Modulation Index :⇒

⇒ In AM system the modulation index is defined as the measure of extent of amplitude variation about an unmodulated maximum carrier. Mathematically, $m_a = \frac{|m(t)|_{max}}{\text{maximum carrier amplitude}} = \frac{|m(t)|_{max}}{V_c}$

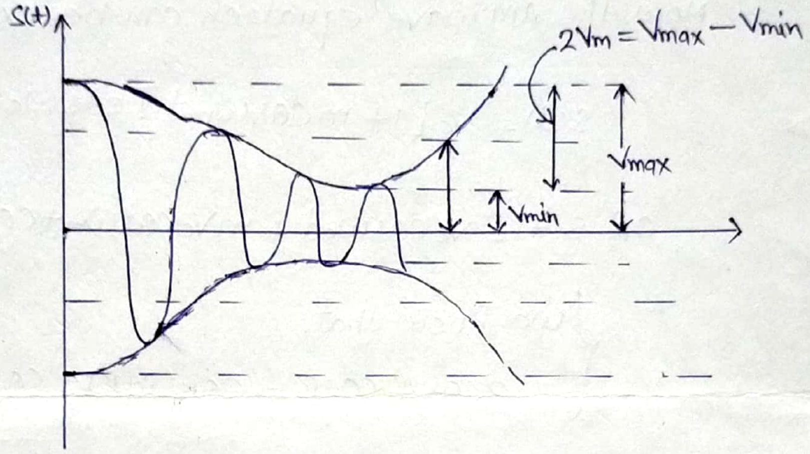
OR

⇒ Ratio of amplitude of message signal to amplitude of carrier signal.

Mathematically,

$$m = \frac{V_m}{V_c}$$

⇒



$(V_c > V_m)$

⇒ $2V_m = V_{max} - V_{min}$

$V_m = \frac{V_{max} - V_{min}}{2}$

⇒ $V_c = V_{max} - V_m$

$= V_{max} - \frac{V_{max} - V_{min}}{2}$

$= \frac{2V_{max} - V_{max} + V_{min}}{2}$

$= \frac{V_{max} + V_{min}}{2}$

⇒ Now,

$$\text{Modulation index } (m) = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

* Modulation index is also known as depth of modulation, degree of modulation or modulation factor. Also modulation index multiplied by 100 is known as percentage modulation.

Physical significance of modulation index :-

⇒ If $m < 1$,

then the maximum amplitude of baseband signal is less than maximum carrier amplitude i.e. $V_m < V_c$, so that the envelop is not reaching the zero amplitude axis of AM wave and the baseband signal may be fully recoverd from the envelope of AM wave.

⇒ If $m = 1$,

then there is 100% modulation. This is the basic condition of modulation.

⇒ If $m > 1$,

then $V_m > V_c$, So that the envelop is reaching the zero amplitude axis of AM wave and shape of base band signal gets distorted.

Power Contain in AM Wave :→

It may be observed from the expression of AM wave that the carrier component of the amplitude modulated wave has the same amplitude as unmodulated carrier. In addition to carrier component, the modulated wave consists of two sideband components. It means that the modulated wave contains more power than the unmodulated carrier.

However, since the amplitudes of two sidebands depend upon the modulation index, therefore the total power of amplitude modulated wave would depend upon the modulation index also.

The general expression of AM wave is

$$s(t) = V_c \cos w_c t + m(t) \cos w_c t \quad \text{---} (1)$$

The total power P_T of the AM wave is the sum of carrier power P_c and side band Powers ($P_{LSB} + P_{USB}$)

$$P_T = P_c + P_{LSB} + P_{USB} \quad \text{----} (II)$$

Now, the carrier power P_c is equal to the mean-squar value of the carrier term $V_c \cos w_c t$, i.e.

~~mean-square value of~~

P_c = mean-squar value of $V_c \cos w_c t$

$$P_c = \frac{1}{2\pi}\int_0^{2\pi} V_c^2 \cos^2 w_c t \cdot dt$$

$$P_c = \frac{V_c^2}{2}$$

{* Since period of the signal $V_c \cos$ is 2π.}

and, side band power P_s is equal to the mean square value of the side band term $m(t) \cos w_c t$, i.e.

