

# Relational Algebra

f(x)

$A \neq B$   $A'$

- is a procedural query language

- fundamental operations

$\underbrace{D_1 \times D_2 \times D_3}_{\uparrow \quad \uparrow}$

- Select

- Project

- Union

- Set difference

- Cartesian Product

- Rename

- Set intersection

- natural join

- division

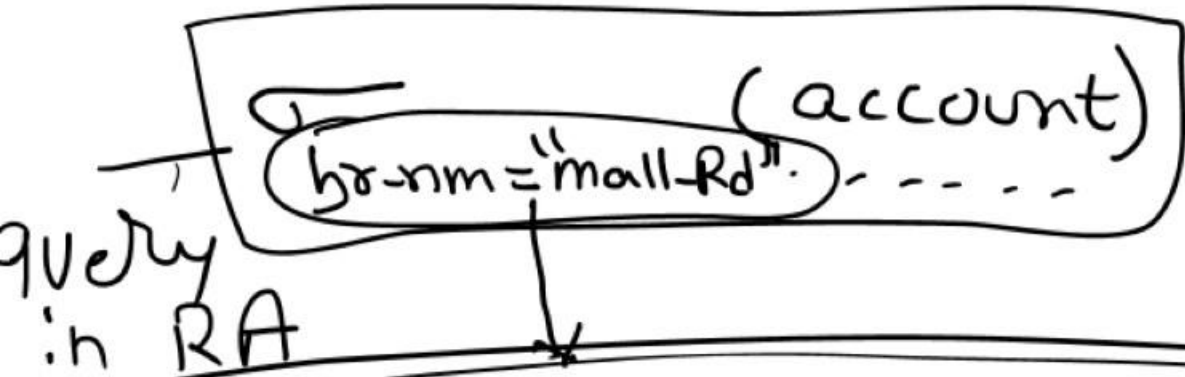
- assignment

unary operations

binary operations

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Select ( $\sigma$ ) — selects tuples that satisfy a given predicate (appears as a subscript to  $\sigma$ )



relation Anonymous

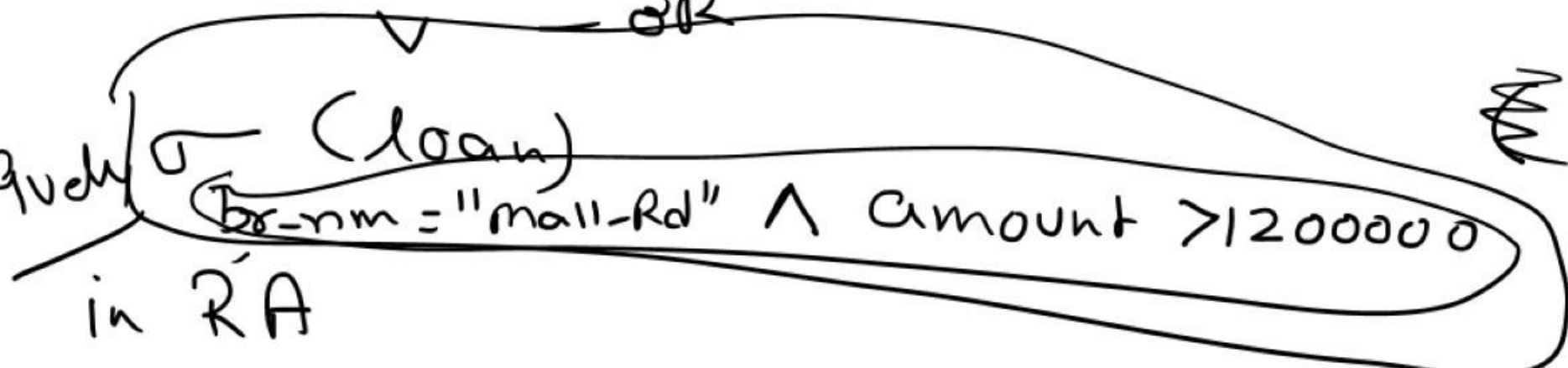
d/p

mall-Rd	A-101	500
mall-Rd	-	-

$=, \neq, <, >, \leq, \geq$  — allowed

$\wedge$  — and AND

$\vee$  — or OR



Project operation ( $\pi$ ) - returns its argument relation with certain att's left out. duplicate rows are eliminated. Att's that we wish to appear in the resultant relation is given as subscript to  $\pi$

write an RA exp to find name of customers who live in Delhi

loan (loan)

(customer)

cust-cty = "Delhi"

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o/p

ln-num	amount

$\pi$  cust-nm

$\pi$  cust-nm

o/p

cust-nm	cust-st	cust-cty
x	a	Delhi
y	b	Delhi
z	c	Delhi

relation

o/p

cust-nm
x
y
z

# Union operation - binary operator

$R_1$

x	y	z
a	c	e
b	d	f

$R_2$

x	y	z
g	i	k
h	j	l

x	y	z

$U$

u	v	w

wrong

$R$

$\pi \rightarrow$

~~$R = R_1 \cup R_2$~~

$$\pi = \pi_1 \cup \pi_2$$

x	y	z
a	c	e
b	d	f
g	i	k
h	j	l

rows/tuples

- Union are taken between compatible relations only

- for  $\pi_1 \cup \pi_2$  to be valid following conditions must hold

(i) Relations  $R$  &  $S$  must be of same arity i.e. they must have same no. of attr's

(ii) domain's of  $i^{\text{th}}$  attr of  $R$  and  $i^{\text{th}}$  attr of  $S$  must be same  $\forall i$

