the average of the grades for a given student
 - Understand the talks given out by politicians and translate them in Chinese
- What are the time / space / speed / performance requirements ?

2. Algorithm Design / Specifications

- Algorithm: Finite set of instructions that, if followed, accomplishes a particular task.
- Criteria to follow:
 - Input: Zero or more quantities (externally produced)
 - Output: One or more quantities
 - Definiteness: Clarity, precision of each instruction
 - Finiteness: The algorithm has to stop after a finite (may be very large) number of steps
 - Effectiveness: Each instruction has to be basic enough and feasible

Algorithm Design Goals

- The two basic design goals that one should strive for in a program are:
 1. Try to save Time
 2. Try to save Space
- A **program that runs faster is a better** program, so saving time is an obvious goal. Like wise,
- a **program that saves space** over a competing program is considered desirable.

4,5,6: Implementation, Testing, Maintenance

- Implementation
 - Decide on the **programming language** to use
 - C, C++, Lisp, Java, Perl, Prolog, assembly, etc. , etc.
 - Write clean, **well documented code**
- Test, test, test....
- Integrate **feedback from users, fix bugs**, ensure compatibility across different versions → Maintenance

3. Algorithm Analysis/Performance

1. Time complexity

- How much time does it take to run the algorithm

2. Space complexity

- How much space is required

I. Time Complexity

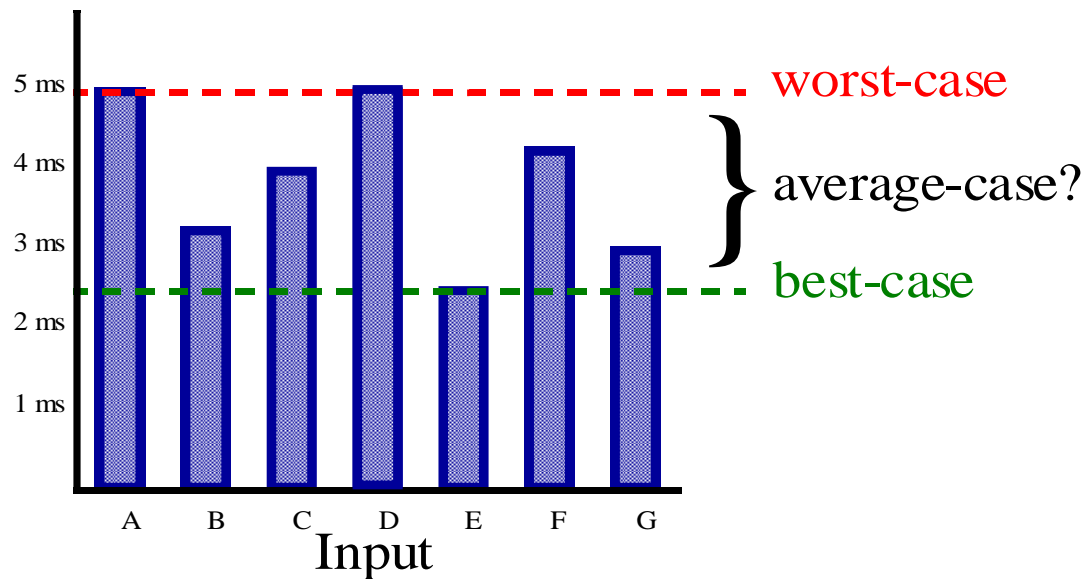
- Often more important than space complexity
 - **space available** (for computer programs!) tends to be larger and larger
 - **time** is still a **problem** for all of us
- 3-4GHz processors on the market
 - still ...
 - researchers estimate that the computation of various transformations for 1 single DNA chain for one single protein on 1 TerraHZ computer would take about 1 year to run to completion
- **Algorithms running time is an important issue**

Running Time

- Problem: prefix averages
 - Given an array X
 - Compute the array A such that $A[i]$ is the average of elements $X[0] \dots X[i]$, for $i=0..n-1$
- Sol 1
 - At each step i , compute the element $X[i]$ by traversing the array A and determining the *sum* of its elements, respectively the average
- Sol 2
 - At each step i update a *sum* of the elements in the array A
 - Compute the element $X[i]$ as sum/i

Big question: Which solution to choose??

