Lecture 9

E-K relationship

Recap: The effect of electric field was discussed on electron and holes. The tilt of the bands is caused by an externally applied electric field. Thus, hole energy increases downward and holes seeking the lowest energy state available that are greatly found at the top of the valence band. In contrast conduction band electron are found at the bottom of the conduction band. An electron in crystal may behave as if it had a mass different from the free electron mass m_0 i.e Effective mass.

E-K diagram

An E-k diagram shows characteristics of a particular semiconductor material. It shows the relationship between the energy and momentum of available quantum mechanical states for electrons in the material. The k vector roughly corresponds to momentum for free particles; is called the crystal momentum. Since momentum is proportional to velocity and kinetic energy is proportional to velocity squared, in free space the relationship between E and k depends on the square of k and is thus a parabola with its bottom at k=0. First, consider a basic E-k band diagram like this one (the x-axis can be either momentum, p, or wavenumber, k since,

$$p = mv = \hbar k \text{------------------------------------}(1)$$

Then, $$E = \frac{1}{2} mV^2 = \frac{1}{2} p^2/m = \hbar^2/2m\, k^2 \text{--------------} \quad (2)$$

Thus the electron energy is parabolic with wave vector **k.** The electron mass is inversely related to the curvature (second derivative) of the (E,K) relationship.

Since $$d^2E/dk^2 = \hbar^2/m \text{----------------------------}(3)$$

Although electrons in solids are not free, most energy bands are close to parabolic at their minima (for conduction band) or maxima for valence band as shown in Figure 1.

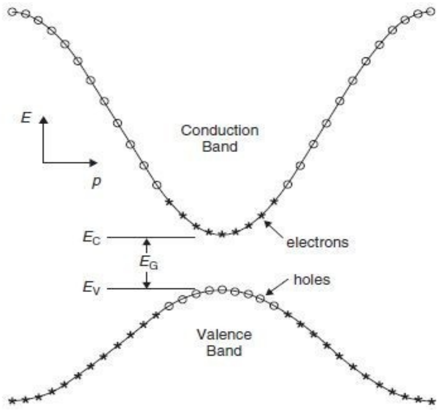
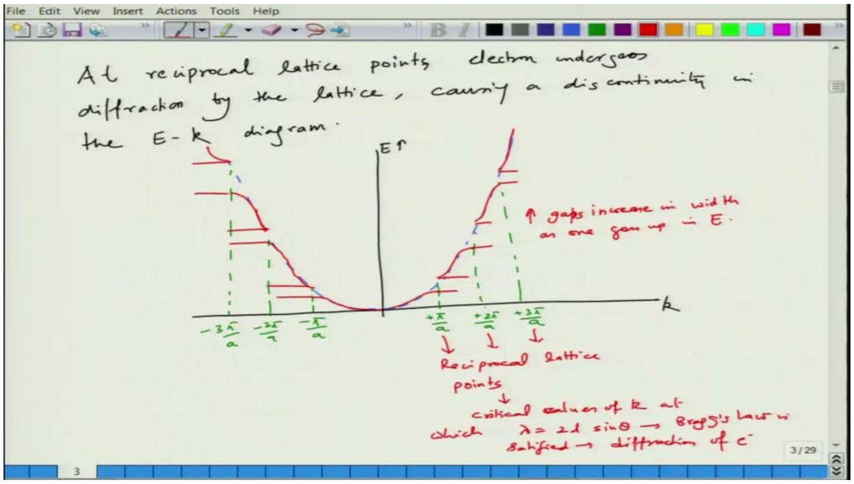


Figure 1

Semiconductors have forbidden energy gap in the energy band diagram in contrast to metals. Essentially, at lattice points, in the reciprocal space at reciprocal lattice points electron undergoes diffraction by the lattice, causing a discontinuity in the E k diagram. For a single one-dimensional solid, we will will have E vs K diagram as below and at the reciprocal lattice points, we will have a diffraction.



Reciprocal lattice points are basically a vector in the reciprocal space. This are + π/ a, + 2 π/ a, + 3 π/ a, and on and forth, - π/ a, - 2 π/ a, - 3 π/ a. We will have these discontinuities at these points that are basically the gaps states. And as these gaps increase in size, increase in width as one goes up in energy. as you increase the energy, the gaps become larger and larger. If this is the case, then we can say that these are essentially energy bands separated by energy gap at a critical value of k. These are the, reciprocal lattice points or we can say critical values of k at which λ is equal to 2 d sin θ, the Bragg's law is satisfied leading to electron diffraction.

Summary

Most energy bands are close to parabolic at their minima (for conduction band) or maxima for valence band. In the reciprocal space at reciprocal lattice points electron undergoes diffraction by the lattice, causing a discontinuity in the E k diagram. These discontinuities at these points are basically the gaps states. As energy increases , the gaps become larger and larger.