Set difference -  $(-)$  ex  $R - S$

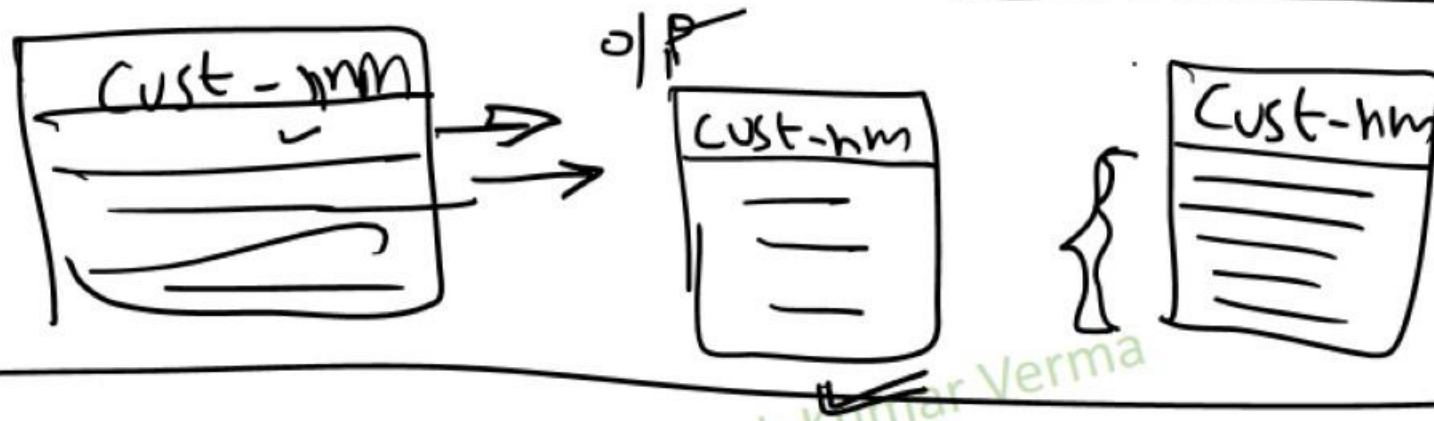


- used to find tuples that are in one relation but are not there in second relation

Union Compatible

RA  $\rightarrow$  find customers name who have an account in the bank but have not taken any loan from the bank.

$$r_{\text{cust-mm}} (\text{depositor}) - r_{\text{cust-hm}} (\text{borrower})$$



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## Cartesian Product ( $\times$ )

- allows us to combine info from any two relation
- relations that are arguments of Cartesian product operation must have distinct names.

~~#~~ ~~o~~  $r_1 = \text{borrower} \times \text{loan}$   
5  $n_1$  no. of tuples  $\rightarrow n_2$  no. of tuples 0

$$r = r_1 \times r_2$$
$$r = n_1 \times n_2 = \text{no. of tuples}$$
$$5 \times 10 = 50 \text{ tuples}$$

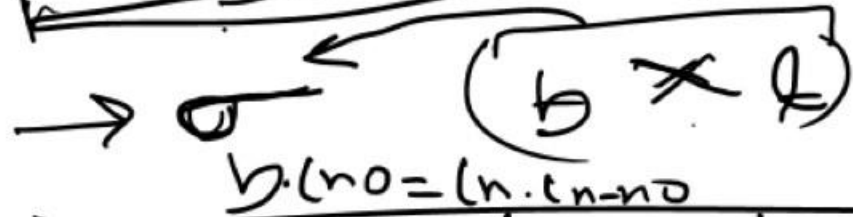
- Cartesian product takes all possible pairings of one tuple from  $r_1$  with one tuple of  $r_2$

borrower

custnm	ln-no
a	L-1
b	L-2
c	L-3

loan

br-nm	ln-num	Amount
x	L-1	5
x	L-2	2
y	L-3	4



custnm	ln-no	br-nm	ln-num	Amt
a	L-1	x	L-1	5
<del>a</del>	<del>L-1</del>	<del>x</del>	<del>L-2</del>	<del>2</del>
a	L-1	y	L-3	4
b	L-2	x	L-1	5
b	L-2	x	L-2	2

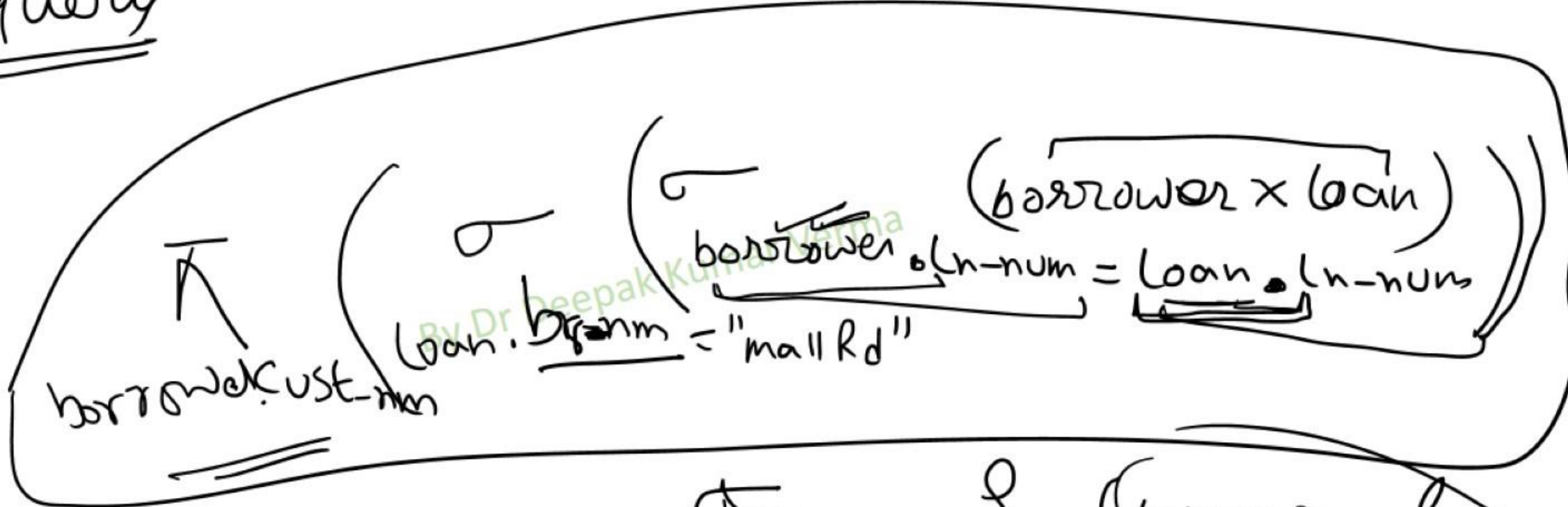
$\rightarrow$  spurious tuples

b	L-2	y	L-3	4
c	L-3	x	L-1	5
c	L-3	x	L-2	2
c	L-3	y	L-3	4



Find names of cust's who have taken loan at ~~all the~~ mall-rd branch.

