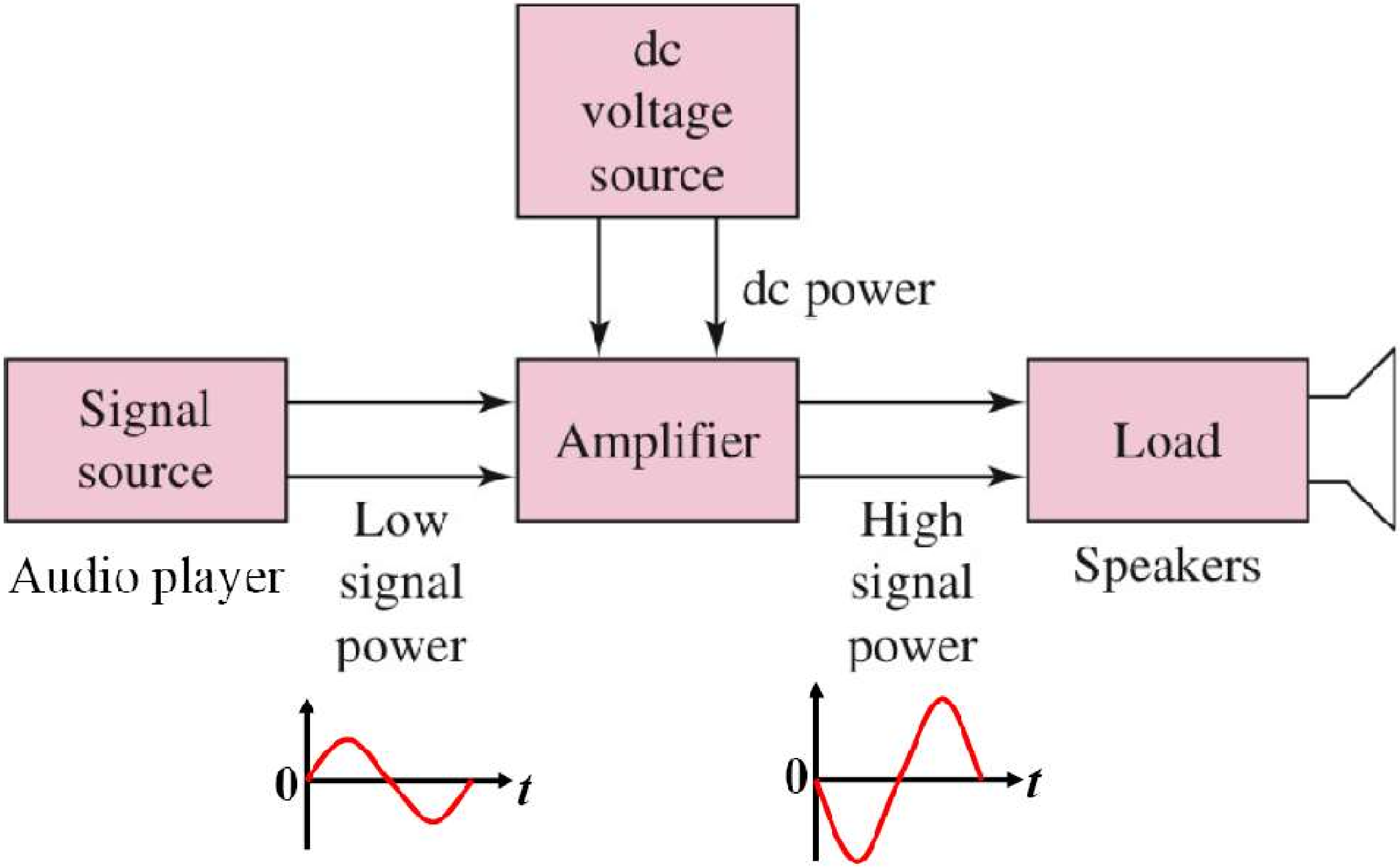
BJT Amplifier & Small-Signal Concept

Linear analog amplifier



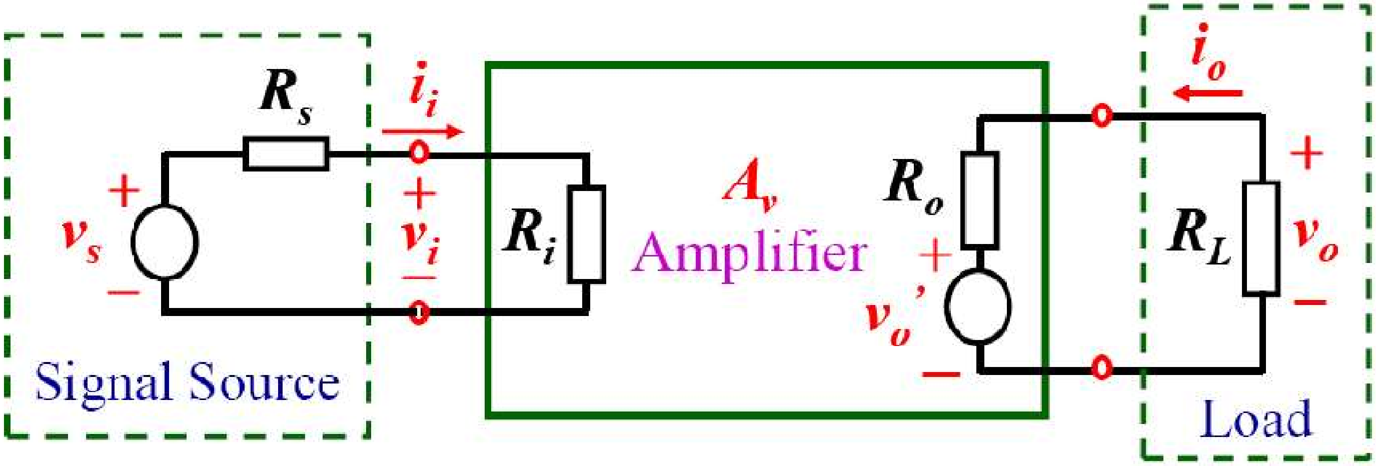
Notation

- Any time-varying signal can be written as a sum of sinusoidal signals of different frequencies & amplitudes (Fourier series) so that a sinusoidal analysis is possible.

Summary of Notation	
Variable	Meaning
i_B, v_{BE}	Total instantaneous values
I_B, V_{BE}	DC values
i_b, v_{be}	Instantaneous ac values
I_b, V_{be}	Phasor values

e.g., $i_B = I_{BQ} + i_b$; $i_C = I_{CQ} + i_c$; $v_{CE} = V_{CEQ} + v_{ce}$

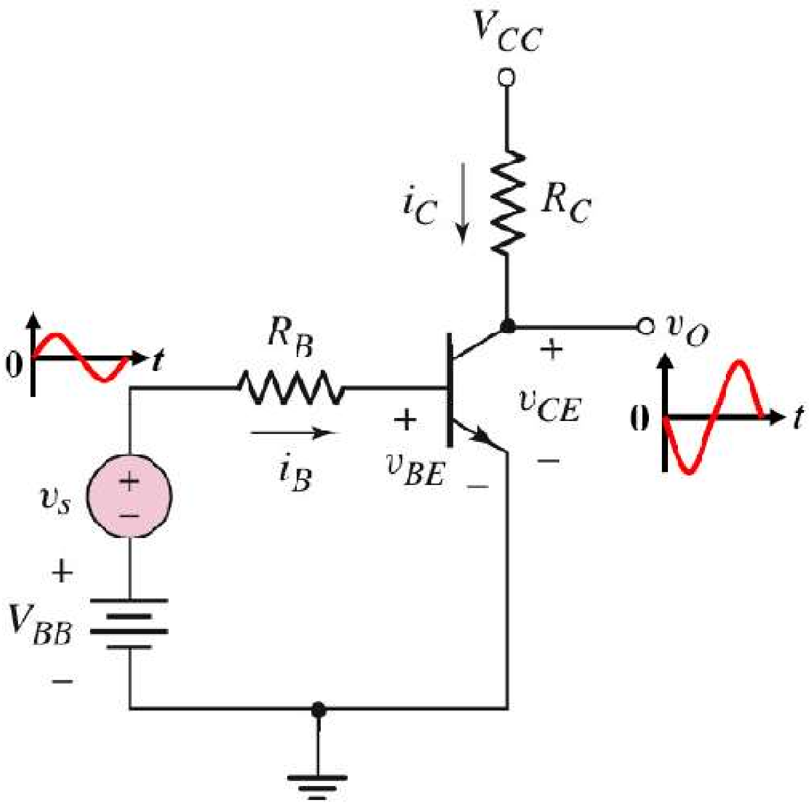
Basic characteristics of an amplifier



- **Amplifier Gain:** $A_v = \dfrac{v_o}{v_s}$
- **Input Resistance:** $R_i = \dfrac{v_i}{i_i}$
- **Output Resistance:** $R_o = \dfrac{v_o}{i_o}\Big|_{v_s=0 \text{ (a short circuit)},\ R_L=\infty \text{ (an open circuit)}}$

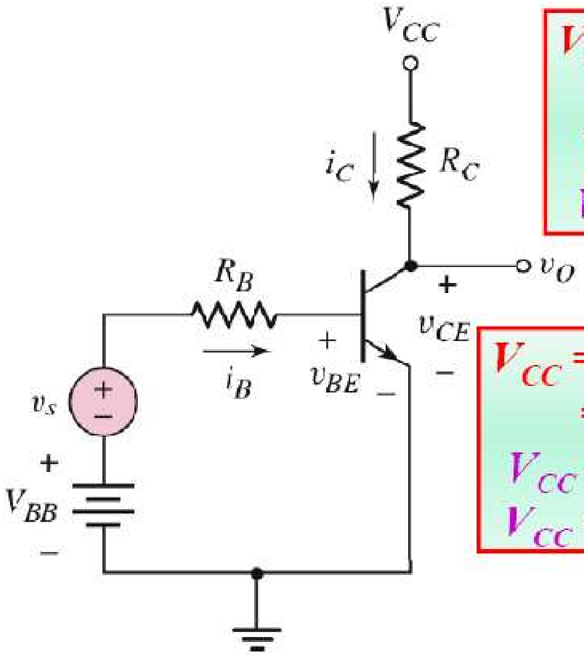
Basic BJT amplifier

- A BJT needs to be biased by a dc voltage source (V_{BB}) at a suitable Q-point to ensure that it is operating in forward-active mode – the precondition for configuring a BJT as an amplifier.
- A BJT amplifier has a time-varying signal source (v_s) in series with the dc voltage source (V_{BB}).
- A change in v_s causes a change in i_B which, in turn, causes a larger change in i_C ($i_C = \beta\ i_B$) & leads to an inverted & amplified signal (v_o) compared to the original v_s.



