Ampere’s Circuital Law and Magnetic Flux Density

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Ampere's Circuital Law

- It states that the line tangent of **H** around a closed path is the same as nett current enclosed by the path

$$\boxed{\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}}$$

Ampere’s Circuital Law

$$I_{enc} = \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$

$$I_{enc} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

Ampere’s Circuital Law

- Above equation derived is Maxwell’s third equation .
- It also show that magnetostatic field is not conservative as

$$\nabla \times \mathbf{H} = \mathbf{J} \neq 0;$$

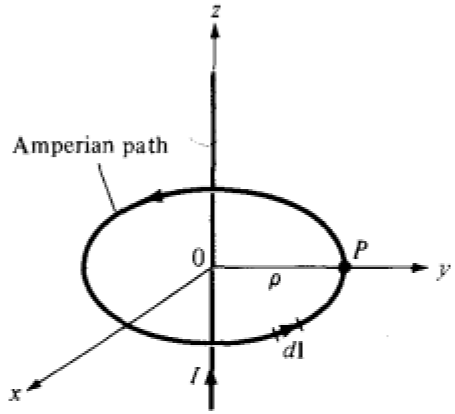
Application of Ampere's Circuital Law

- **<u>Infinite line current</u>**

Consider an infinitely long filamentary current I along the z axis. To determine **H** at the observation point P, we allow a closed path to pass through P k/a 'Amperian Path'. We choose a concenteric circle as an amperian path which show **H** is constant provided ρ is constant. Since the path encloses the whole current I,according to ampere's law

$$I = \int H_\phi \mathbf{a}_\phi \cdot \rho \, d\phi \, \mathbf{a}_\phi = H_\phi \int \rho \, d\phi = H_\phi \cdot 2\pi\rho$$