RA query



~~SELECT~~ ln bl (borrower x loan)  
(bl)  
borrower.ln-num = loan.ln-num

# Rename operation ( $\rho$ )

$\rho_r(E)$   $\rightarrow$  relation or RA expression  
 $\downarrow$   
newname

$\square$  I/O - relation/table

$\rho_r(A_1, A_2, \dots, A_n)(E)$

custn, Ln-no.

$\rho_b(\text{borrower})$

$\rho_b(c, L)(\text{borrower})$

b

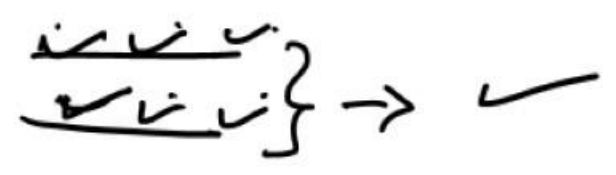
$\rho_{br}(\text{borrower} \times \text{loan})$

Find the largest acc balance in the bank.



balance which not less i.e. they are highest

Relation

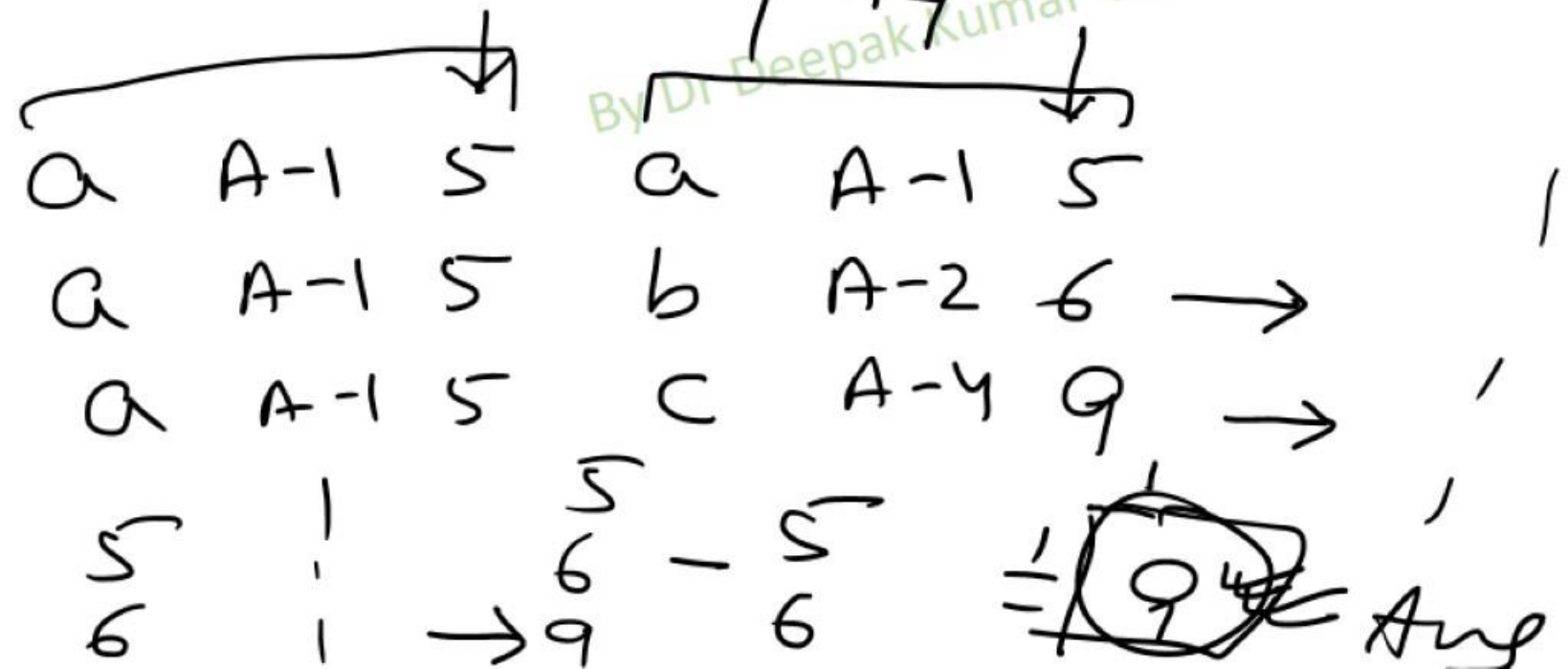


account

br-nm	acc-no	bal
a	A-1	5
b	A-2	6
c	A-4	9

d

br-nm	acc-no	bal
a	A-1	5
b	A-2	6
c	A-4	9



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# Additional operations

1) Set Intersection ( $\cap$ )

$$R \cap S = \boxed{R - (R - S)}$$

tuples which are present in both R & S

find cust's have taken both loans from the bank and are also having account.