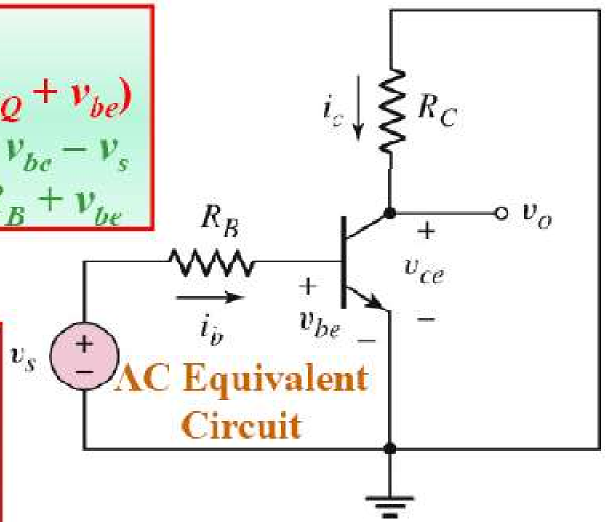
Common-Emitter Amplifier

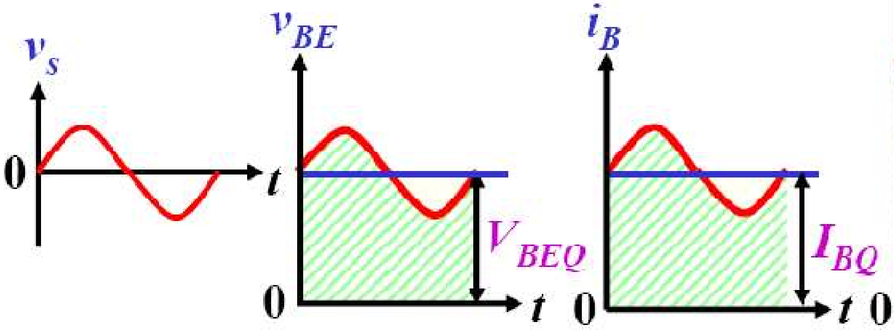
Graphical analysis of BJT amplifiers

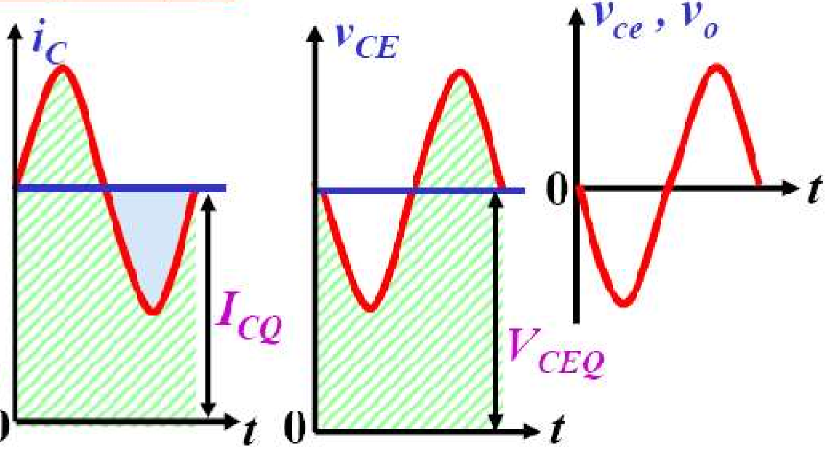


$$\begin{aligned} V_{BB} + v_s &= i_B R_B + v_{BE} \\ &= (I_{BQ} + i_b)R_B + (V_{BEQ} + v_{be}) \\ V_{BB} - I_{BQ}R_B - V_{BEQ} &= i_b R_B + v_{be} - v_s \\ V_{BB} &= I_{BQ}R_B + V_{BEQ}\ ;\ v_s = i_b R_B + v_{be} \end{aligned}$$

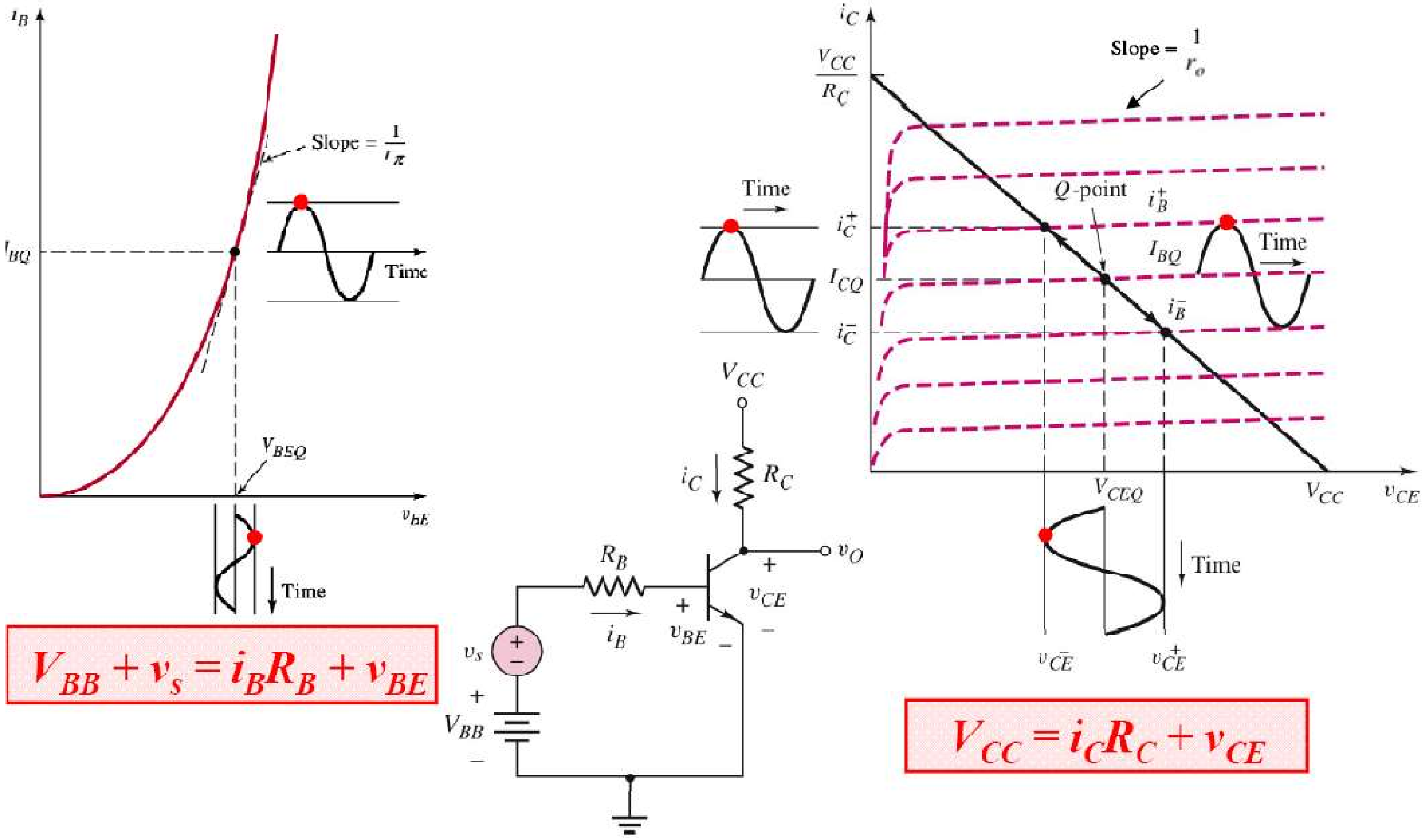
$$\begin{aligned} V_{CC} &= i_C R_C + v_{CE} \\ &= (I_{CQ} + i_c)R_C + (V_{CEQ} + v_{ce}) \\ V_{CC} - I_{CQ}R_C - V_{CEQ} &= i_c R_C + v_{ce} \\ V_{CC} &= I_{CQ}R_C + V_{CEQ}\ ;\ v_{ce} = -i_c R_C \end{aligned}$$







Graphical analysis of BJT amplifiers (Cont’d)



