# **Chapter 2**

### Kinetics of Homogeneous Reactions

## **The General Mole Balance Equation**



# Rate expression

$$\frac{1}{V}\frac{dN_{A}}{dt} = r_{A}$$
$$\frac{dN_{j}}{dt} = r_{j}V$$

$$F_{j} = C_{j} \cdot v$$

$$\frac{\text{moles}}{\text{time}} = \frac{\text{moles}}{\text{volume}} \cdot \frac{\text{volume}}{\text{time}}$$

$$\frac{dF_{A}}{dV} = r_{A}$$

Constant-volume batch reactor

Constant-pressure batch reactor.

$$\frac{1}{V}\frac{dN_{A}}{dt} = \frac{d(N_{A}/V)}{dt} = \frac{dC_{A}}{dt} = r_{A}$$

$$N_{A} = C_{A}V$$

$$\frac{1}{V}\frac{dN_{A}}{dt} = \frac{1}{V}\frac{d(C_{A}V)}{dt} = \frac{dC_{A}}{dt} + \frac{C_{A}}{V}\frac{dV}{dt} = r_{A}$$

$$r_{A} = \frac{dC_{A}}{dt} + \frac{C_{A}d\ln V}{dt}$$

# Thermodynamics

### Thermodynamics provides:

- heat liberated or absorbed for a given extent of reaction
- maximum possible conversion
- Equilibrium Constants from Thermodynamics standard free energy  $\Delta G^{O}$  for

 $aA \rightarrow rR + sS$ ,  $\Delta H_{rT} \begin{cases} \text{positive, endothermic} \\ \text{negative, exothermic} \end{cases}$ 

 $\Delta G^{\circ} = rG^{\circ}_{\mathsf{R}} + sG^{\circ}_{\mathsf{S}} - aG^{\circ}_{\mathsf{A}} = -\mathbf{R}T\ln K$ 

$$-\mathbf{R}T\ln\frac{\left(\frac{f}{f^{o}}\right)_{R}^{r}\left(\frac{f}{f^{o}}\right)_{S}^{s}}{\left(\frac{f}{f^{o}}\right)_{A}^{a}} \qquad \Delta n = r + s - a$$

$$K_{f} = \frac{f_{R}^{r} f_{S}^{s}}{f_{A}^{a}}, \quad K_{p} = \frac{p_{R}^{r} p_{S}^{s}}{p_{A}^{a}}, \quad K_{y} = \frac{y_{R}^{r} y_{S}^{s}}{y_{A}^{a}}, \quad K_{C} = \frac{C_{R}^{r} C_{S}^{s}}{C_{A}^{a}}$$

Fugacity and pressure are identical

$$f^{\circ} = p^{\circ} = 1$$
 atm.

#### **Equilibrium Conversion.**

$$\frac{d(\ln K)}{dT} = \frac{\Delta H_r}{\mathbf{R}T^2}$$

K provides maximum attainable conversion

$$\ln \frac{K_2}{K_1} = -\frac{\Delta H_r}{\mathbf{R}} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

# Effect of temperature on equilibrium conversion as predicted by thermodynamics (pressure fixed.)



## Equilibriums define in three ways

- Thermodynamic view: System is in equilibrium with its surroundings of given temperature and pressure.
   If ∆G<sup>O</sup> <0, lowest possible value</li>
- Statistical mechanism: System consist of the greatest no. of equally likely molecular configuration which are macroscopically indistinguishable.
- Chemical Kinetics view: Rate of change of all the forward reaction and reverse reaction are equal

- Chemical Kinetics: Study of the rate and mechanism by which one chemical species converted to another
- **Rate:** reactant consumed per unit time
- Mechanism: sequence of individual chemical event. Whose overall result produce the observed reaction.
- Postulate theories are available

# The Rate Equation

single-phase reaction aA + bB = rR + sS



rates of reaction of all materials are related  $\frac{-r_A}{a} = \frac{-r_B}{b} = \frac{r_R}{r} = \frac{r_S}{s}$ 

# Rate of reaction is influenced by the composition and the energy of the material



#### **CONCENTRATION-DEPENDENT TERM:**

- hold the temperature of the system constant
- Based on the form and number of kinetic equations used to describe the progress of reaction

# Single and Multiple Reactions

- Single reaction: Single rate equation are chosen to represent the progress
  - Multiple reactions: more than one kinetic expression is needed

$$A \rightarrow R \rightarrow S$$

- Series reactions:
- Parallel reactions: Reactions of two type



- More complicated scheme: A + B → R
  - $R + B \rightarrow S$
- Reaction proceeds in parallel with respect to B
- Series with respect to A, R, and S.