✓ depositor  $\cap$  borrower ✓

---

names of cust

---

Natural join -  $\bowtie$

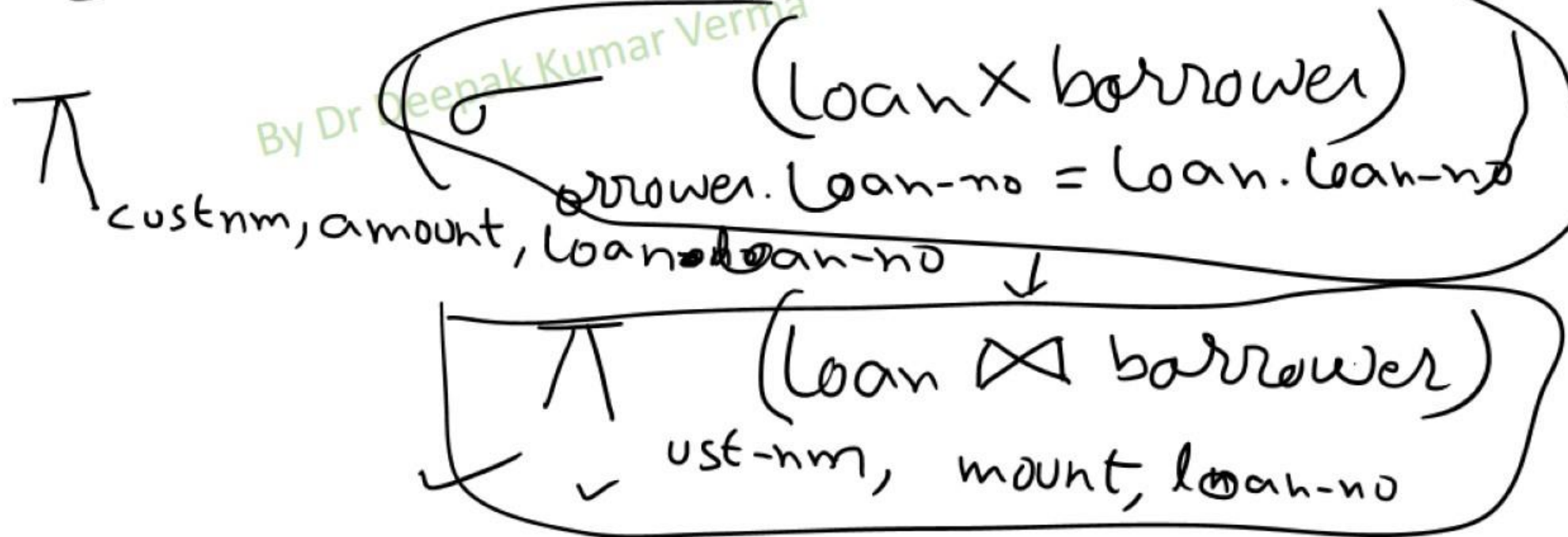
- binary operation - it allows us to combine certain selections and a Cartesian product into one operation
- natural join forms a Cartesian product of its two arguments, performs selection forcing equality on those attributes that appear in both relations and finally removes duplicates

$n, S$

$$\sigma = \quad (n \times S)$$

---

Find names of all cust's who have a loan at the bank & find the amount of the loan.



Division Operation  $\div$

Suited for queries that include phrase "for all"

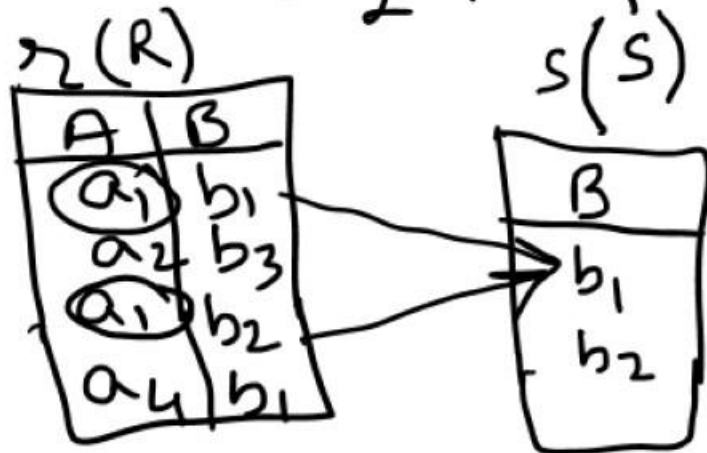
Ex. find all customers who have an account at all branches located in Kanpur.

$$\pi_1 = \pi_{brnm} \left( \sigma_{brcty = \text{"kanpur"}} \right) \quad (\text{branch})$$

$$\pi_2 = \pi_{custnm, brnm} (\text{depositor} \bowtie \text{account})$$

We need to find customers who appear in  $\pi_2$  With every branch name in  $\pi_1$

ex say



$$\pi_2 \div S =$$

R-S
A
$a_1$



## Views

- It is not desirable for all users to see the entire logical schema
- There may be certain security reasons that require that only certain part of the logical schema to be visible to the users
- Any relation that is not part of logical model but is made visible to a user as virtual relation is referred to as a view

Create view  $V$  as  $\langle \text{query expression} \rangle$

ex - branch(branch, brcty, assets)

→ Create view br as  $\sigma_{\text{branch, assets}}$  (branch)

→ Create view allcusts as

$\pi_{\text{branch, custnm}}(\text{depositor} \bowtie \text{account})$

$\cup$   
 $\pi_{\text{branch, custnm}}(\text{borrower} \bowtie \text{loan})$

## Tuple relational calculus

- Is a non procedural query language
- Query in TRC is expressed as

$$\{t \mid P(t)\}$$

- Set of all tuples  $t$  s.t. predicate  $P$  is true for  $t$
- Several tuple variables may appear in a formula
- Tuple variable is said to be a free variable unless it is quantified by a  $\exists$  or  $\forall$

ex  $t \in \text{loan} \wedge \exists s \in \text{customer} (t[\text{brnm}] = s[\text{brnm}])$

$\hookrightarrow$  free variable  $\quad \hookrightarrow$  bounded variable

- A tuple relational calculus formula is built out of atoms.
- An atom has one of the following forms

(i)  $s \in r$ ,  $s$  is a tuple variable,  $r$  is a relation

(ii)  $s[x] \Theta u[y]$ ,  
 $s, u$  - tuple variables  
 $x$  - is attr of  $s$