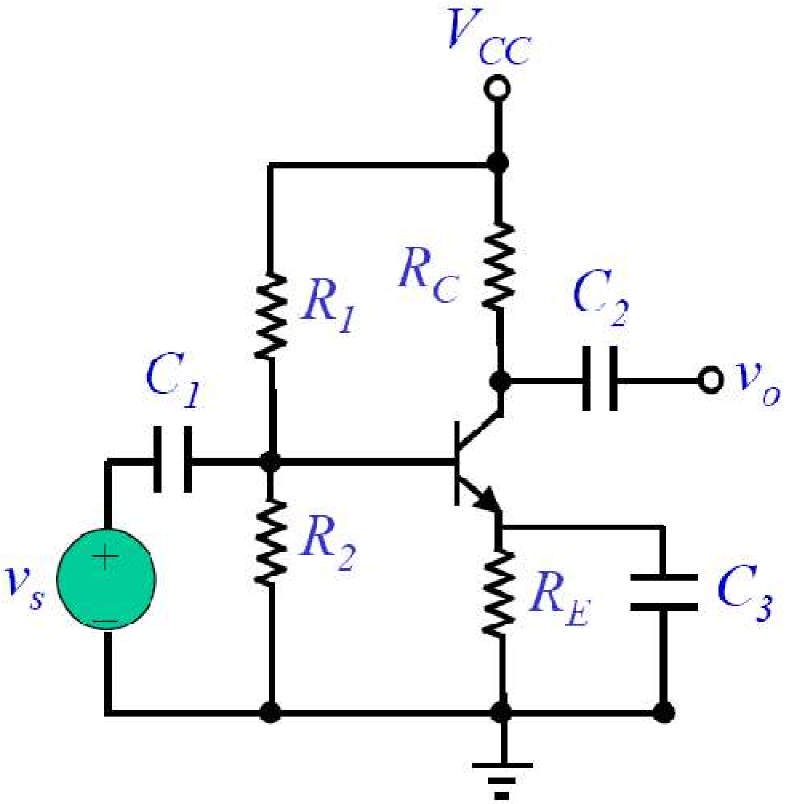
$$V_{BB} + v_s = i_B R_B + v_{BE}$$

$$V_{CC} = i_C R_C + v_{CE}$$

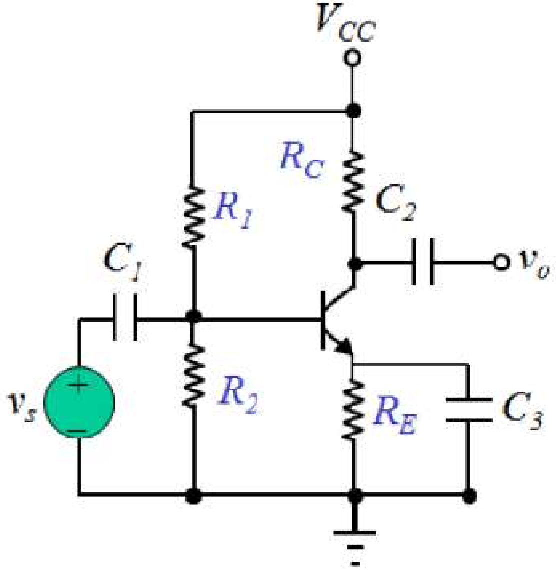
Analysis of BJT amplifiers

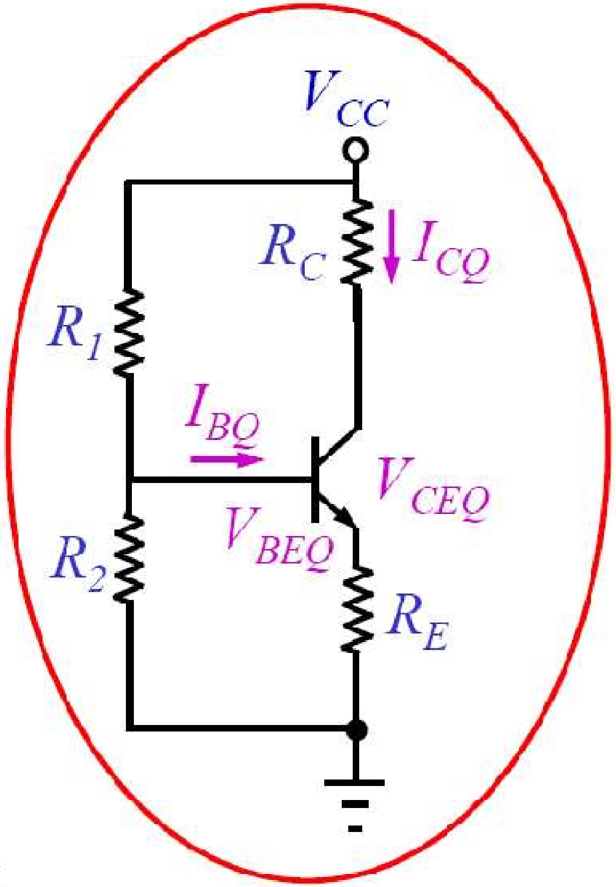
Function of each component:

- ☐ Capacitors: Acting as an open circuit for a dc operation but a short circuit for an ac operation (If $f = 10$ kHz & $C = 10\ \mu$F, then $|Z_C| = (2\pi f C)^{-1} = 8\ \Omega$, which is usually smaller than $R_{TH} = R_1 // R_2$)
- ☐ R_1, R_2, R_C & R_E: Setting dc biasing Q-point
- ☐ R_C: Converting i_c variation into v_{ce} (or v_o) variation (signal conversion)



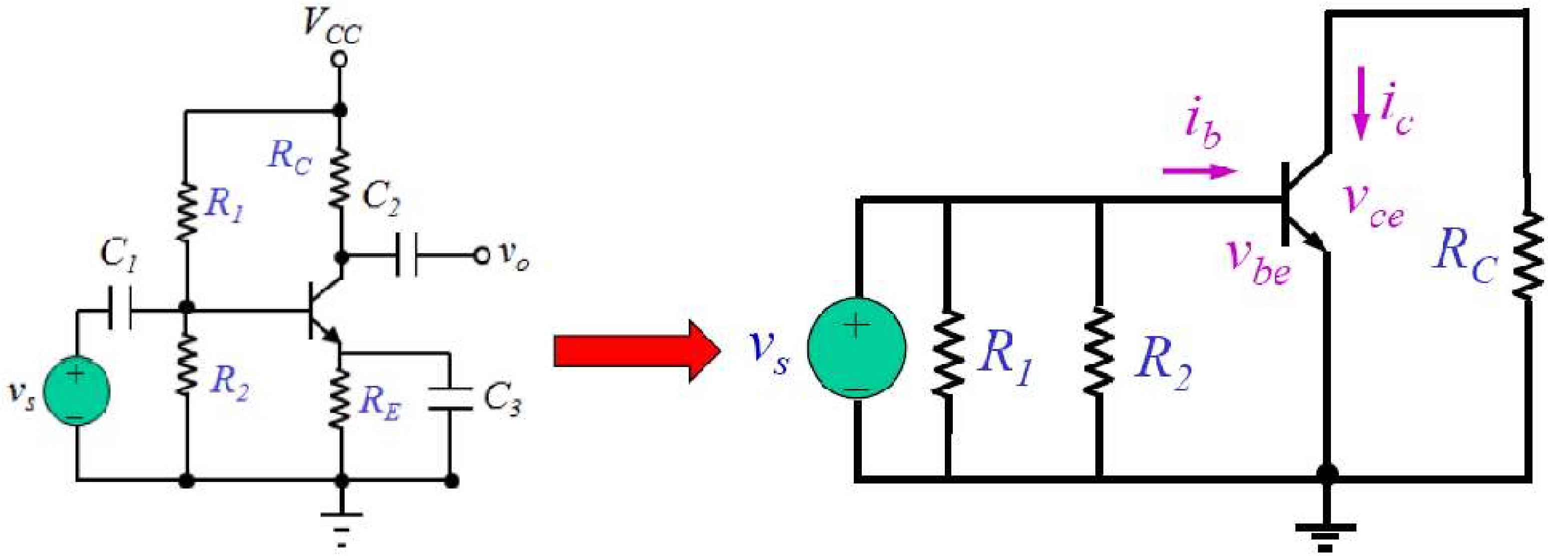
dc analysis and equivalent circuit





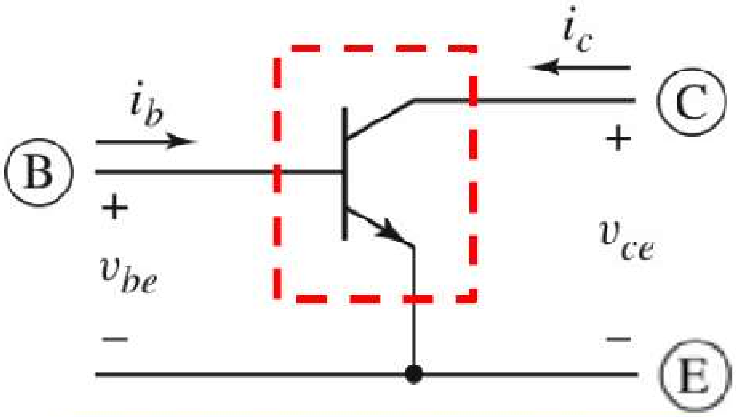
- To determine the dc biasing Q-point
  - Reduce all signal sources to zero
  - Open all capacitors
  - Draw & analyze the dc equivalent circuit

ac analysis and equivalent circuit

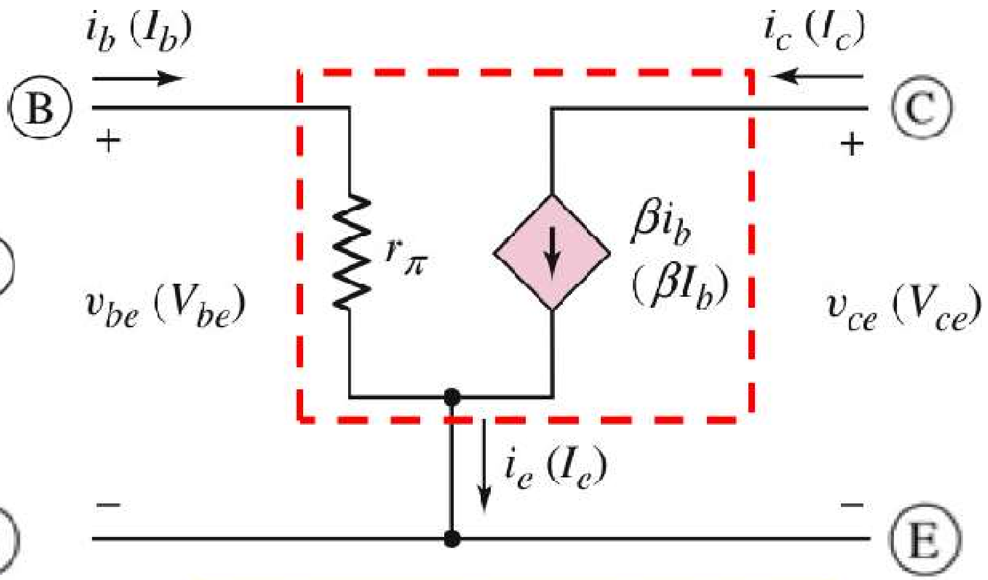


- To determine the ac characteristics (e.g., small-signal voltage gain, input & output impedances, frequency response, etc.)
  - Reduce all dc voltage sources to zero
  - Short all capacitors
  - Draw & analyze the ac equivalent circuit

Small-signal hybrid-π equivalent circuit



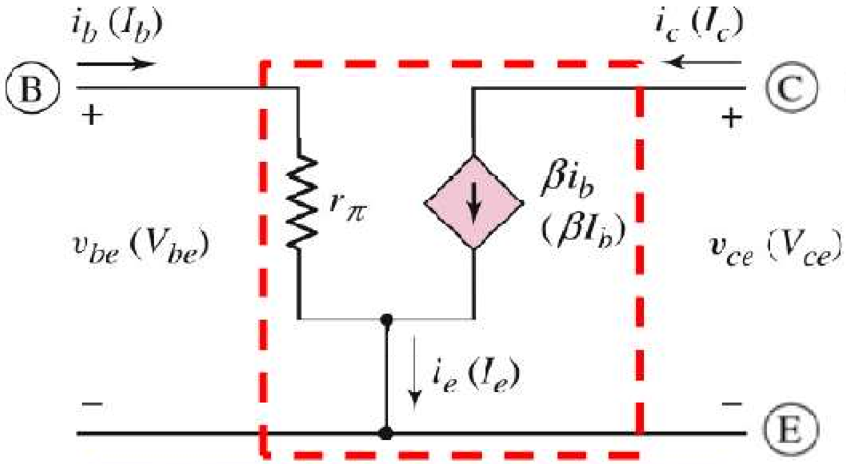
BJT as a small-signal, two-port network



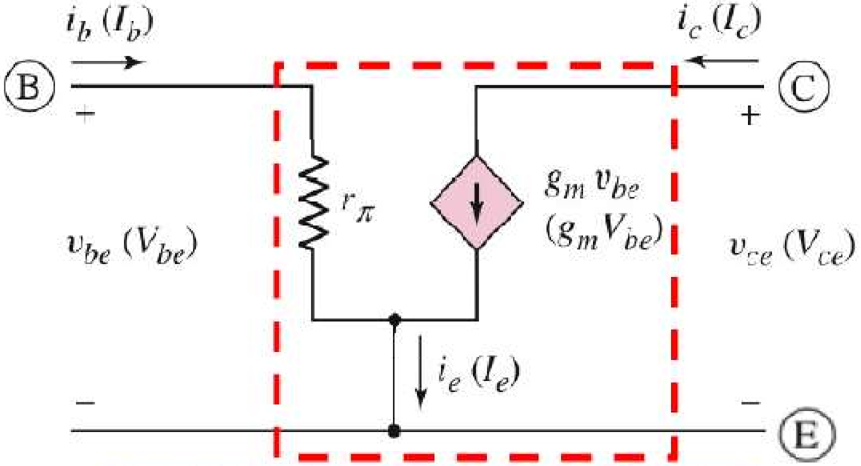
Small-signal hybrid-π equivalent circuit

