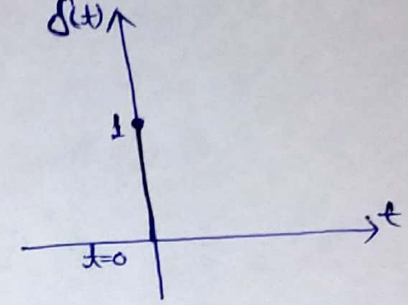
Laplace Transform of Common Functions:⇒

1. Delta Function:-

The delta function is defined as

$$\delta(t) = 1 \quad \text{for } t=0$$
$$= 0 \quad \text{otherwise}$$



according to laplace transform

$$F(s) = \int_0^\infty f(t).e^{-st}\,dt$$

here,

$f(t) = \delta(t)$ and it value is 1 only at $t=0$, therefore

$$F(s) = \int_0^\infty 1.\,e^{0}\,dt$$

$$= 1$$

$$\therefore \; \delta(t) \xleftrightarrow{L.T.} 1$$

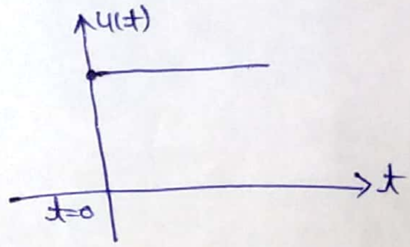
ROC:-

Since 's' term is absent in eq. ROC is entire s-pla

2. Unit Step Function:-

The unit step function is defined as

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



Now, we have

$$F(s) = \int_0^\infty 1.\,e^{-st}\,dt$$

$$= \left[-\frac{e^{-st}}{s}\right]_0^\infty$$

$$= -\frac{1}{s}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{s}$$

$$\therefore \; u(t) \longleftrightarrow \frac{1}{s}$$

ROC:-

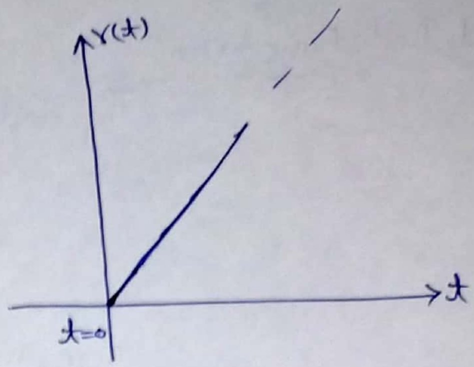
the L.T. of u(t) is 1/s and it is for the range Re(s)> , that means for $\sigma > 0$

3. Unit Ramp Function :⇒

Unit ramp signal is defined as

$r(t) = t$ for $t \geq 0$

$= 0$ otherwise



We have,

$$F(s) = \int_0^\infty t \cdot e^{-st} \, dt$$

$$= \left[\frac{e^{-st}}{s^2} (-st-1) \right]_0^\infty \qquad \left\{ \int t\, e^{at} dt = \frac{e^{at}}{a^2} (at-1) \right.$$

$$= -\left[\frac{e^{-st}}{s^2} (st+1) \right]_0^\infty = -\frac{1}{s^2} \left[e^{-st} \cdot st + e^{-st} \right]_0^\infty$$

$$= -\frac{1}{s^2} \left\{ \left[e^{-st} \cdot st \right]_0^\infty + \left[e^{-st} \right]_0^\infty \right\}$$

$$= -\frac{1}{s^2} \left\{ \left[\frac{st}{e^{st}} \right]_0^\infty + \left[\frac{1}{e^{st}} \right]_0^\infty \right\} = -\frac{1}{s^2} [0 - 1] = \frac{1}{s^2}$$

$$\therefore \; r(t) \xleftrightarrow{L.T.} \frac{1}{s^2}$$

ROC :-

here L.T. of $r(t)$ is $1/s^2$ and it is valid for the range $Re\{s\} > 0$ i.e. means $\sigma > 0$.

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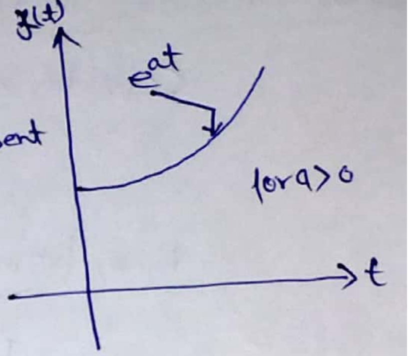
4. Positive Side Growing exponential Function :⇒