-  $y$  is attr of  $u$

-  $\ominus$  is comparison operation  $<, >, \leq, \geq, =, \neq$

-  $x, y$  - attr's domain is s.t. they can be compared

(iii)  $S[x] \ominus c$ , where  $c$  is a constant

• Formulae are build from atoms using following rules

• An atom is a formula

• If  $P_1$  is a formula then are  $\neg P_1$  and  $(P_1)$

• If  $p_1$  and  $p_2$  are formula then so are

$P_1 \vee P_2, P_1 \wedge P_2, P_1 \Rightarrow P_2$

If  $p_1(s)$  is a formula containing free tuple variable  $s$  and  $r$  is a relation then

$\exists s \in r (P_1(s))$

$\forall s \in r (P_1(s))$

are also formulae

Example: find brnm, lnum, amount for loans of 1200000 rupees

$$\{t \mid t \in \text{loan} \wedge t[\text{amount}] > 1200000\}$$

Find lnum for each loan of amount greater than 1200000 rupees

$$\{t \mid \exists s \in \text{loan} (t[\text{lnum}] = s[\text{lnum}] \wedge s[\text{amount}] > 1200000)\}$$

Find names of all customers who have loan from mall-road branch

$$\{t \mid \exists s \in \text{borrower} (t[\text{custnm}] = s[\text{custnm}] \wedge \exists u \in \text{loan} (u[\text{lnum}] = s[\text{lnum}] \wedge u[\text{brnm}] = \text{"mall-rd"}))\}$$

$p \Rightarrow q$ , means if p is true then q must be true, used for division operation

Find all customers who have an account at all branches located in kanpur

$$\{t \mid \forall u \in \text{branch} (u[\text{brcty}] = \text{"kanpur"} \Rightarrow \exists s \in \text{depositor} (t[\text{custnm}] = s[\text{custnm}] \wedge \exists w \in \text{account} (w[\text{accnum}] = s[\text{accnum}] \wedge w[\text{brnm}] = u[\text{brnm}]))))\}$$

## Safety of an expression

$\{t \mid \neg (t \in \text{loan})\}$  - may produce infinite many tuples

- Such expressions are not desirable in database
- For safe expressions we define the domain of a formula  $\text{dom}(p)$
- $\text{Dom}(p)$  is the set of all values referenced by  $p$  i.e. values mentioned in  $p$  itself also the values that appear in tuples of relation mentioned in  $p$ .
- $\{t \mid p(t)\}$  is safe if all values appearing in the result are from  $\text{dom}(p)$

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## Domain relational Calculus

- Uses domain variables
- Domain variables takes values from domain of attributes
- A DRC expression is of the form

$$\{ \langle x_1, x_2, \dots, x_n \rangle \mid P(x_1, x_2, \dots, x_n) \}$$

domain variable      formula composed of atoms

Example:

Find brnm, lnum, amount for loans over 1200000 rupees

$$\{ \langle b, l, a \rangle \mid \langle b, l, a \rangle \in \text{loan} \wedge a > 1200000 \}$$

Find names of customers who have an account in NOIDA, lives in Delhi and balance more than 500000 rupees

$$\{ \langle cn \rangle \mid \exists cs ( \langle cn, "DELHI", cs \rangle \in \text{customer} \wedge \\ \exists an ( \langle cn, an \rangle \in \text{depositor} \wedge \\ \exists bn, bal ( \langle an, bn, bal \rangle \in \text{account} \wedge \\ bal > 500000 \wedge \exists as ( \langle bn, "NOIDA", as \rangle \in \text{branch} ) ) ) ) \}$$

Find the names of all customers who have an account at all branches located in kanpur

$$\{ \langle c \rangle \mid \forall x, y, z (\langle x, y, z \rangle \in \text{branch}) \wedge y = \text{"kanpur"} \\ \Rightarrow \exists a, b (\langle x, a, b \rangle \in \text{account} \wedge (c, a) \in \text{depositor}) \}$$

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## Assertions

An assertion is a predicate expressing a condition that we wish the database always to satisfy.

Domain constraints and referential integrity constraints are special forms of assertions.

There are many constraints that we cannot express using domain constraints & referential integrity constraints.

Ex. – sum of all loan amounts for each branch must be less than the sum of all account balance at the branch

Ex. every loan has at least one customer who maintains an account with a min bal of \$1000.00

```
Creat assertion <assertion_name> check <predicate>
```

```
Create assertion bal_constraint check  
    (Not exists (select * from laon
```

```
Where not exists (select * from barrower, depositor, account
```

```
Where loan.loan_num = borrower.loan_num
```

```
and
```

```
borrower.cust_nm = depositor.cust_nm
```

```
and
```

```
depositor.acc_num = acc.acc_num
```

```
and
```

```
account.bal>=1000)))
```

When an assertion is created system tests it for validity. If the assertion is valid then any further modification to the database is allowed only if, it does not cause the assertion to be violated.



## Triggers

- A trigger is a statement that is executed automatically by the system as a side effect of a modification to the db
- for trigger two requirements, 1. Specify the conditions under which the trigger is to be executed, 2. Specify the actions to be taken when the trigger executes.

Ex. instead of allowing –ve acc balances, the bank deals with overdrafts by setting the acc bal to zero, & creating an loan in the amount of the overdraft. This loan is given a ln number identical to the acc num of the overdrawn account.

Defin trigger overdraft on update of account t

(if new T.bal<0

then (insert into loan values

(T.br\_nm,T.acc\_num, -new T.bal)

insert into borrower

(select cust\_nm, acc\_num from depositor

Where T.acc\_num = depositor.acc\_num)

Update accounts

Set s.bal = 0

Where s.acc\_num = T.acc\_num) )

New – used so that value of T.bal after update should be used if it is omitted the value before the update is used.

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## Functional dependencies

- Functional dependency is a generalization of the notion of key
- FD are constraints on the set of legal relations.