- $\beta = i_c / i_b = I_c / I_b$ = common-emitter current gain
- $r_\pi = v_{be} / i_b = V_{be} / I_b = V_T / I_{BQ} = \beta V_T / I_{CQ}$ = small-signal resistance, where $V_T = kT / e$ = thermal voltage

Small-signal hybrid-π equivalent circuit (Cont'd)



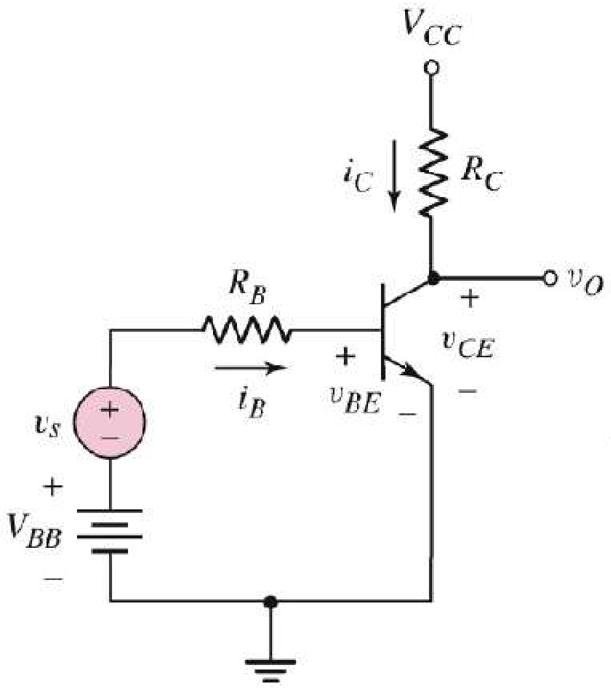
With current gain parameter

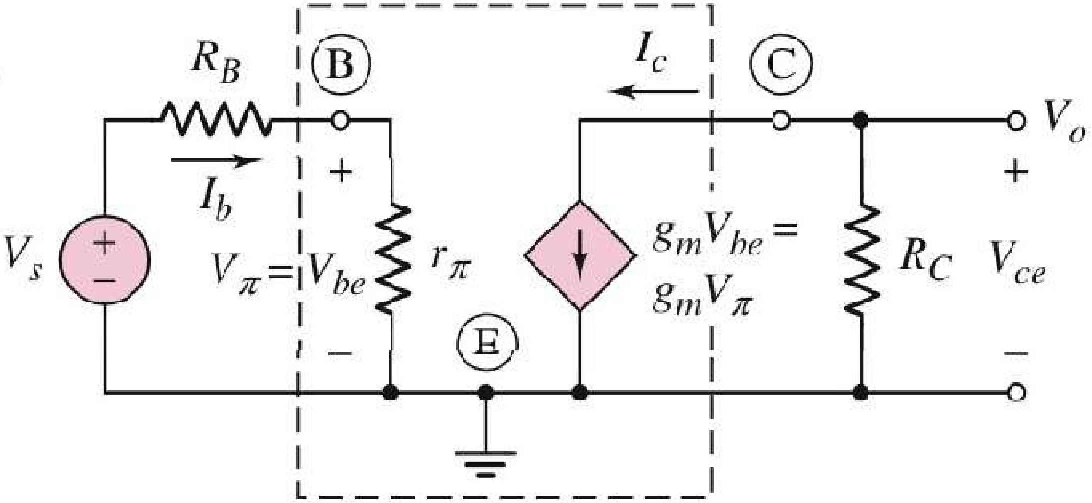


With transconductance parameter

- $g_m = \beta / r_\pi = I_{CQ} / V_T$ = transconductance
- i_c is assumed to be independent of v_{ce}, which is not the case in practice & the assumption will be released later to include the "Early Effect".

Small-signal voltage gain





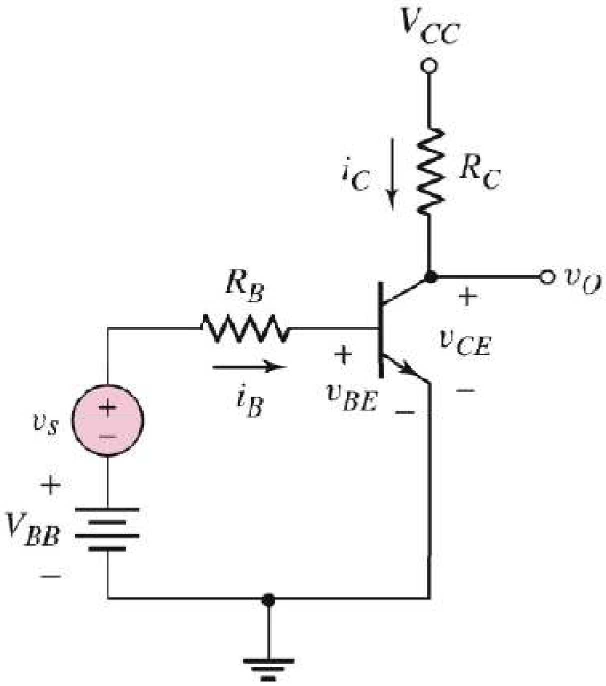
$$V_\pi = V_{be} = V_s\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

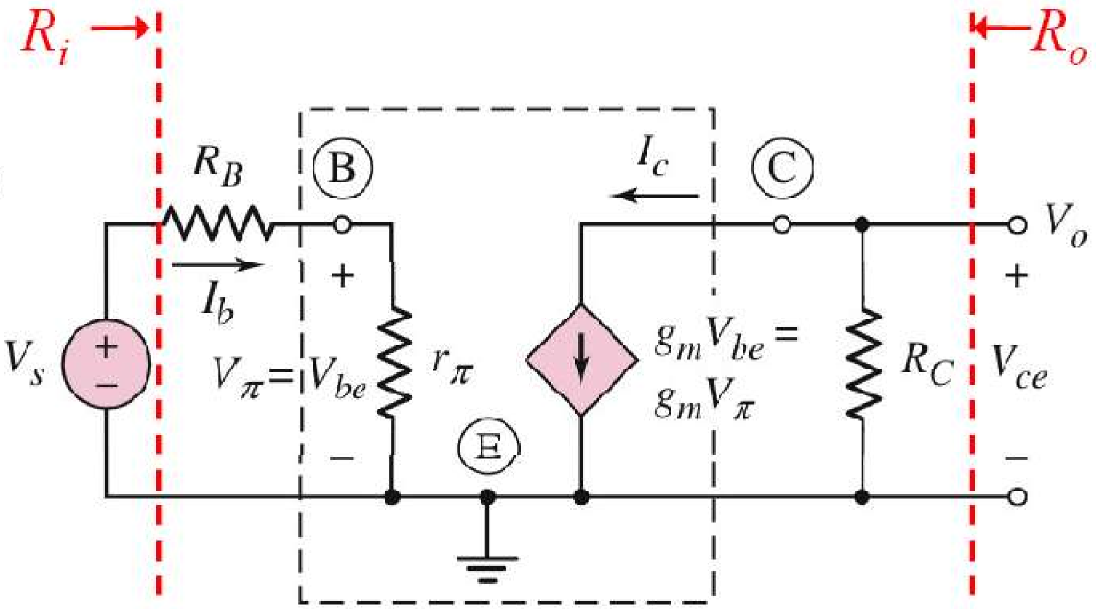
$$V_o = V_{ce} = -(g_m V_\pi)R_C = -(g_m V_{be})R_C$$

Small-signal voltage gain:

$$A_v = \frac{V_o}{V_s} = -(g_m R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

Input and output resistances





Input resistance: $R_i = R_B + r_\pi$

Output resistance: $R_o = R_C$

[Setting $V_s = 0$ (short), then $V_\pi = 0$ & $g_m V_\pi = 0$ (open)]

Example: dc circuit analysis

- Calculate the small-signal voltage gain, input resistance & output resistance of the BJT amplifier circuit at 300 K. Assume that the BJT & circuit parameters are: $\beta = 100$, $V_{CC} = 12$ V, $V_{BE} = 0.7$ V, $R_C = 6$ kΩ, $R_B = 50$ kΩ & $V_{BB} = 1.2$ V.

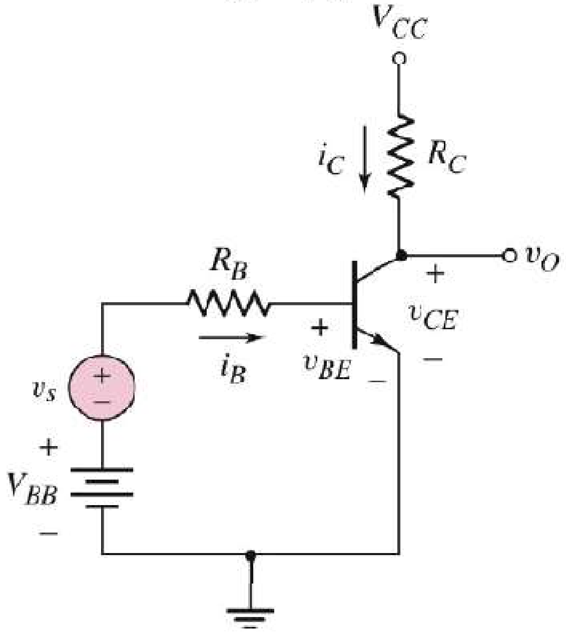
Step 1: Perform dc analysis to determine the dc biasing Q-point:

$$I_{BQ} = \frac{V_{BB} - V_{BE}(\text{on})}{R_B} = \frac{1.2 - 0.7}{50\text{k}} = 10\,\mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = (100)(10\mu) = 1\text{mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C = 12 - (1\text{m})(6\text{k}) = 6\,\text{V}$$

Since $I_{BQ} > 0$ A, $V_{BB} > V_{BE}(\text{on})$ & $V_{CEQ} > V_{BE}(\text{on})$, the BJT amplifier is biased in forward-active mode about the Q-point ($V_{CEQ} =$ 6 V, $I_{CQ} = 1$ mA, $I_{BQ} = 10\ \mu$A).