Running time



Suppose the program includes an *if-then* statement that may execute or not: → **variable running time**

Typically **Algorithms are measured** by their **worst case**

Space Complexity:

- Space complexity = The **amount of memory required by an algorithm to run to completion**
- The space need by a program has the following components:
- **Instruction space:** Instruction space is the space needed to store **the compiled version of the program instructions.**
- **Data space:** Data space is the space needed to store **all constant and variable values.** Data space has two components:
 - Space needed by constants and simple variables in program.

2. Space Complexity

- Space needed by **dynamically allocated objects such as arrays and class instances.**
- **Environment stack space:** The environment stack is used **to save information needed to resume execution of partially completed functions.**
- Some algorithms may be more efficient if data completely loaded into memory
 - Need to look also at **system limitations**
 - E.g. Classify 2GB of text in various categories [politics, tourism, sport, natural disasters, etc.]
 - can I afford to load the entire collection?

Space Complexity (cont'd)

1. **Fixed part:** The size required to store certain data/variables, that is **independent of the size of the problem:**
 - e.g. name of the data collection
 - same size for classifying 2GB or 1MB of texts
2. **Variable part:** Space needed by variables, whose size is **dependent on the size of the problem:**
 - e.g. actual text
 - load 2GB of text VS. load 1MB of text

Space Complexity (cont'd)

- $S(P) = c + S(\text{instance characteristics})$
 - $c = \text{constant}$
- Example:

```
void float sum (float* a, int n)  
{  
    float s = 0;  
    for(int i = 0; i < n; i++) {  
        s+ = a[i];  
    }  
    return s;  
}
```

Space?

one word for n , one for a [passed by reference!], one for $i \rightarrow$ constant space!

Experimental Approach

- Write a program that implements the algorithm
- Run the program with data sets of varying size.
- Determine the actual running time
- Problems?

Experimental Approach

- It is necessary to implement and test the algorithm in order to determine its running time.
- Experiments can be done only on a limited set of inputs, and may not be indicative of the running time for other inputs.
- The same hardware and software should be used in order to compare two algorithms. – condition very hard to achieve!

Use a Theoretical/Analytical Approach

- Based on **high-level description of the algorithms**, rather than language dependent implementations
- Makes possible an **evaluation of the algorithms** that is independent of the hardware and software environments
→ **Generality**

Algorithms (example)

Describe an algorithm for finding the maximum value in a finite sequence of integers.

Solution:

- Set the temporary maximum equal to the first integer in the sequence.
- Compare the next integer in the sequence to the temporary maximum, and if it is larger than the temporary maximum, set the temporary maximum equal to this integer.
- Repeat the previous step if there are more integers in the sequence
- Stop when there are no integers left in the sequence.
- The temporary maximum at this point is the largest integer in the sequence.

Algorithms (example)

Describe an algorithm for finding the maximum value in a finite sequence of integers.

Solution:

Procedure max($a_1, a_2, a_3, \dots, a_n$: integers)

max = a_1

for $i=2$ **to** n

if max < a_i

then max = a_i

output max

Algorithm Example

- Example: Describe an algorithm for finding the maximum value in a finite sequence of integers.

(or)

find the maximum element of an array.

1. **Algorithm** `arrayMax(A, n)`:

Input: An array A storing n integers.

Output: The maximum element in A.

2. `currentMax` \leftarrow `A[0]`
3. **for** `i` \leftarrow 1 **to** `n - 1` **do**
4. **if** `currentMax` < `A[i]`
5. **then** `currentMax` \leftarrow `A[i]`
6. **return** `currentMax`

Low Level Algorithm Analysis

- Based on **primitive operations** (low-level computations independent from the programming language)
- E.g.:
 - Make an addition = 1 operation
 - Calling a method or returning from a method = 1 operation
 - Index in an array = 1 operation
 - Comparison = 1 operation etc.
- **Method**: Inspect the pseudo-code and **count the number of primitive operations executed by the algorithm**