FD  $\alpha \rightarrow \beta$  holds on R if, in any legal relation  $\mathcal{R}(R)$   $\forall$  Pairs of tuples  $t_1$  &  $t_2$  in  $\mathcal{R}$  s.t.  $t_1[\alpha] = t_2[\alpha]$  then it is also the case that  $t_1[\beta] = t_2[\beta]$ .

Using the above defined FD notion, we say that K is a Superkey of R if  $K \rightarrow R$ . That is, K is a Superkey if, Whenever  $t_1[K] = t_2[K]$  then it is also the case that  $t_1[R] = t_2[R]$ . In other words  $t_1 = t_2$ .

A	B	C	D
a1	b1	c1	d1
a1	b2	c1	d2
a2	b2	c2	d2
a2	b3	c2	d3
a3	b3	c2	d4

$A \rightarrow C$  satisfied  
 $C \rightarrow A$  not satisfied  
 $AB \rightarrow D$  satisfied

Trivial FD's

$A \rightarrow A$   
 $AB \rightarrow A$

In general FD of the form  $\alpha \rightarrow \beta$   
is trivial if  $\beta \subseteq \alpha$

We use assertions in SQL to enforce FD's

FD's are used in two ways

- To specify constraints on the set of legal relations
- To test relations to see whether the latter are legal under a given set of FD's

# Extraneous att's on the left side of a FD

db-colleges

FD  $\alpha \rightarrow \beta$

ex  $\checkmark$   $(\text{Rollno}, \text{name}) \rightarrow \text{address, branch, section}$   
 $\swarrow$   $\searrow$  L  $\rightarrow$  R  
 $\rightarrow$  extraneous att in this FD  
Rollno  $\rightarrow$  address, branch, section

suppose  $\alpha \rightarrow \beta$  and  $A \subseteq \alpha$  s.t.  $(\alpha - A) \rightarrow \beta$   
 $\downarrow$   $\downarrow$   
maybe more than one att.  $\swarrow$  then A is an extraneous att  
in FD  $\alpha \rightarrow \beta$

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Left irreducible F)

CK relation can have many CK's, one of them considered to be PK.

Suppose  $\alpha \rightarrow \beta$  holds R and X is the set of extraneous attributes on left side of  $\alpha \rightarrow \beta$  then  $(\alpha - X) \rightarrow \beta$  is called left irreducible

Prime att / key att

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An attribute  $A \in R$  is called a Prime (or key) att if it forms part of any of the CK's of R.



non prime / non key att -

Att's those does not form part of any of the CK's

Logically implied FD's or logically inferred FD's

A FD  $\alpha \rightarrow \beta \notin F$  is said to be logically implied by  $F$  iff it is satisfied by each legal relation  $r(R)$  that satisfies  $F$

Say  $R(A, B, C)$  - be a relation  
and

$F = \{A \rightarrow B, B \rightarrow C\}$   
-  $\underbrace{\hspace{10em}}_{\text{infer/imply/deduce}} \rightarrow A \rightarrow C$

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## Closure of a FD set

Suppose  $F$  is the set of FD's holding on  $R$  and  $F_1$  is the complete set of FD's logically implied by  $F$  then closure of  $F$  (denoted by  $F^+$ ) is defined as union of  $F$  &  $F_1$

$$F^+ = F \cup F_1$$

---

## Rules for logical inference of FD's

### Armstrong rules (Axioms)

# Rule 1 - Reflexivity rule

if  $\beta \subseteq \alpha$  then  $\alpha \rightarrow \beta$  holds  
this will generate Trivial FD's

$AB \rightarrow \textcircled{B}$ ,  $AB \rightarrow A$ ,  $A \rightarrow A$ ,  $B \rightarrow B$

grandson  $\alpha \rightarrow \textcircled{\beta}$



Rule 2 - Augmentation rule. if  $\alpha \rightarrow \beta$  then  
 $\alpha\gamma \rightarrow \beta\gamma$   $A \rightarrow B$   $\uparrow$   $R(A, B, C, D)$   
 $AD \rightarrow BD$

Rule 3 - Transitivity rule  
if  $\alpha \rightarrow \beta$ ,  $\beta \rightarrow \gamma$  then  $\alpha \rightarrow \gamma$

# Additional rules

Rule 4 - Union rule. - if  $\alpha \rightarrow \beta$  &  $\alpha \rightarrow \gamma$   
then  $\alpha \rightarrow \beta\gamma$   $\alpha \rightarrow \gamma\beta$   
(4<sup>th</sup>)

Rule 5 - Decomposition rule if  $\alpha \rightarrow \beta\gamma$  then  
 $\alpha \rightarrow \beta, \alpha \rightarrow \gamma$

L cannot be decomposed

$AB \rightarrow C$   
 $AB \rightarrow D$   
 $AB \rightarrow CD$  ✓  
 $AB \rightarrow CD$   
 $AB \rightarrow C$  ✓  
 $AB \rightarrow D$  ✓

✓  $[AB] \rightarrow C$   
 $A \rightarrow C$   
 $B \rightarrow C$  Wrong

$A \rightarrow C$   
 $B \rightarrow D$   
↑  
 $AB \rightarrow CD$  Wrong  
cannot union on L side of a FD.