Example: dc circuit analysis (Cont’d)

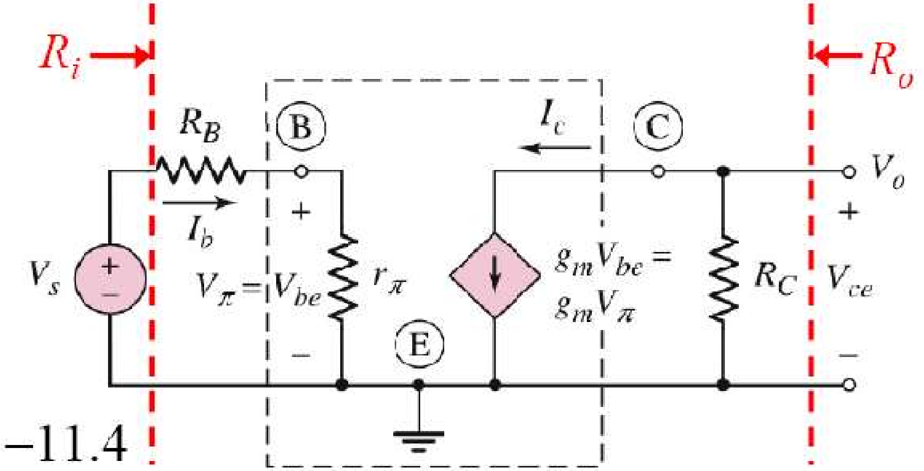
Step 2: Perform ac analysis using small-signal hybrid-π equiv. cct:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{\beta (kT/e)}{I_{CQ}} = \frac{(100)\left[(86\times10^{-6}e)(300)/e\right]}{1\mathrm{m}} = 2.6\,\mathrm{k}\Omega$$

$$g_m = \frac{\beta}{r_\pi} = \frac{100}{2.6\mathrm{k}} = 38.5\,\mathrm{mA/V}$$

$$A_v = \frac{V_o}{V_s} = -(g_m R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

$$= -(38.5\mathrm{m})(6\mathrm{k})\left(\frac{2.6\mathrm{k}}{2.6\mathrm{k}+50\mathrm{k}}\right) = -11.4$$



The BJT amplifier is capable of amplifying the input signal amplitude by 11.4 times. The –ve sign indicates a phase reversal of 180º.

$R_i = R_B + r_\pi = 50\mathrm{k} + 2.6\mathrm{k} = 52.6\,\mathrm{k}\Omega$

$R_o = R_C = 6\,\mathrm{k}\Omega$

Example: dc circuit analysis (Cont'd)

Further Discussion:

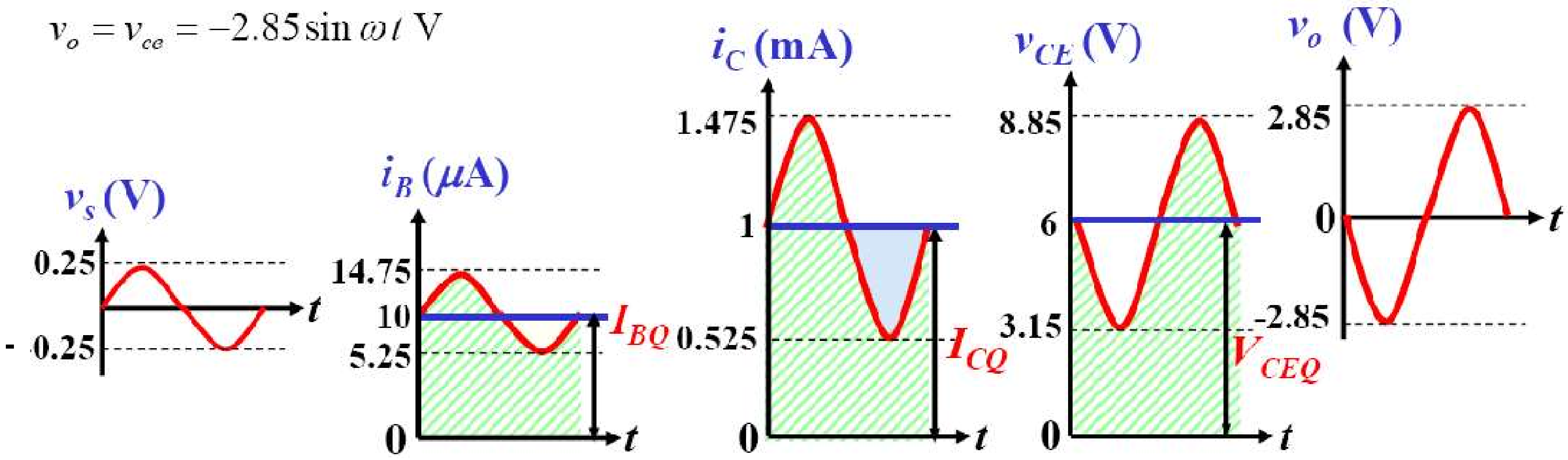
Let $v_s = 0.25 \sin \omega t$ V

$$i_b = \frac{v_s}{R_B + r_\pi} = \frac{0.25 \sin \omega t}{50\text{k} + 2.6\text{k}} = 4.75 \sin \omega t \ \mu\text{A} \quad \Rightarrow \quad i_B = I_{BQ} + i_b = 10 + 4.75 \sin \omega t \ \mu\text{A}$$

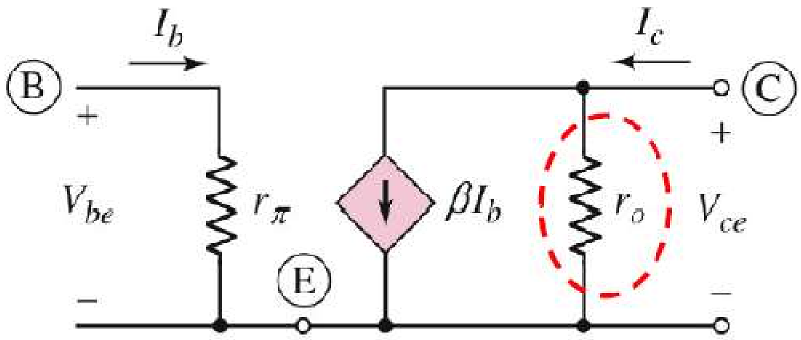
$$i_c = \beta i_b = (100)(4.75\mu \sin \omega t) = 0.475 \sin \omega t \text{ mA} \quad \Rightarrow \quad i_C = I_{CQ} + i_c = 1 + 0.475 \sin \omega t \text{ mA}$$

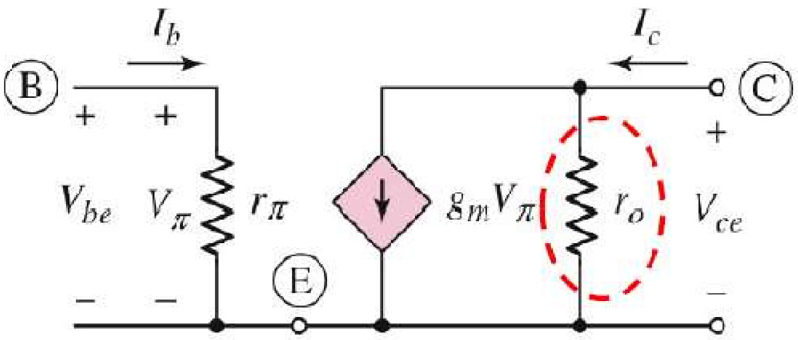
$$v_{ce} = -i_c R_C = -(0.475\text{m})(6\text{k}) \sin \omega t = -2.85 \sin \omega t \text{ V} \quad \Rightarrow \quad v_{CE} = V_{CEQ} + v_{ce} = 6 - 2.85 \sin \omega t \text{ V}$$

$$v_o = v_{ce} = -2.85 \sin \omega t \text{ V}$$



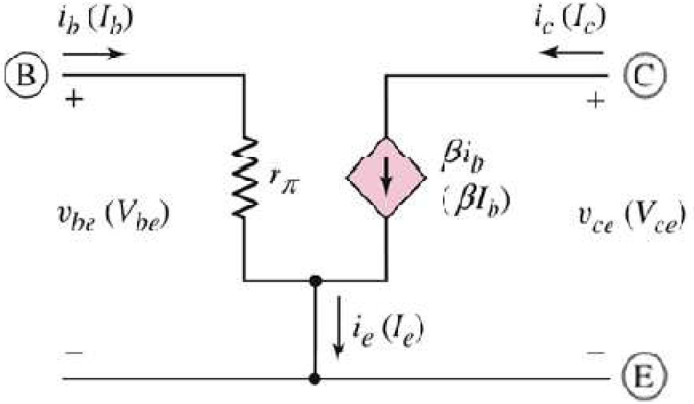
Small-signal hybrid-π equivalent circuit with Early effect



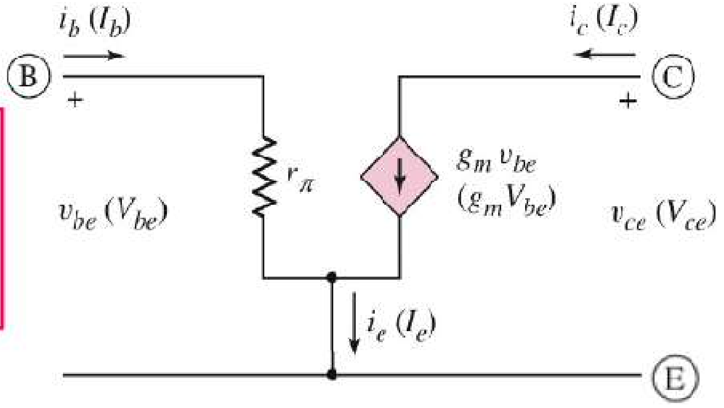


$$r_o = \left.\frac{\partial v_{CE}}{\partial i_C}\right|_{Q-\text{point}} \cong \frac{V_A}{I_{CQ}}$$

- ❑ **r_o = small-signal transistor output resistance**
- ❑ **V_A = early voltage ($50 < V_A < 300$)**