P_s = mean-squar value of $m(t) \cos w_c t$

$$= \frac{1}{2\pi}\int_0^{2\pi} [m(t) \cos w_c t]^2 dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} m^2(t) \cos^2 w_c t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}[2\cos^2 w_c t]\, m^2(t)\, dt$$

OR

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} m^2(t)\, dt + \frac{1}{2\pi}\int_0^{2\pi} m^2(t) \cos 2w_c t\, dt$$

In AM generation, a Band pass filter or a tuned circuit tuned to carrier frequency ω_c is used to filter out the second integral term. Therefor

$$P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} m^2(t)\,dt$$

P_s = mean sequar value of $\frac{1}{2} m^2(t)$

OR $P_s = \frac{1}{2}\overline{m^2(t)}$

$\Rightarrow$ OR $P_s = \frac{\left[\frac{m.V_c}{2}\right]^2}{2} = \frac{m^2 V_c^2}{8}$

The total sideband power P_s is due to the contributions of the upper and lower sideband.

Therefore, the total power P_t of the AM signal

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$P_t = \frac{V_c^2}{2} + \frac{m^2 V_c^2}{8} + \frac{m^2 V_c^2}{8}$$

$$P_t = \frac{V_c^2}{2} + \frac{m^2 V_c^2}{4}$$

$$P_t = \frac{V_c^2}{2}\left[1 + \frac{m^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{m^2}{2}\right] \quad \text{OR} \quad P_t = P_c + P_c\frac{m^2}{2}$$

Current Relationship in AM Wave:-

* In AM, it is generally more convenient to measure the AM transmission current than power.
* In this case, the modulation index may be calculated from the values of unmodulated and modulated current in AM transmission.

Let I_c be the r.m.s. value of the carrier of an AM transmission and I_t be the r.m.s. value of the total or modulated carrier of an AM transmitter. Let R be the antenna resistance through which these currents flow.

Now,

$$P_t = P_c\left(1+\frac{m^2}{2}\right)$$

where,

$$P_t = I_t^2 . R$$

and $P_c = I_c^2 . R$

(R = Antenna Resistance) [Take R=1KΩ, when not given]

from the above eq.

$$\frac{P_t}{P_c} = 1+\frac{m^2}{2}$$

$$\Rightarrow \frac{I_t^2 . R}{I_c^2 . R} = 1+\frac{m^2}{2}$$

$$\frac{I_t}{I_c} = \sqrt{1+\frac{m^2}{2}}$$

$$I_t = I_c\sqrt{1+\frac{m^2}{2}}$$

Transmission Efficiency of AM Wave :-

In AM wave, the amount of useful message Power(P_s) may expressed by a term known as transmission efficiency(η).

Hence transmission efficiency of AM wave may be defined as the percentage of total power contributed by sidebands.

Mathematically,

$$\text{Transmission Efficiency}(\eta) = \frac{P_s}{P_t} \times 100 \text{ or } \frac{\text{Useful P}}{\text{Total P}}$$

---

we know that the total modulated power of an AM wave is expressed as

$$P_t = P_c\left[1 + \frac{m^2}{2}\right] = P_c + \underbrace{P_c \frac{m^2}{2}}$$

side band Power OR useful Power

→ The large carrier power P_c is a wastes ~~from the~~ power, because it does not carry any information or message.

Now,

$$\eta = \frac{P_c \cdot \frac{m^2}{2}}{P_c\left(1 + \frac{m^2}{2}\right)} \times 100$$

$$\eta = \frac{\frac{m^2}{2}}{1 + \frac{m^2}{2}} \times 100$$

$$\eta = \frac{m^2}{2 + m^2} \times 100$$

---

* For 100% modulation i.e. m=1

$$\eta = \frac{1}{2+1} = \frac{1}{3} = 33.33\%$$

Therefore the maximum transmission efficiency of the AM is only 33.33%. This means only-third of the total power is carried by side-bands and the rest two-third is wasted.