It is defined as

$$f(t) = e^{at} . u(t) \quad \text{for } a>0$$

Multiplication by $u(t)$ indicats that e^{at} is present in the range $t = 0 \to \infty$



we have,

$$F(S) = \int_0^\infty f(t) . e^{-st} dt$$

$$= \int_0^\infty e^{at} . e^{-st} dt$$

$$= \int_0^\infty e^{-(s-a)t} dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)} \right]_0^\infty$$

$$= \frac{-1}{(s-a)} \left[\frac{1}{e^{(s-a)t}} \right]_0^\infty$$

$$= -\frac{1}{s-a} [0-1]$$

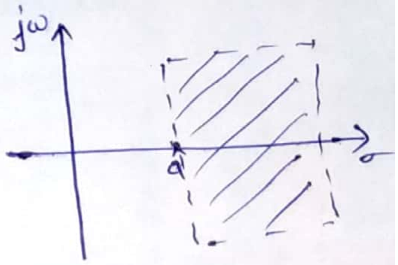
$$= \frac{1}{(s-a)}$$

$$\therefore \quad e^{at} . u(t) \xleftrightarrow{L.T.} \frac{1}{s-a}$$

ROC :-

The L.T. is $\frac{1}{s-a}$. It has pole as $+a$. therefor ROC is $Re\{s-a\} > 0$, that means $Re(s) > a$. But $Re(s)$ means σ

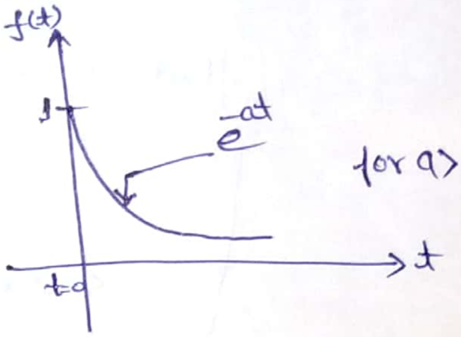
$\therefore$ ROC is $\sigma > a$



5. Positive side decaying exponential function :⇒

It is defined as

$$f(t) = e^{-at} . u(t) \quad \text{for } a>0$$



we have,

$$F(s) = \int_0^\infty e^{-at} . e^{-st} dt$$

$$= \int_0^\infty e^{-(s+a)t} . dt$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_0^\infty$$

$$= \frac{1}{-(s+a)} \left[\frac{1}{e^{(s+a)t}} \right]_0^\infty$$

$$= \frac{1}{-(s+a)} [0-1]$$

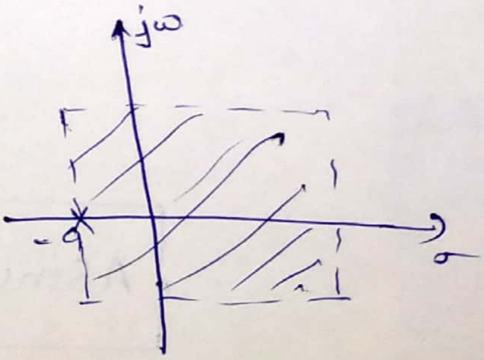
$$= \frac{1}{s+a}$$

$$\therefore \quad e^{-at} . u(t) \xleftrightarrow{L.T.} \frac{1}{s+a}$$

ROC :-

The ROC is, $Re\{s+a\} > 0$; that means $Re(s) > -a$

$\therefore$ ROC is $\sigma > -a$



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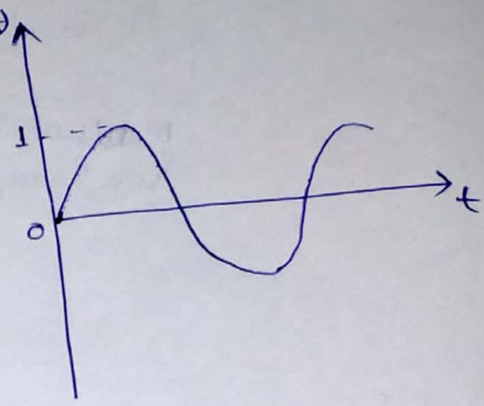
6. Sinusoidal Function: ⇒

Consider an input signal,

$$f(t) = \sin\omega t$$



Now, from laplace transform

$$F(S) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty \sin\omega t . e^{-st}\, dt$$

$$\left\{ \text{we have the trigonometric identity} \quad \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right\}$$

Therefore,

$$F(S) = \int_0^\infty \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right] e^{-st}\, dt$$

$$= \frac{1}{2j}\left[\int_0^\infty e^{j\omega t} . e^{-st} dt - \int_0^\infty e^{-j\omega t} . e^{-st} dt\right]$$