$$\boxed{\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi}$$



Application of Ampere's Circuital Law

- **Infinite sheet of current**

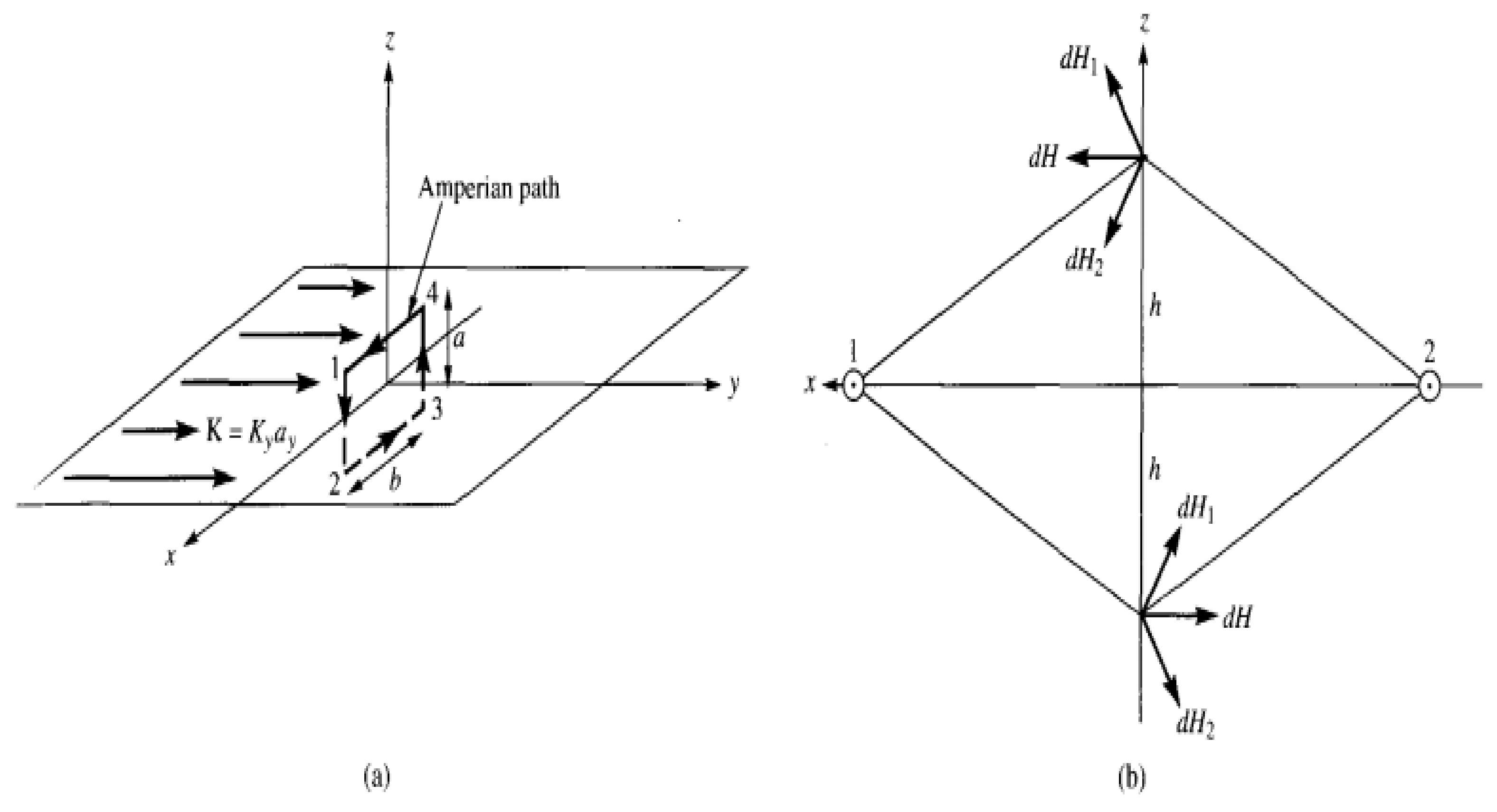
Consider an infinite current sheet in the zero plane.If the sheet has a uniform current density **K**=Ky**ay** A/m as shown in fig, applying ampere's law to the rectangular closed path1-2-3-4-1 gives

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} = K_y b$$

To evaluate the integral, we regard the infinite sheet as comprising of filaments,**dH** above or below the sheet due to pair of filamentay current can be found using

$$\boxed{\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi}$$

Application of Ampere's Circuital Law



Application of Ampere’s Circuital Law

- As evident from the figure b , the resultant **dH** has only x component.Also,**H** on one side of the sheet is the –ve of that on the other side.During the infinite extend of the sheet, the sheet can be regarded as consisting of such filamentary pairs so that characteristics of **H** for pair are the same for infinite current sheet, that is

$$\mathbf{H} = \begin{cases} H_o \mathbf{a}_x & z > 0 \\ -H_o \mathbf{a}_x & z < 0 \end{cases}$$

Application of Ampere's Circuital Law

$$\oint \mathbf{H} \cdot d\mathbf{l} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1\right) \mathbf{H} \cdot d\mathbf{l}$$
$$= 0(-a) + (-H_o)(-b) + 0(a) + H_o(b)$$
$$= 2H_o b$$

$$\mathbf{H} = \begin{cases} \frac{1}{2} K_y \mathbf{a}_x, & z > 0 \\ -\frac{1}{2} K_y \mathbf{a}_x, & z < 0 \end{cases}$$

Application of Ampere's Circuital Law

In general, for an infinite sheet of current density **K** A/m,

$$\boxed{\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{a}_n}$$

where $\mathbf{a}_n$ is a unit normal vector directed from the current sheet to the point of interest.

