Laplace :⇒

The laplace operator is one of the mathematical tools used for the solution of linear ordinary integro-differential equations. (Mastly continuous- time systms are described by integro-differential equations)

The solution of linear differential equation by the laplace transformation methods. It converts the linear differential equation in to an algebraic equation in s-domain. It is then possible to manipulate the algebraic equation by simple algebraic rules to obtain the expression in suitable forms. Then the final solution is obtained by taking the inverse laplace transform.

$$f(t) \xleftrightarrow{L.T.} F(S)$$

Laplace Transformation :⇒

The laplace transformation method is a powerful technique for solving linear differential equations.

The laplace transformation of a time function $f(t)$ is defined as

$$F(s) = L\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}\,dt$$

for $t = -\infty$ to ∞

where,

Integration is carried from $-\infty \rightarrow +\infty$

s is the laplace transformation variable or laplace operator, which is complex variable and it is expressed as

$$s = \sigma + j\omega$$

here,
σ is real and positive part and $j\omega$ is imaginary part, in which ω is angular frequency.

* In the time function, t represents time; s in the laplace transform that must represent the dimension inverse of time i.e., frequency. this reason that the transformed variable is complex frequency

Advantage of Laplace transformation :⇒

1. Solution of continuous time systems can be easily obtained.
2. Initial conditions are automatically incorporated without the necessity of first determing the genral solution.
3. It gives complete solution in one operation.

Region of Convergence :⇒

It is also called Existance of Laplace Transform.

* Equation attains some finite value is called as region of convergence (ROC).

According to the definition of Laplace transform

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

Now we have,

$$s = \sigma + j\omega$$

$$\therefore \; e^{-st} = e^{-\sigma t} \cdot e^{-j\omega t}$$

We have,

$$e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

therefore,

$e^{-j\omega t}$ is lies always in the range +1 to −1. So we can modify the condition of existance of laplace transform as

$$\int_0^\infty |f(t)\, e^{-\sigma t}|\, dt < \infty$$

It gives sufficient condition for existance of Laplace transform.

Definition:-

The range of values of σ for which above equation attains some finite value is called region of convergence (ROC).

Bilateral Laplace Transform :⇒

It is defined as,

$$\therefore F(S) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

The laplace transform of signal $f(t)$ is $F(S)$ and it is denoted by,

$$f(t) \xleftrightarrow{L} F(S)$$

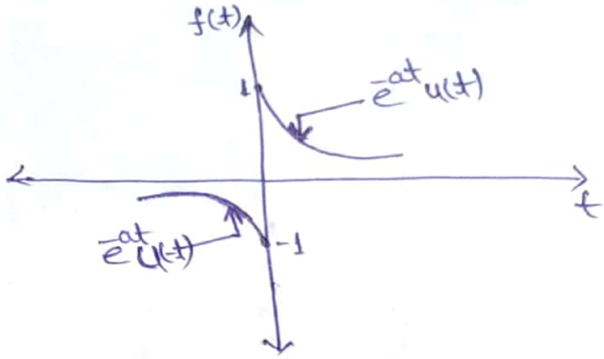
the limits of integration are from $-\infty$ to $+\infty$; so it is called as double sided or bilateral laplace transform.

ROC :⇒ If signal is both sided then ROC is an intersection of two ROCs. we will solve some numericals then related to the concept of ROCs.

Laplace transform of two sided exponential signal :⇒

consider two sided exponential signal given by,

$$f(t) = e^{-at}u(t) - e^{at}u(-t)$$



we have,

$$F(S) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} [e^{-at}u(t) - e^{at}u(-t)] . e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at} . e^{-st} u(t)\, dt - \int_{-\infty}^{\infty} e^{at} . e^{-st} u(-t)\, dt$$

In the first integration, we have $u(t)$. so limits of integration will be from $0 \to \infty$ and in second integration, we have $u(-t)$. so limit of integration will be from $-\infty$ to 0

Therefore,

$$F(s) = \int_0^{\infty} e^{-at} \cdot e^{-st}\, dt - \int_{-\infty}^{0} e^{at} \cdot e^{-st}\, dt$$

$$= \int_0^{\infty} e^{-(a+s)t}\, dt - \int_{-\infty}^{0} e^{(a-s)t}\, dt$$

$$= \left[\frac{e^{-(a+s)t}}{-(a+s)} \right]_0^{\infty} - \left[\frac{e^{(a-s)t}}{(a-s)} dt \right]_{-\infty}^{0}$$

$$= -\frac{1}{(a+s)} \left[\frac{1}{e^{(a+s)t}} \right]_0^{\infty} - \frac{1}{(a-s)} \left[e^{(a-s)t} \right]_{-\infty}^{0}$$

$$= -\frac{1}{(a+s)} [0-1] - \frac{1}{(a-s)} [1-0]$$

$$= \frac{1}{(a+s)} - \frac{1}{(a-s)}$$

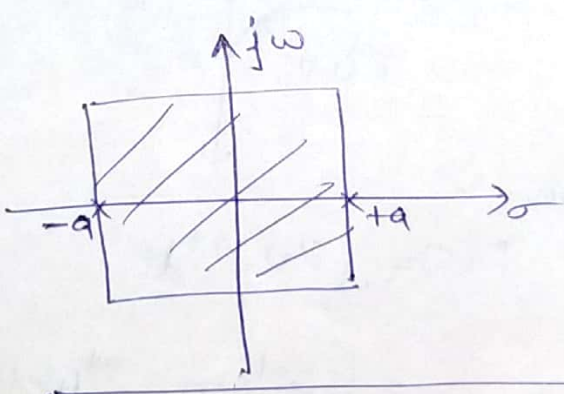
$$= \frac{(a-s) - a - s}{(a+s)(a-s)} = \frac{-2s}{(a+s)(a-s)} = \frac{2s}{(s+a)(s-a)}$$

<u>ROC</u>

$Re\{s+a\} > 0$ and, $Re\{s-a\} < 0$

$\therefore \sigma + a > 0$ $\qquad \therefore \sigma - a < 0$

$\therefore \sigma > -a$ $\qquad \therefore \sigma < +a$



$$\boxed{\therefore \text{ROC}: -a < \sigma < +a}$$