# Rule 6 - Pseudotransitivity Rule

if  $\alpha \rightarrow \beta$  &  $\beta \underline{\underline{\delta}} \rightarrow \gamma$   
then,  $\alpha \delta \rightarrow \gamma$

ex  ~~$R(A, B, C)$~~ ,  $F = \{A \rightarrow B, B \rightarrow C\}$

$F^+$  =  $\left\{ \begin{array}{l} A \rightarrow B, B \rightarrow C, A \rightarrow C, A \rightarrow A, B \rightarrow B, C \rightarrow C, \\ AB \rightarrow B, AB \rightarrow A, BC \rightarrow B, BC \rightarrow C, CA \rightarrow C, \\ CA \rightarrow A, ABC \rightarrow AB, ABC \rightarrow BC, ABC \rightarrow CA, \\ \dots \dots \dots \end{array} \right\}$

# Algorithm to determine closure of FD set F

(i)  $F^+ = F$

(ii) Repeat

Save the value of  $F^+$

- use Armstrong's reflexivity rule to logically infer FD's from  $F^+$  and those FD's to  $F^+$

- use Armstrong's Augmentation rule to logically infer FD's from  $F^+$  & add those to  $F^+$

- To each FD pair of the form ~~add FD~~  $\{\alpha \rightarrow \beta, \beta \rightarrow \gamma\}$  in  $F^+$  add

Until  $F^+ = \text{save-}F^+ = F^+$  no more new FD's appear

# Cover of an FD set F

Closure, Cover

A FD set say  $G$  is said to be Cover of FD set  $F$

if

$$F \subseteq G^+$$

$\leftarrow R(A, B, C)$

ex

$$F = \{A \rightarrow B, AB \rightarrow BC\}$$

$$G = \{A \rightarrow B, A \rightarrow C\}$$

ideally  $G^+$

$\alpha \rightarrow \beta$   
 $\alpha \gamma \rightarrow \beta \gamma$

$$A \rightarrow C$$

Augment by B

$$\Rightarrow AB \rightarrow BC$$

now  $A \rightarrow B$  &  $A \rightarrow C$  of  $G$  are in  $F$

hence  $G$  is cover of  $F$

## Equivalent FD Sets

Two set of FD's  $F$  &  $G$  are said to be equivalent if both  $F$  &  $G$  are cover of each other

i.e.  $F \subseteq G^+$  and  $G \subseteq F^+$

therefore  $F^+ = G^+$

hence  $F \equiv G$

$F^+, F_c$

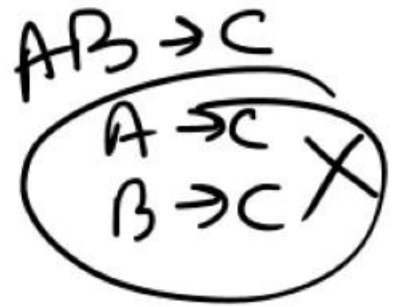
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✓ Canonical cover (minimal cover) of a FD set

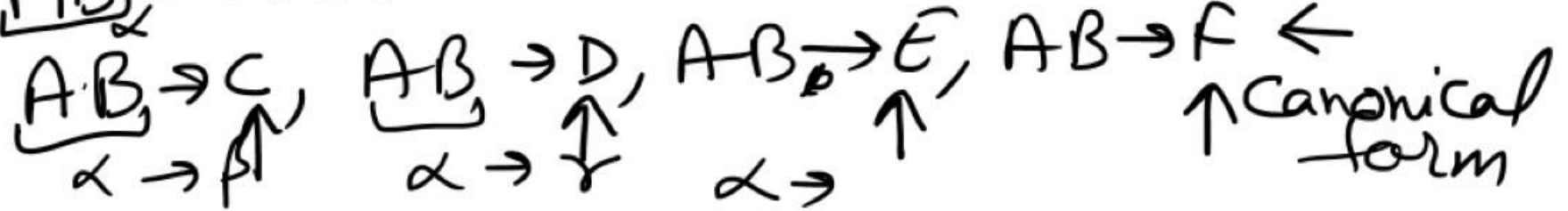
A FD set  $F_c$  is said to be Canonical Cover (minimal cover) of a FD set  $F$  iff  $F_c \equiv F$  and satisfies the

following 3 conditions

(i) each FD in  $F_c$  must be in Canonical form



$AB \rightarrow CDEF$



must have only one att on its right side

(ii) All FD's in  $F_c$  must be in left irreducible form i.e. no FD in  $F_c$  should have any extraneous att.

(iii) NO FD in  $F_c$  should be extraneous.  ~~$AC \rightarrow BC$~~

# Algo to determine Canonical cover / minimal cover

$\mathcal{F}$  a FD Set

(i)  $\underline{\mathcal{F}_c} = \underline{\mathcal{F}}$

(ii) For each FD of the form  $A \rightarrow B_1, B_2, \dots, B_n$  in  $\mathcal{F}_c$  replace by  $n$  FD's  
 $\{A \rightarrow B_1, A \rightarrow B_2, \dots, A \rightarrow B_n\}$

(iii) for each FD  $\alpha \rightarrow \beta \in \mathcal{F}_c$  and for each attr  $A \in \alpha$

if  $\{ \{ \mathcal{F}_c - \{ \alpha \rightarrow \beta \} \} \cup \{ (\alpha - A) \rightarrow \beta \} \}^+ = \mathcal{F}_c^+$   
if that happens then

$$\underline{\mathcal{F}_c} = \{ \mathcal{F}_c - \{ \alpha \rightarrow \beta \} \} \cup \{ (\alpha - A) \rightarrow \beta \}$$

(iv) for each FD  $\alpha \rightarrow \beta \in F_c$   
if  $\{F_c - \{\alpha \rightarrow \beta\}\}^+ = F_c^+$

then  $\underline{F_c} = F_c - \{\alpha \rightarrow \beta\}$ ;

\* A set of FD may have more than one Canonical cover.

~~$F_c = \{ \dots \}$  ,  $F_c = \{ \dots \}$~~

All the canonical covers will be equivalent to one given FD set.

---

closure of an att set  $\alpha$  under FD set  $F$ .