### Elementary and Non-elementary Reactions

#### **Elementary reactions:**

- Stoichiometric equation  $A + B \rightarrow R$
- Rate of disappearance of A is given by  $-r_{A} = kC_{A}C_{B}$

rate expression:  $H_2 + Br_2 \rightarrow 2HBr$ 

$$r_{\rm HBr} = \frac{k_1 [\rm H_2] [\rm Br_2]^{1/2}}{k_2 + [\rm HBr] / [\rm Br_2]}$$

Overall effect of a sequence of elementary reactions reason for observing only a single reaction rather than two or more elementary reactions is that the amount of intermediates formed is negligibly small.

#### Postulate:

- That the rate-controlling mechanism involves the collision or interaction of a single molecule of A with a single molecule of B
- Number of collisions of molecules A with B is proportional to the rate of reaction.
- At a given temperature the number of collisions is proportional to the concentration of reactants in the mixture.

# Molecularity and Order of Reaction Molecularity:

- number of molecules involved in the reaction
- have the values of one, two, or occasionally three;

Must be an integer because it refers to the mechanism Note: that the molecularity refers only to an elementary reactions

$$-r_{A} = kC_{A}^{a}C_{B}^{b}\dots C_{D}^{d}$$
$$a + b + \dots + d = n$$

### Order of the reaction:

- where a, b, ..., d are not necessarily related to the stoichiometric coefficients.
- can have a fractional value

a<sup>th</sup> order with respect to A b<sup>th</sup> order with respect to B n<sup>th</sup> order overall

# Difference between Molecularity and Order of reaction

Molecularity	Order of reaction
It is defined as number of molecules or atoms or ions participating in rate determining step.	Order is defined as sum of powers of concentration terms in a rate equation.
It is a theoretical quantity, which can be determined through reaction mechanism.	Order of reaction can be determined experimentally.
It is always whole number or integer. It can never be fraction or zero.	Order of reaction is zero, fraction , integer & whole number.

**Rate Constant, k**  

$$r_{A} = k C_{A}^{n}$$
  
 $r_{A} = k (\frac{mol}{lit})^{n};$   
 $\frac{1}{time} = (\frac{mol}{lit})^{n-1}$ 

Dimensions of the rate constant k for the

nth-order reaction

 $(time)^{-1}(concentration)^{1-n}$ 

first-order reaction

 $(time)^{-1}$ 

### **Representation of an Elementary Reaction**

- Elementary reactions are often represented by an equation showing both the molecularity and the rate constant
- Biomolecular irreversible reaction with second-order rate constant k<sub>1</sub>

$$2A \xrightarrow{k_1} 2R \qquad -r_A = r_R = k_1 C_A^2$$

• Rate is measured  $B + 2D \rightarrow 3T$ 

$$-r_{\rm B} = k_{\rm B}C_{\rm B}C_{\rm D}^2$$
$$-r_{\rm D} = k_{\rm D}C_{\rm B}C_{\rm D}^2$$

$$r_{\rm T} = k_{\rm T} C_{\rm B} C_{\rm D}^2$$

• from the stoichiometry

$$-r_{\rm B} = -\frac{1}{2}r_{\rm D} = \frac{1}{3}r_{\rm T}$$
  $k_{\rm B} = \frac{1}{2}k_{\rm D} = \frac{1}{3}k_{\rm T}$ 

### Representation of a Non-elementary Reaction

Stoichiometry does not match its kinetics

Stoichiometry:  $N_2 + 3H_2 \rightleftharpoons 2NH_3$ Rate:  $r_{NH_3} = k_1 \frac{[N_2][H_2]^{3/2}}{[NH_3]^2} - k_2 \frac{[NH_3]}{[H_2]^{3/2}}$ 

 try to develop a multistep reaction model to explain the kinetics

### Kinetic Models for Non-elementary Reactions

Kinetics of the reaction (non-elementary):