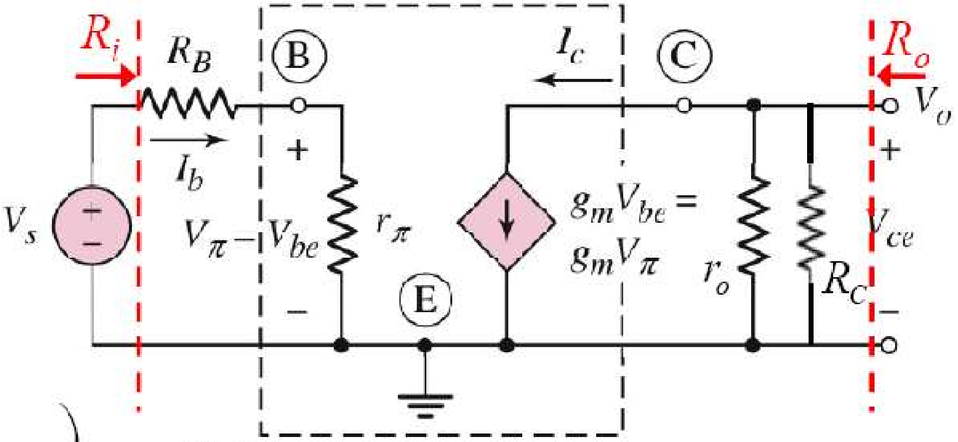


No Early Effect



Example: determine BJT amplifier parameters

- Determine the small-signal voltage gain, input resistance, and output resistance of the BJT amplifier circuit in previous example with the early effect. Assume that the early voltage is 50V.



$$r_o = \frac{V_A}{I_{CQ}} = \frac{50}{1\text{m}} = 50\,\text{k}\Omega$$

$$A_v = \frac{V_o}{V_s} = -g_m(r_o // R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

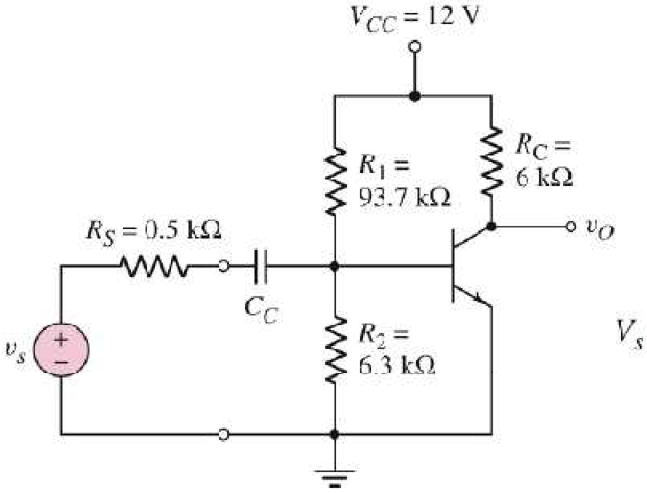
$$= -(38.5\text{m})(50\text{k}//6\text{k})\left(\frac{2.6\text{k}}{2.6\text{k} + 50\text{k}}\right) = -10.2$$

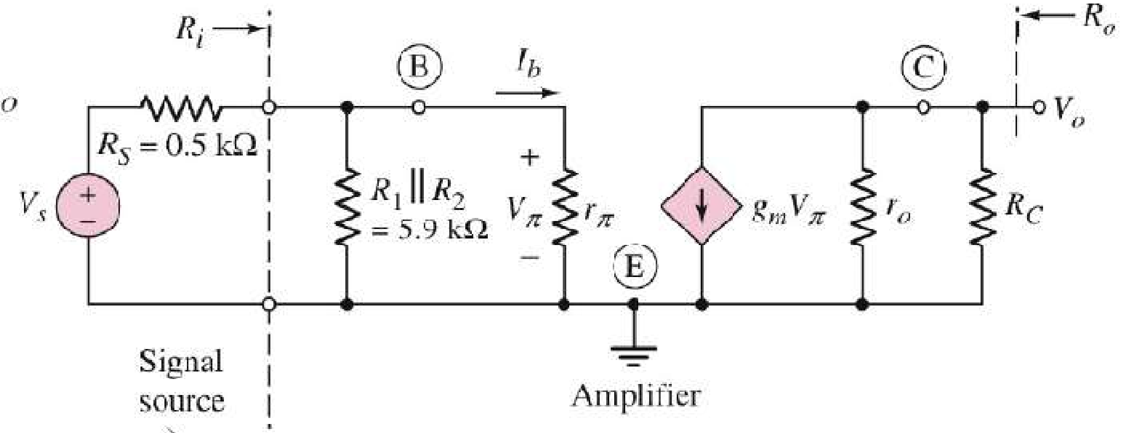
$$R_i = R_B + r_\pi = 50\,\text{k} + 2.6\,\text{k} = 52.6\,\text{k}\Omega$$

$$R_o = r_o // R_C = 50\,\text{k} // 6\,\text{k} = 5.4\,\text{k}\Omega$$

r_o reduces both $|A_v|$ from 11.4 to 10.2 & R_o from 6 to 5.4 kΩ.

Common-emitter amplifiers (with voltage-divider biasing & coupling capacitor)



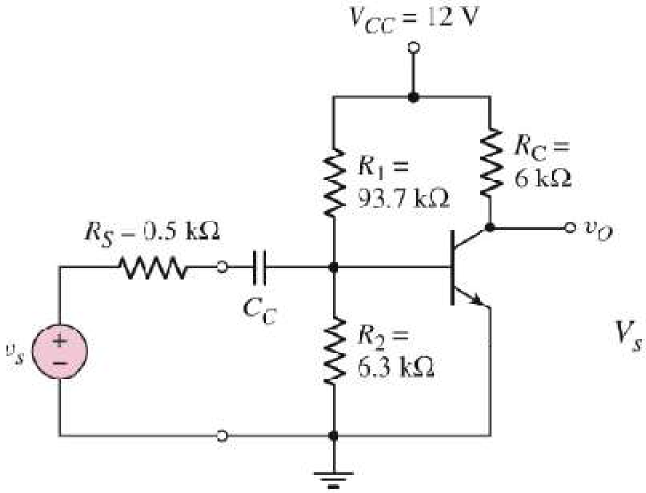


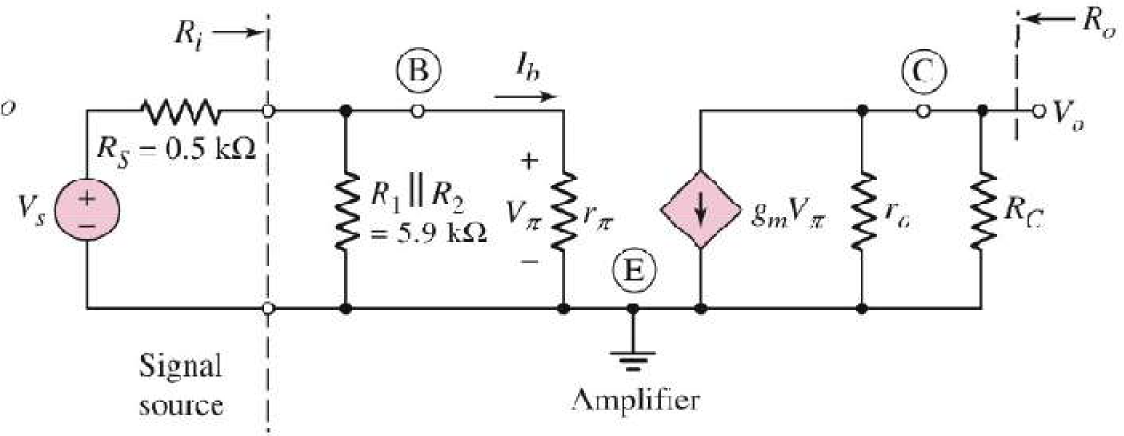
$$V_\pi = V_{be} = V_s \left(\frac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S} \right)$$

$$V_o = -g_m V_\pi (r_o // R_C) = -g_m V_s \left(\frac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S} \right) (r_o // R_C)$$

Small-signal voltage gain: $A_V = \dfrac{V_o}{V_s} = -g_m \left(\dfrac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S} \right) (r_o // R_C)$

Common-emitter amplifiers (with voltage-divider biasing & coupling capacitor)- Cont'd



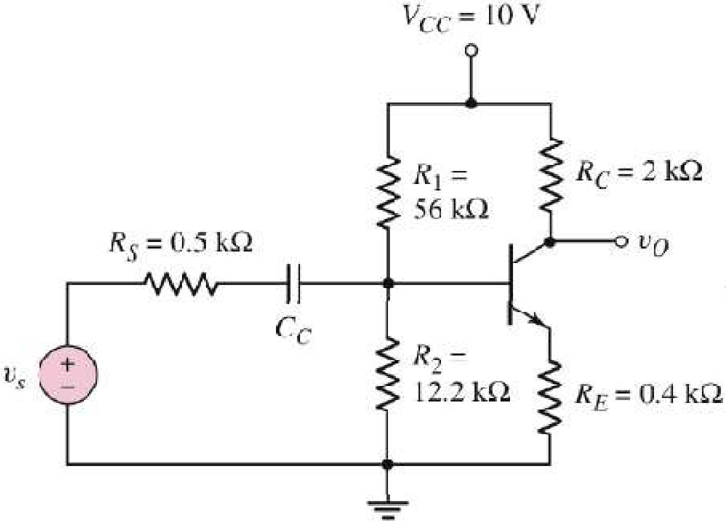


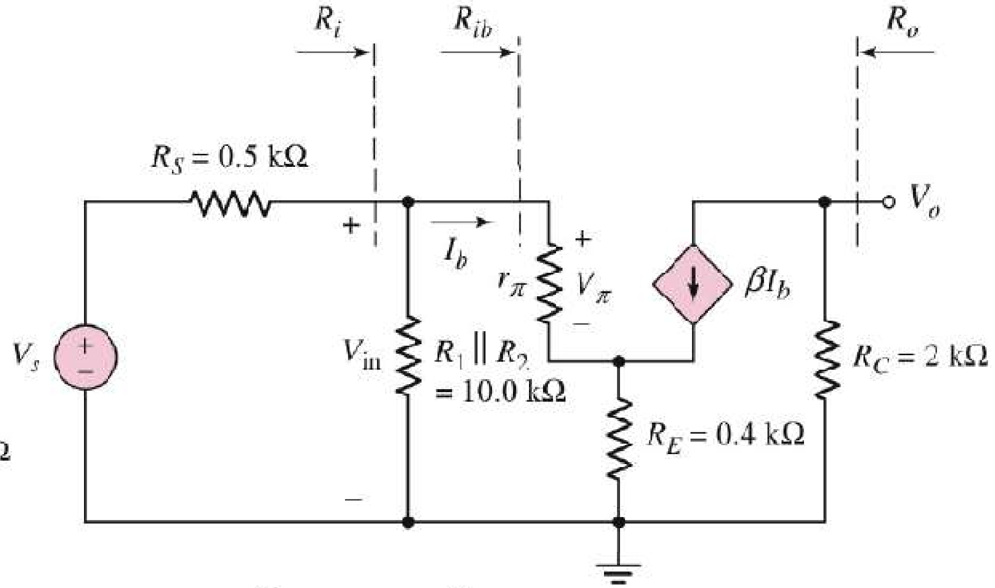
Input resistance: $R_i = R_1 // R_2 // r_\pi$