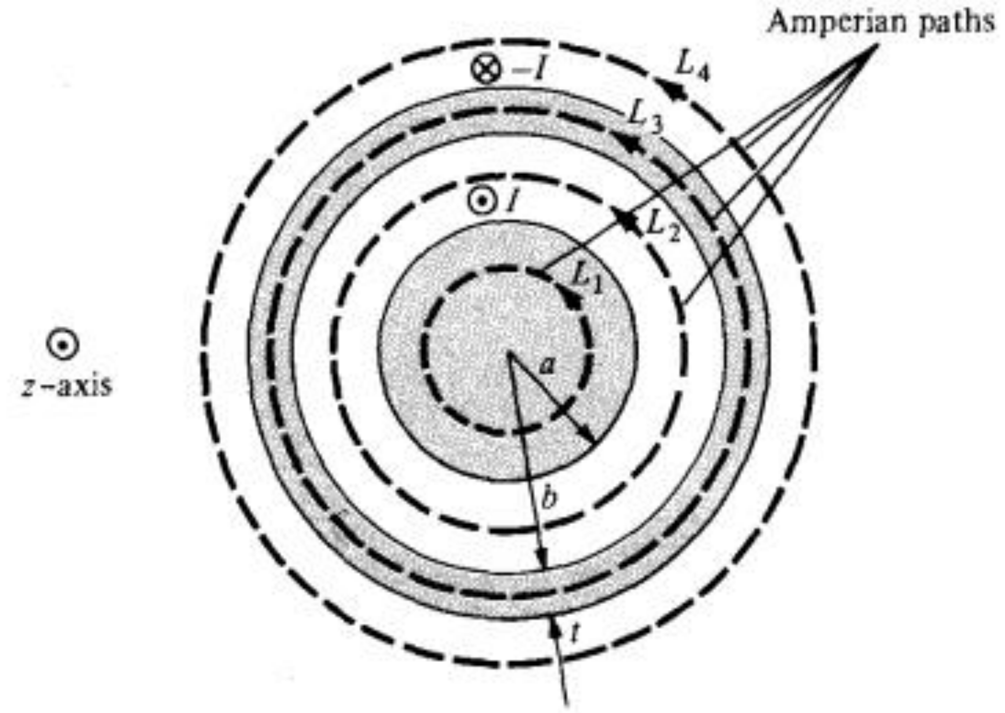
Application of Ampere's Circuital Law

- **<u>Infinitely long Coaxial transmission line</u>**

Consider an infinitely long transmission line consisting of two concenteric cylinder having there axes along the z axis as shown in Fig.

The inner conductor has radius a and carries current I,while the other conductor has inner radius b and thickness t and carries current –I.We want to determine **H** everywhere, assuming that current is uniformly distributed in both conductors

Application of Ampere’s Circuital Law



Application of Ampere's Circuital Law

For region $0 \le \rho \le a$, we apply Ampere's law to path L_1, giving

$$\oint_{L_1} \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{S}$$

Since the current is uniformly distributed over the cross section,

$$\mathbf{J} = \frac{I}{\pi a^2}\,\mathbf{a}_z, \qquad d\mathbf{S} = \rho\, d\phi\, d\rho\, \mathbf{a}_z$$

$$I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{S} = \frac{I}{\pi a^2} \iint \rho\, d\phi\, d\rho = \frac{I}{\pi a^2}\,\pi\rho^2 = \frac{I\rho^2}{a^2}$$

Application of Ampere's Circuital Law

$$H_\phi \int dl = H_\phi \, 2\pi\rho = \frac{I\rho^2}{a^2}$$

or

$$H_\phi = \frac{I\rho}{2\pi a^2}$$

For region $a \leq \rho \leq b$, we use path L_2 as the Amperian path,

$$\oint_{L_2} \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}} = I$$

$$H_\phi 2\pi\rho = I$$

or

$$H_\phi = \frac{I}{2\pi\rho}$$

Application of Ampere's Circuital Law

For region $b \leq \rho \leq b + t$, we use path L_3, getting

$$\oint \mathbf{H} \cdot d\mathbf{l} = H_\phi \cdot 2\pi\phi = I_{\text{enc}}$$

$$I_{\text{enc}} = I + \int \mathbf{J} \cdot d\mathbf{S}$$

$$\mathbf{J} = -\frac{I}{\pi[(b+t)^2 - b^2]}\mathbf{a}_z$$

Application of Ampere's Circuital Law

Thus

$$I_{enc} = I - \frac{I}{\pi[(b + t)^2 - t^2]} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{\rho} \rho \, d\rho \, d\phi$$