 $A_2 + B_2 \rightarrow 2AB$ 

- Cannot measure or observe the intermediates formed because they are only present in very minute quantities
- May postulate a series of elementary steps to explain the kinetics
   A₂ ≈ 2A\*

$$A^* + B_2 \rightleftharpoons AB + B^*$$

$$A^* + B^* \rightleftharpoons AB$$

we may postulate different types of intermediates

### **Types of intermediates**

Free Radicals

 $CH_3\cdot,\quad C_2H_5\cdot,\quad I\cdot,\quad H\cdot,\quad CCl_3\cdot$ 

- Ions and Polar Substances  $N_3^-$ ,  $Na^+$ ,  $OH^-$ ,  $H_3O^+$ ,  $NH_4^+$
- Molecules

• A, B, R S

Transition Complexes

Based on these four kinds of intermediates Postulated reaction schemes can be of two types.

a. Non-chain Reactions b. Chain Reactions

### Free radicals, chain reaction mechanism

unstable and highly reactive

$$CH_3$$
,  $C_2H_5$ ,  $I$ ,  $H$ ,  $CCl_3$ .

- The reaction  $H_2 + Br_2 \rightarrow 2HBr$
- Experimental rate  $r_{\text{HBr}} = \frac{k_1[\text{H}_2][\text{Br}_2]^{1/2}}{k_2 + [\text{HBr}]/[\text{Br}_2]}$
- Introduces and involves the intermediates  $H \cdot and Br \cdot$

 $Br_2 \rightleftharpoons 2Br \cdot$  Initiation and termination  $Br \cdot + H_2 \rightleftharpoons HBr + H \cdot$  Propagation  $H \cdot + Br_2 \rightarrow HBr + Br \cdot$  Propagation

# Molecular intermediates, non-chain mechanism

Enzyme-catalvzed fermentation reactions:

Experimental rate

 $A \xrightarrow{\text{with}} R$ 

$$r_{\rm R} = \frac{k[A][E_0]}{[M] + [A]}$$

Intermediate: concentration of intermediate not negligible

 $A + enzyme \rightleftharpoons (A \cdot enzyme)^*$  $(A \cdot enzyme)^* \rightarrow R + enzyme$ 

Proposed by Michaelis and Menten

(A. enzyme)\* as intermediate

### Transition complex, non-chain mechanism

- Numerous collisions between reactant molecules result in a wide distribution of energies among the individual molecules
- Result in strained bonds, unstable forms of molecules, or unstable association of molecules
- Decompose to give products, or by further collisions return to molecules in the normal state

$$A \rightarrow R + S$$

- Energized and unstable form for the reactant, A\*
  - $A + A \rightarrow A^* + A$  Formation of energized molecule
  - $A^* + A \rightarrow A + A$  Return to stable form by collision
    - $A^* \rightarrow R + S$  Spontaneous decomposition into products

## **Testing Kinetic Models**

- Search for the correct mechanism
- Reaction may proceed by more than one mechanism
- More than one mechanism can be consistent with kinetic data
- Requires an extensive knowledge of the chemistry
- Test the correspondence between experiment and a proposed mechanism
- Hypothesize the existence of either of two types of intermediates

## **Testing Kinetic Models**

• Type 1.

Intermediate X usually present at such small concentration that its rate of change in the mixture can be taken to be zero

[X] is small and  $\frac{d[X]}{dt} \approx 0$ 

Steady-state approximation

## • Type 2.

Where a homogeneous catalyst of initial concentration  $C_o$  is present in two forms, either as free catalyst C or combined in an appreciable extent to form intermediate X,

$$\begin{bmatrix} C_0 \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} + \begin{bmatrix} X \end{bmatrix} \qquad \qquad \frac{dX}{dt} = 0$$

$$\begin{pmatrix} \text{reactant} \\ A \end{pmatrix} + \begin{pmatrix} \text{catalyst} \\ C \end{pmatrix} \stackrel{1}{\rightleftharpoons} \begin{pmatrix} \text{intermediate} \\ X \end{pmatrix} \qquad \qquad K = \frac{k_1}{k_2} = \frac{\begin{bmatrix} X \end{bmatrix}}{\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} C \end{bmatrix}}$$

