Analysis and Design of Algorithms

UNIT-1

Recurrence Relations

Content

- Solving Recurrence Relations
  - Substitution Method

SUBSTITUTION METHOD

2. SUBSTITUTION METHOD

- The substitution method comprises of 3 steps-

1. “Making a good guess” method or Guess the form of the solution.
2. then use induction or Verify by induction method.
3. Solve for constants or to find the constants and show that solution works.

- Examples:
  - $T(n) = 2T(n/2) + \Theta(n)$ → $T(n) = \Theta(n \log n)$

2. SUBSTITUTION METHOD

- *The substitution method can be used to establish either upper or lower bounds on a recurrence.*
- We substitute the guessed solution for the function, when applying the inductive hypothesis to smaller values. Hence the name "substitution method".
- This method is powerful, but we must be able to guess the form of the answer in order to apply it.
  - In this method – "To guess the solution, play around with small values for insight".
  - That is in this method first one start "guess a solution and prove by induction".

Substitution method - An example

- **T(n) = 2T((n/2)) +n**

We **guess** that the solution is **T(n)=0(n lg n).**

i.e. to show that **T(n) ≤ c n lg n** , for some constant c> 0 and n ≥ m.

Assume that this bound holds for [n/2]. So , we get

T(n) ≤ 2(**c * (n/2) lg((n/2)**)) + n →T(n/2)=**c * (n/2) lg((n/2)**

≤ cn lg(n/2) + n

= cn lg n – cn lg 2 + n

= cn lg n – cn + n

≤ cn lg n

where , the last step holds as long as c≥ 1.