$$= I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

Substituting this , we have

$$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

Application of Ampere’s Circuital Law

For region $\rho \geq b + t$, we use path L_4, getting

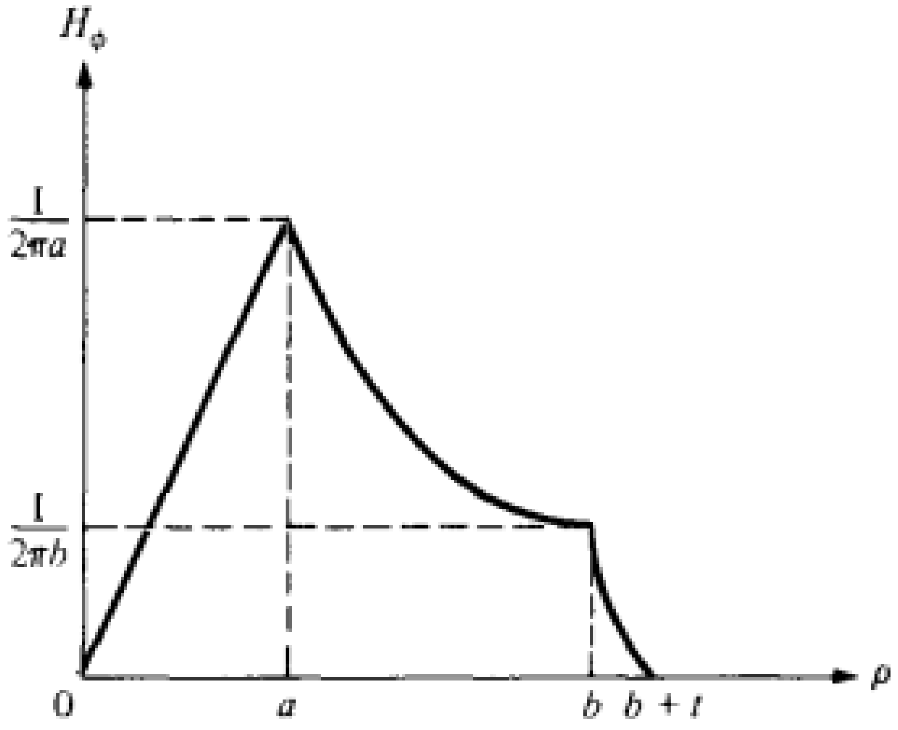
$$\oint_{L_4} \mathbf{H} \cdot d\mathbf{l} = I - I = 0$$

or

$$H_\phi = 0$$

$$\mathbf{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\,\mathbf{a}_\phi, & 0 \leq \rho \leq a \\[2ex] \dfrac{I}{2\pi\rho}\,\mathbf{a}_\phi, & a \leq \rho \leq b \\[2ex] \dfrac{I}{2\pi\rho}\left[1 - \dfrac{\rho^2 - b^2}{t^2 + 2bt}\right]\mathbf{a}_\phi, & b \leq \rho \leq b + t \\[2ex] 0, & \rho \geq b + t \end{cases}$$