Trial-and-error procedure involved in searching for a mechanism

## SEARCH FOR THE REACTION MECHANISM

Irreversible reaction:

$$A + B = AB$$

Rate of formation of product

$$r_{AB} = kC_B^2$$
. . . independent of  $C_A$ 

 Chemistry of the reaction suggests that the intermediate consists of an association of reactant molecules and that a chain reaction does not occur

If this were an elementary reaction, r<sub>AB</sub> = kC<sub>A</sub>C<sub>B</sub> = k[A][B]
 Start with simple two-step models, and if these are unsuccessful we will try more complicated three-, four-, or five-step models

# Trial – Error Method Model 1

- Hypothesize a two-step reversible scheme
- intermediate substance A<sup>\*</sup><sub>2</sub>
- Involves four elementary reactions  $2A \stackrel{k_1}{\longleftarrow} A_2^*$



$$A_2^* \xrightarrow{k_2} 2A$$

$$A_2^* + B \xrightarrow[k_4]{k_3} A + AB \qquad A_2^* + B \xrightarrow[k_4]{k_3} A + AB$$

$$A + AB \xrightarrow{k_4} A_2^* + B$$

$$r_{AB} = k_3[A_2^*][B] - k_4[A][AB]$$
$$r_{A_2^*} = \frac{1}{2}k_1[A]^2 - k_2[A_2^*] - k_3[A_2^*][B] + k_4[A][AB]$$

### **Two-step reversible scheme**

- Concentration of A<sup>\*</sup><sub>2</sub> is always extremely small
- Rate of change is zero

$$r_{A_2^*} = 0$$

$$[\mathbf{A}_{2}^{*}] = \frac{\frac{1}{2}k_{1}[\mathbf{A}]^{2} + k_{4}[\mathbf{A}][\mathbf{A}\mathbf{B}]}{k_{2} + k_{3}[\mathbf{B}]}$$

▶ If k<sub>2</sub>, is very small, this expression reduces

$$r_{AB} = \frac{\frac{1}{2}k_1k_3[A]^2[B] - k_2k_4[A][AB]}{k_2 + k_3[B]}$$

$$r_{\rm AB} = \frac{1}{2} k_1 [\rm A]^2$$

## **Two-step reversible scheme**

If k<sub>4</sub>, is very small 
$$r_{AB} = \frac{(k_1 k_3 / 2k_2)[A]^2[B]}{1 + (k_3 / k_2)[B]}$$

Neither of these special forms matches the experimentally

So another model needs to be tried

Model 2.  

$$B + B \xrightarrow{1} B_{2}^{*}$$

$$A + B_{2}^{*} \xrightarrow{3}_{4} AB + B$$

### SEARCH FOR A MECHANISM FOR THE ENZYME SUBSTRATE REACTION

- **Reactant:** Substrate
- Enzyme: Molecular weight (mw > 10 000) protein-like substance
  A R
- Many of these reactions exhibit the following behaviour:
  - **1.** A rate proportional to the concentration of enzyme introduced into the mixture  $[E_o]$ .
  - **2.** At low reactant concentration the rate is proportional to the reactant concentration, [A]
  - **3.** At high reactant concentration the rate levels off and becomes independent of reactant concentration.

### **ENZYME SUBSTRATE REACTION**

Michaelis and Menten

$$A + E \stackrel{1}{\underset{2}{\rightleftharpoons}} X$$
$$X \stackrel{3}{\xrightarrow{3}} R + E$$