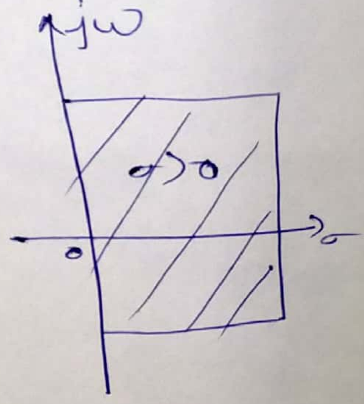
$$= \frac{1}{2j}\left\{L[e^{j\omega t}] - L[e^{-j\omega t}]\right\}$$

$$\left\{ \begin{array}{l} \text{But, the standard laplace transform pairs,} \\ e^{\pm at} u(t) \longleftrightarrow \frac{1}{S \mp a}; \quad ROC: \sigma > \pm a \\ \therefore L\{e^{j\omega t}\} \longrightarrow \frac{1}{S - j\omega}; \quad ROC: \sigma > j\omega \\ \text{Similarly,} \\ L\{e^{-j\omega t}\} \longrightarrow \frac{1}{S + j\omega}; \quad ROC: \sigma > -j\omega \end{array} \right\}$$

$$F(S) = \frac{1}{2j}\left[\frac{1}{S - j\omega} - \frac{1}{S + j\omega}\right]$$

$$= \frac{1}{2j}\left[\frac{2j\omega}{s^2 + \omega^2}\right]$$

$$= \frac{\omega}{s^2 + \omega^2}; \quad ROC: \sigma > 0$$



$$\boxed{A \sin\omega t . u(t) \xleftrightarrow{L.T.} \frac{A\omega}{s^2 + \omega^2}}$$

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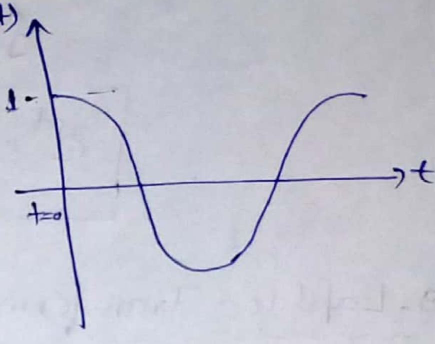
7. Cosinusoidal Function :⇒

Consider a signal,



$$f(t) = \cos\omega t$$

from laplace transformation,

$$F(s) = \int_0^\infty f(t) \cdot e^{-st}\, dt$$

$$= \int_0^\infty \cos\omega t \cdot e^{-st}\, dt$$

we know that,

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

Therefore,

$$F(s) = \int_0^\infty \left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right] e^{-st}\, dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{j\omega t} \cdot e^{-st}\, dt + \int_0^\infty e^{-j\omega t} \cdot e^{-st}\, dt\right]$$

$$= \frac{1}{2}\left\{L[e^{j\omega t}] + L[e^{-j\omega t}]\right\}$$

But the standard laplace transform pair

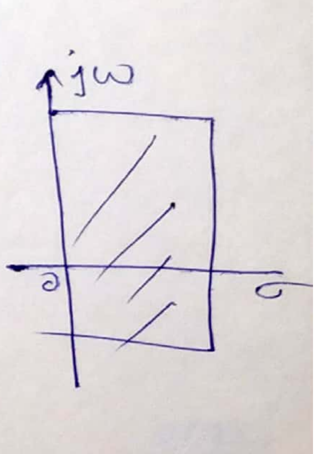
$$e^{\pm at} u(t) \xleftrightarrow{L.T.} \frac{1}{s \mp a}; \quad ROC: \sigma > \pm a$$

$$\therefore L\{e^{\pm j\omega t}\} \longrightarrow \frac{1}{s \mp j\omega}; \quad ROC\ \sigma \pm j\omega$$

$$F(s) = \frac{1}{2}\left[\frac{1}{s - j\omega} + \frac{1}{s + j\omega}\right]$$

$$= \frac{1}{2}\left[\frac{2s}{s^2 + \omega^2}\right]$$

$$= \frac{s}{s^2 + \omega^2}; \quad ROC: \sigma > 0$$



$$\boxed{A\cos\omega t \cdot u(t) \xleftrightarrow{L.T.} \frac{A.s}{s^2 + \omega^2}}$$

8. Laplace Transform of Damped Sine Wave :⇒

$$f(t) = e^{\mp at} \sin\omega t$$

$$\boxed{e^{\mp at} \sin\omega t \xleftrightarrow{L.T.} \frac{\omega}{(s \pm a)^2 + \omega^2}}$$

ROC: $\sigma > \mp a$

9. Laplace Transform of Damped cosine wave :⇒

$$f(t) = e^{\mp at} \cos\omega t$$

$$\boxed{e^{\mp at} \cos\omega t \xleftrightarrow{L.T.} \frac{(s \pm a)}{(s \pm a)^2 + \omega^2}}$$

ROC: $\sigma > \mp a$