Application of Ampere's Circuital Law

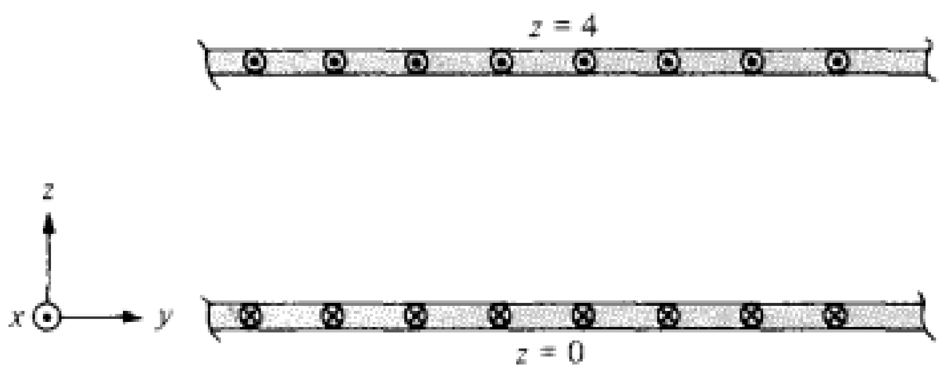


Numerical

Planes $z = 0$ and $z = 4$ carry current $\mathbf{K} = -10\mathbf{a}_x$ A/m and $\mathbf{K} = 10\mathbf{a}_x$ A/m, respectively. Determine $\mathbf{H}$ at

(a) $(1, 1, 1)$

(b) $(0, -3, 10)$



Solution

(a) At (1, 1, 1), which is between the plates ($0 < z = 1 < 4$),

$$\mathbf{H}_o = 1/2\, \mathbf{K} \times \mathbf{a}_n = 1/2\,(-10\mathbf{a}_x) \times \mathbf{a}_z = 5\mathbf{a}_y \text{ A/m}$$

$$\mathbf{H}_4 = 1/2\, \mathbf{K} \times \mathbf{a}_n = 1/2\,(10\mathbf{a}_x) \times (-\mathbf{a}_z) = 5\mathbf{a}_y \text{ A/m}$$

Hence,

$$\mathbf{H} = 10\mathbf{a}_y \text{ A/m}$$

(b) At (0, −3, 10), which is above the two sheets ($z = 10 > 4 > 0$),

$$\mathbf{H}_o = 1/2\,(-10\mathbf{a}_x) \times \mathbf{a}_z = 5\mathbf{a}_y \text{ A/m}$$

$$\mathbf{H}_4 = 1/2\,(10\mathbf{a}_x) \times \mathbf{a}_z = -5\mathbf{a}_y \text{ A/m}$$

Hence,

$$\mathbf{H} = 0 \text{ A/m}$$

Magnetic Flux density

The magnetic flux density $\mathbf{B}$ is similar to the electric flux density $\mathbf{D}$. As $\mathbf{D} = \varepsilon_o \mathbf{E}$ in free space, the magnetic flux density $\mathbf{B}$ is related to the magnetic field intensity $\mathbf{H}$ according to

$$\boxed{\mathbf{B} = \mu_o \mathbf{H}}$$

where μ_o is a constant known as the *permeability of free space*. The constant is in henrys/meter (H/m) and has the value of

$$\boxed{\mu_o = 4\pi \times 10^{-7} \text{ H/m}}$$

Magnetic Flux density

The magnetic flux through a surface S is given by

$$\boxed{\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}}$$

where the magnetic flux Ψ is in webers (Wb) and the magnetic flux density is in webers/square meter (Wb/m^2) or teslas.

Magnetic Flux density

- The **magnetic flux line** is the path to which B is tangential at every point in a magnetic field. It is the line along which the needle of a magnetic compass will orient itself if placed in the magnetic field.
- Unlike electric flux lines, magnetic flux lines always close upon themselves.
- This is due to the fact that *it is not possible to have isolated magnetic poles (or magnetic charges).*

An **isolated magnetic** charge does not exist.

Thus the total flux through a closed surface in a magnetic field must be zero; that is,

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{S} = 0}$$

- *This is **gauss law of magnetism**.*

Magnetic Flux density

By applying the divergence theorem to eq. (7.33), we obtain

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B} \, dv = 0$$

or

$$\boxed{\nabla \cdot \mathbf{B} = 0}$$