1

Two assumptions

 $[\mathbf{E}_0] = [\mathbf{E}] + [\mathbf{X}]$ 

$$\frac{d\mathbf{X}}{d\mathbf{t}} \cong \mathbf{0}$$

First expression

$$\frac{d[X]}{dt} = k_1[A][E] - k_2[X] - k_3[X] = 0$$

Second expression

$$\frac{d[\mathbf{R}]}{dt} = k_3[\mathbf{X}]$$

### **SEARCH FOR A MECHANISM**

- Eliminating [E]  $[X] = \frac{k_1[A][E_0]}{(k_2 + k_3) + k_1[A]}$
- Substituting in second expression

$$\frac{d[\mathbf{R}]}{dt} = \frac{k_1 k_3 [\mathbf{A}] [\mathbf{E}_0]}{(k_2 + k_3) + k_1 [\mathbf{A}]} = \frac{k_3 [\mathbf{A}] [\mathbf{E}_0]}{[\mathbf{M}] + [\mathbf{A}]}$$

$$(\mathbf{M}) = \left(\frac{k_2 + k_3}{k_1}\right) \text{ is called}$$
the Michaelis constant

 Comparing with experiment, we see that this equation fits the three reported facts

$$\frac{-d[A]}{dt} = \frac{d[R]}{dt} \begin{cases} \propto [E_0] \\ \propto [A] \text{ when } [A] \ll [M] \\ \text{is independent of } [A] \text{ when } [A] \ge [M] \end{cases}$$

### TEMPERATURE-DEPENDENT TERM OF A RATE EQUATION

### Arrhenius' Law:

 $r_i = f_1$ (temperature)  $\cdot f_2$ (composition)

 $= k \cdot f_2(\text{composition})$ 

 $k = k_0 e^{-\mathbf{E}/\mathbf{R}T}$  $K_o =$  Frequency or pre-exponential factor E = Activation energy of the reaction.

At two different temperatures

$$\ln \frac{r_2}{r_1} = \ln \frac{k_2}{k_1} = \frac{\mathbf{E}}{\mathbf{R}} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

E stays constant

# Comparison of Theories with Arrhenius' Law

• Collision and transition state theories for the temperature dependency of the rate constant

$$k = k_0' T^m e^{-\mathbf{E}/\mathbf{R}T}, \qquad 0 \le m \le 1$$

- More complicated versions m can be as great as 3 or 4
- Temperature dependency for Collision theories

$$\mathsf{K} = \mathsf{k}_{\mathsf{o}} T^{1/2} e^{-E/RT}$$

• Temperature dependency for transition theories

$$K = k_o T e^{-E/RT}$$

# Comparison of Theories with Arrhenius' Law

Exponential term is so much more

temperature-sensitive than the pre-exponential term

Arrhenius' law is a good approximation to the temperature dependency of both collision and transition-state theories

$$k = k_0 e^{-\mathbf{E}/\mathbf{R}T}$$

### Activation Energy and Temperature Dependency



# Arrhenius' law

Plot of In k vs 1/T gives a straight line

- Large slope for large E and small slope for small E
- Reactions with high activation energies are very temperature-sensitive
- Iow activation energies are relatively temperatureinsensitive
- Frequency factor k<sub>o</sub>, does not affect the temperature sensitivity

## Shift in controlling mechanism of reaction

#### **Multiple reactions**

Change in the observed activation energy with temperature shift in the controlling mechanism of reaction indicates a Parallel reactions Series reactions **E**<sub>obs</sub> rises Increase in temperature **E**<sub>obs</sub> falls **E**<sub>obs</sub> falls Decrease in temperature **E**<sub>obs</sub> rises Mech. 1 R А Mech. 2 High E Low E ln k Low E

High T

1/T

High E

1/T

Low T

High T

Low T

### SEARCH FOR THE ACTIVATION ENERGY OF A PASTEURIZATION PROCESS

**Example :** Milk is pasteurized if it is heated to 63°C for 30 min, but if it is heated to 74°C it only needs 15 s for the same result. Find the activation energy of this sterilization process.

Solution :

Arrhenius temperature dependency for the process

$$\ln \frac{r_2}{r_1} = \ln \frac{t_1}{t_2} = \frac{\mathbf{E}}{\mathbf{R}} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

Given

Substituting values

 $t_1 = 30 \text{ min}$  at a  $T_1 = 336 \text{ K}$  $t_2 = 15 \text{ sec}$  at a  $T_2 = 347 \text{ K}$   $\ln \frac{30}{0.25} = \frac{\mathbf{E}}{8.314} \left(\frac{1}{336} - \frac{1}{347}\right)$ 

from which the activation energy

 $\mathbf{E} = 422\ 000\ \mathrm{J/mol}$