Output resistance: $R_o = r_o // R_C$

[Setting $V_s = 0$ (short), then $V_\pi = 0$ & $g_m V_\pi = 0$ (open)]

Common-emitter amplifiers (with voltage-divider biasing & coupling capacitor & emitter resistor)





$$V_o = -(\beta I_b) R_C$$

$$V_{in} = I_b r_\pi + (I_b + \beta I_b) R_E$$

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta) R_E$$

$$R_i = R_1 // R_2 // R_{ib}$$

$$V_{in} = V_s \left(\frac{R_i}{R_S + R_i} \right)$$

$$A_v = \frac{V_o}{V_s} = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} \left(\frac{R_i}{R_i + R_S} \right)$$

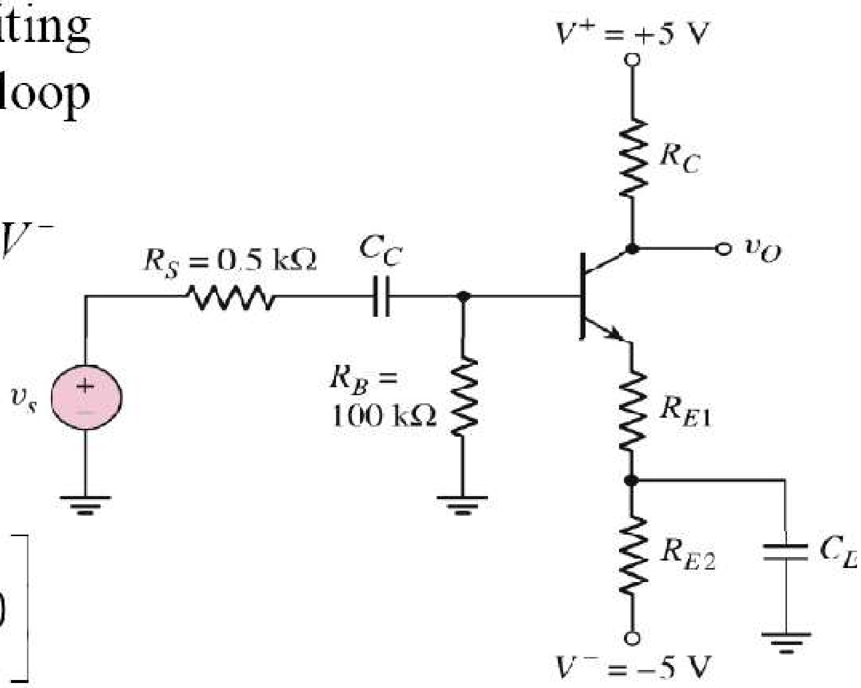
DC & AC load lines

- DC load line is used to find Q-point
- AC load line is used to determine graphically the operation of a BJT amplifier
- DC and AC load lines are essentially different since capacitors appear as an open circuit for a de operation but a short circuit for an ac operation

DC load line

- The DC load line is found by writing a dc KVL equation for the C-E loop as follows:

$$V^+ = I_C R_C + V_{CE} + I_E (R_{E1} + R_{E2}) + V^-$$



- Noting that $I_E - [\,(1+\beta)/\beta\,]\, I_C$,

$$V_{CE} = (V^+ - V^-) - I_C\left[R_C + \left(\frac{1+\beta}{\beta}\right)(R_{E1} + R_{E2})\right]$$

- If $\beta >> 1$, the $(1+\beta)/\beta \approx 1$. The slope of the dc load line is $\dfrac{-1}{R_C + R_{E1} + R_{E2}}$

AC load line

- The ac load line is found by writing an ac KVL equation for the C-E loop as follows:

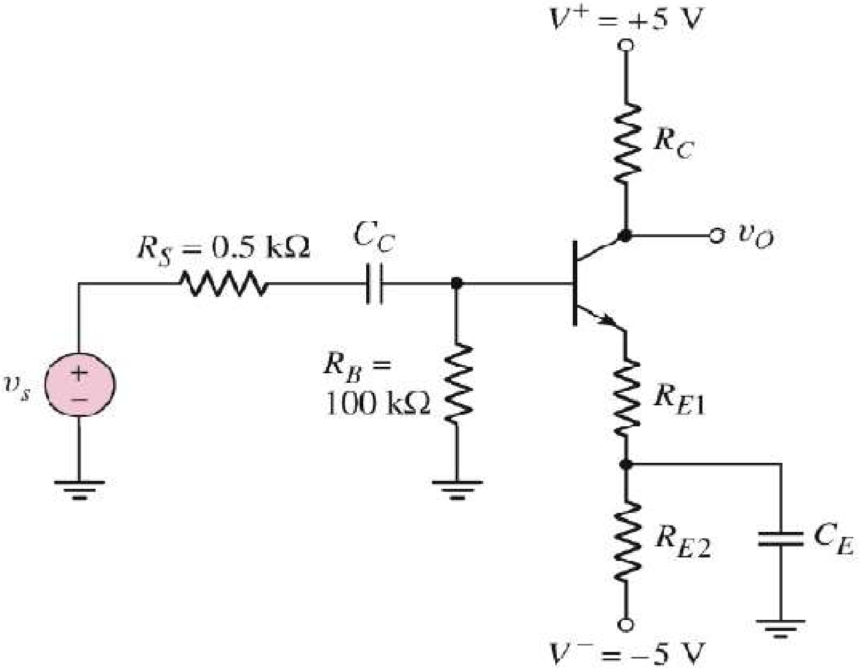
$$i_c R_C + v_{ce} + i_e R_{E1} = 0$$

- Assuming $(i_c \cong i_e)$

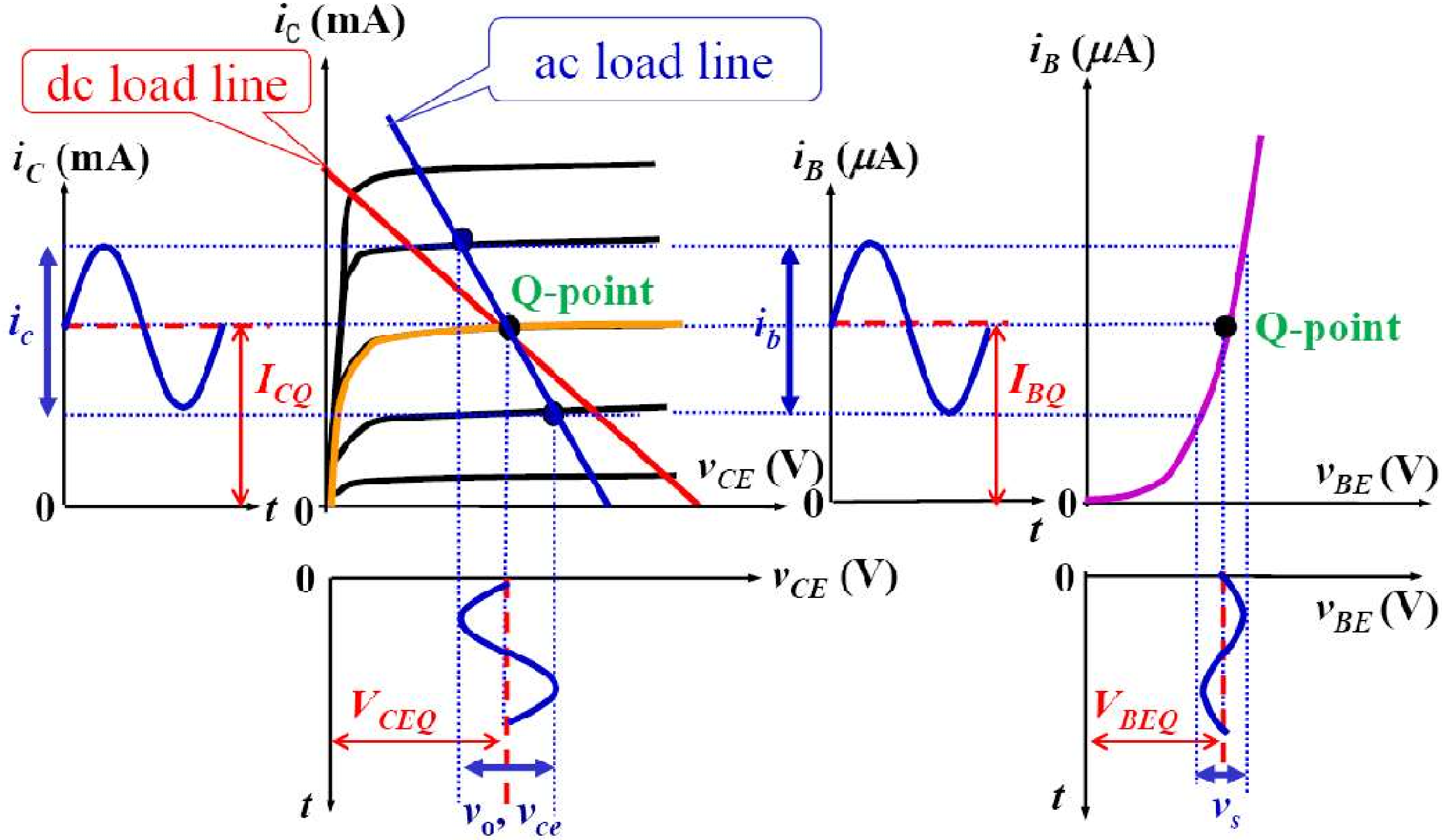
$$v_{ce} = -i_c (R_C + R_{E1})$$

- The slope of the ac load line is

$$\frac{-1}{R_C + R_{E1}}$$

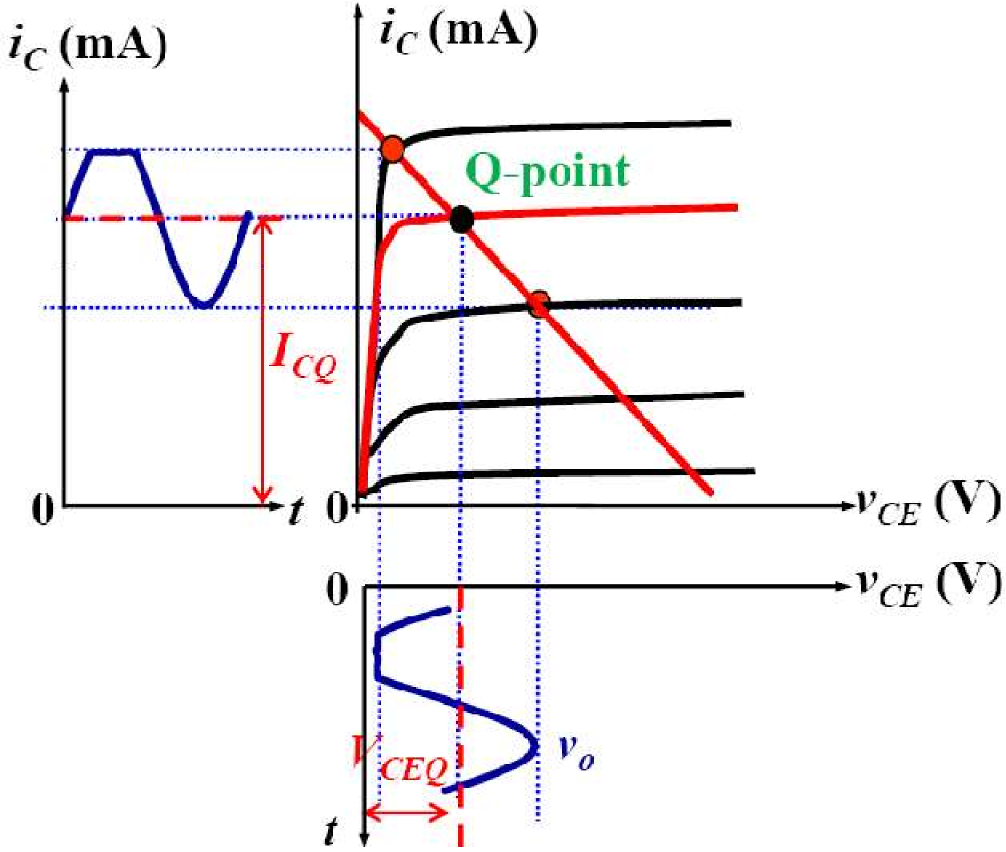


Maximum output symmetrical swing

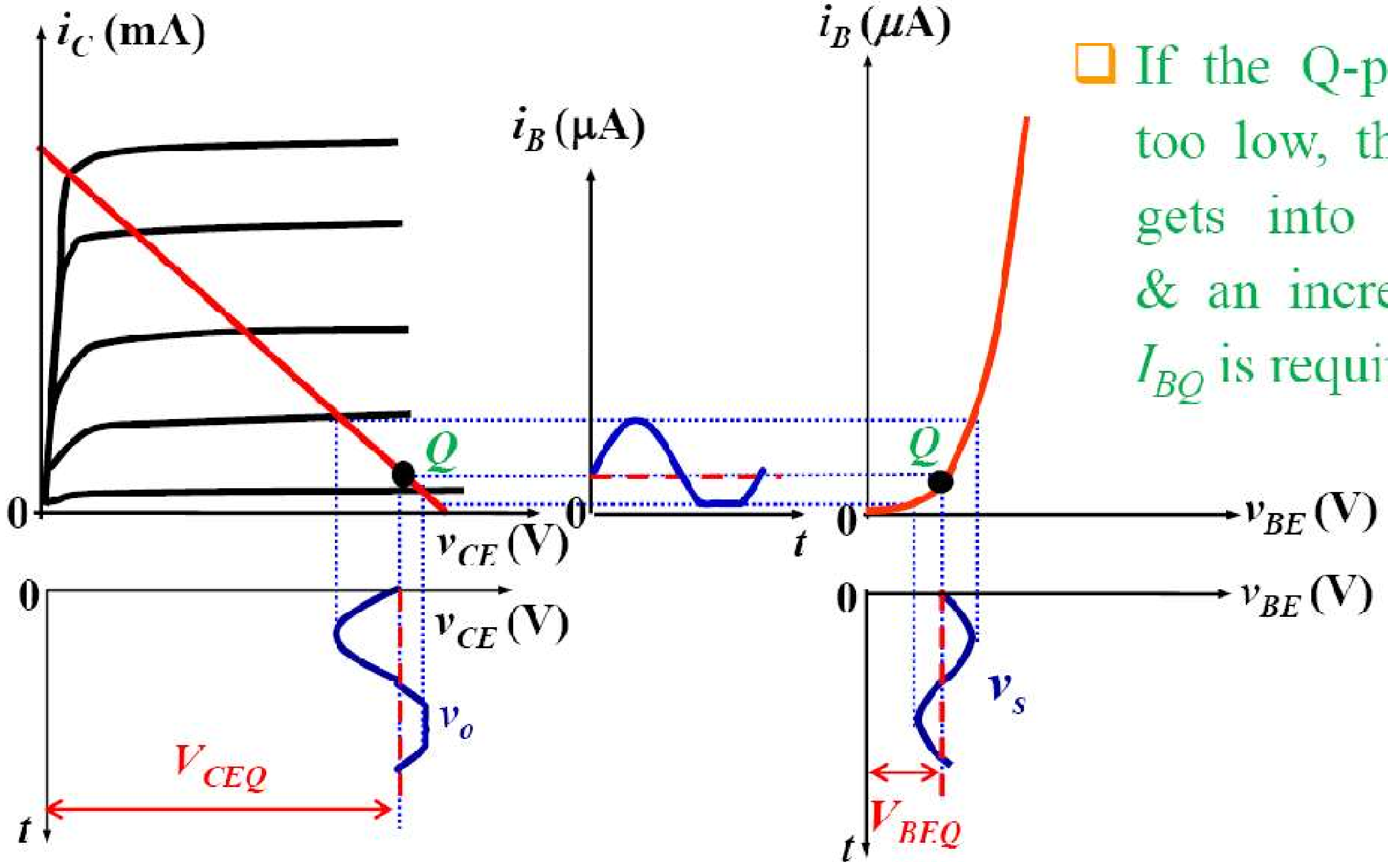


Saturation

- If the Q-point is not set properly, the BJT may get into saturation or cutoff, resulting in nonlinear distortion.
- If the Q-point is set too high, the BJT gets into saturation, & a reduction in I_{BQ} is required.
- Even with a proper Q-point setting, if the signal amplitude is too large, distortion will also result, & a reduction of signal amplitude is required.



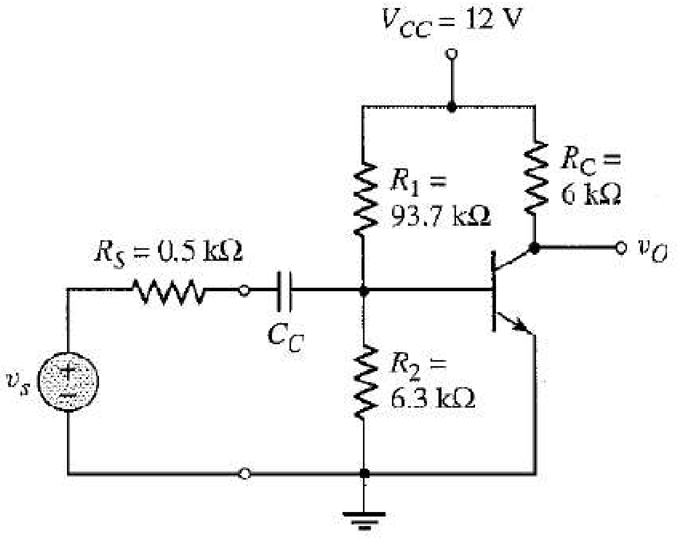
Cutoff



- ☐ If the Q-point is too low, the BJT gets into cutoff, & an increase in I_{BQ} is required.

Example: Calculate BJT circuit parameters

- Determine the small-signal voltage gain, input resistance, and output resistance of the circuit shown in attached Figure. Assume the transistor parameters are: β=100, $V_{BE(on)}$ =0.7V, and V_A=100V



AC Solution: The small-signal hybrid-π parameters for the equivalent circuit are

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{(0.95)} = 2.74\,\text{k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.95}{0.026} = 36.5\,\text{mA/V}$$

and

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.95} = 105\,\text{k}\Omega$$

Assuming that C_C acts as a short circuit, Figure 6.29 shows the small-signal equivalent circuit. The small-signal output voltage is

$$V_o = -(g_m V_\pi)(r_o | R_C)$$

The dependent current $g_m V_\pi$ flows through the parallel combination of r_o and R_C, but in a direction that produces a negative output voltage. We can relate the control voltage V_π to the input voltage V_s by a voltage divider. We have

$$V_\pi = \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} \right) \cdot V_s$$

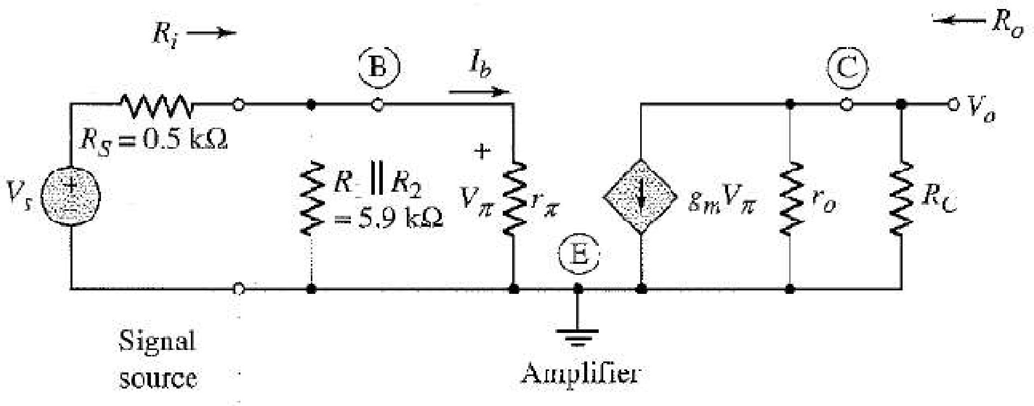


Figure 29 The small-signal equivalent circuit, assuming the coupling capacitor is a short circuit

We can then write the small-signal voltage gain as

$$A_v = \frac{V_o}{V_s} = -g_m \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} \right) (r_o \| R_C)$$

or

$$A_v = -(36.5) \left(\frac{5.9 \| 2.74}{5.9 \| 2.74 + 0.5} \right) (105 \| 6) = -163$$

We can also calculate R_i, which is the resistance to the amplifier. From Figure 29, we see that

$$R_i = R_1 \| R_2 \| r_\pi = 5.9 \| 2.74 = 1.87 \text{ k}\Omega$$

The output resistance R_o is found by setting the independent source V_s equal to zero. In this case, there is no excitation to the input portion of the circuit so $V_\pi = 0$, which implies that $g_m V_\pi = 0$ (an open circuit). The output resistance looking back into the output terminals is then

$$R_o = r_o \| R_C = 105 \| 6 = 5.68 \text{ k}\Omega$$