$R(A, B, C, D, E, F, G, H, I)$

$FD = \{ \underline{AB} \rightarrow C, \underline{CD} \rightarrow EF, \underline{F} \rightarrow GHI \}$

$AB \rightarrow ?$

$\{AB\}^+ = AB$

$\underline{\{AB\}^+} \rightarrow \underline{ABC}$

$\{ABD\}^+$

$\{ABD\}^+ = ABD$

$AB \rightarrow C$

$\{ABD\}^+ \rightarrow \underline{ABCD}$

~~$AB \rightarrow C$~~

$CD \rightarrow EK$

$\{ABD\}^+ \rightarrow ABCDEF$

$ABD \rightarrow F, F \rightarrow \underline{GHI}$

$ABD \rightarrow GHI$

$\{ABD\}^+ \rightarrow \underline{ABCDEFGHI}$

$\{ABD\}^+ \rightarrow R$

$\{ABD\}^+ \rightarrow \underline{CK}$

$\{CD\}^+$

$\{CD\}^+ \rightarrow \underline{CD}$

$CD \rightarrow EF$

$CD \rightarrow CDEF$

$\{CD\}^+ \rightarrow \underline{CDEFGHI}$



Algo to determine closure  $d_b$  ( $\alpha \subseteq R$ ) under  
FD set  $f$  on  $R$ .

---

$\alpha^+ := \alpha$  by reflexivity rule

Repeat

save\_ $\alpha^+ := \alpha^+$

For each FD  $\underline{\gamma} \rightarrow \underline{\beta}$  in  $f^+$

~~if each FD~~  
if  $\gamma \subseteq \alpha^+$  ( $\therefore \alpha^+ \rightarrow \beta$ ) then

$\alpha^+ := \alpha^+ \cup \beta$

Until (save\_ $\alpha^+ = \alpha^+$ )

no change is made in  
the last iteration  
save\_ $\alpha^+$  is closure  $d_b$   $\alpha$ .

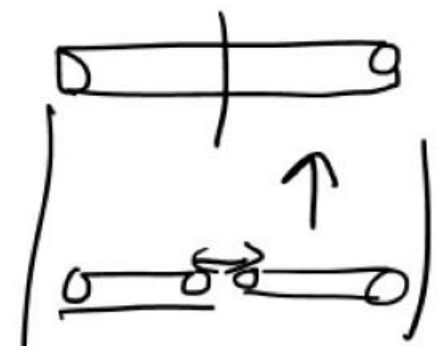
# Lossless join decomposition

ex

R

A	B	C
a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	b <sub>1</sub>	c <sub>2</sub>
a <sub>3</sub>	b <sub>2</sub>	c <sub>1</sub>

$$FD = \{ \overset{\checkmark}{A \rightarrow B}, \overset{\checkmark}{A \rightarrow C}, \overset{\times}{A \rightarrow BC} \}$$



$$R_1 \cap R_2 = A \quad R_1 \cap R_2 \rightarrow R_1 \quad A \rightarrow R_1$$

$$R_1 \cap R_2 \rightarrow R_2 \quad A \rightarrow R_2$$

decomposition I

$$R_1 \text{ CK } \underline{\underline{=}} \underline{\underline{A}}$$

$\checkmark$ A	B
a <sub>1</sub>	b <sub>1</sub>
a <sub>2</sub>	b <sub>1</sub>
a <sub>3</sub>	b <sub>2</sub>

$A \rightarrow B$

$\checkmark$ A	C
a <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	c <sub>2</sub>
a <sub>3</sub>	c <sub>1</sub>

$A \rightarrow C$

$$R_1 \times R_2$$

$$A \rightarrow R_1$$

$$A \rightarrow \underline{\underline{AC}}$$

Lossless

R'

A	B	C
a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	b <sub>1</sub>	c <sub>2</sub>
a <sub>3</sub>	b <sub>2</sub>	c <sub>1</sub>

join decomposition

decomposition  $\Pi + R$

$R_1$

A	B
$a_1$	$b_1$
$a_2$	$b_1$
$a_3$	$b_2$

$R_2$  CK = (BC)

B	C
$b_1$	$c_1$
$b_1$	$c_2$
$b_2$	$c_1$

BC  $\rightarrow$  B  
 (BC)  $\rightarrow$  C  
 Prime att  
 $R_1 \bowtie R_2$

$R'$

A	B	C
$a_1$	$b_1$	$c_1$
$a_1$	$b_1$	$c_2$
$a_2$	$b_1$	$c_1$
$a_2$	$b_1$	$c_2$
$a_3$	$b_2$	$c_1$

Spurious tuples

$A \rightarrow B$  Lossy join decomposition

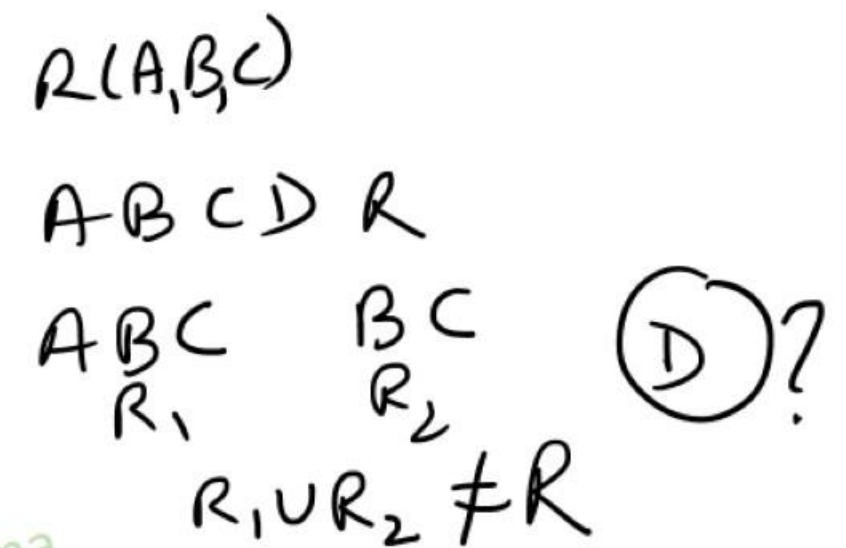
$R_1 \cap R_2 = (B) \rightarrow \times CK$

$R_1 \cap R_2 \rightarrow R_1$   
 or  $R_1 \cap R_2 \rightarrow R_2$

The decomposition  $\{R_1, R_2\}$  of schema  $R$

where  $\underline{R_1} \cup \underline{R_2} = R$   
 $\downarrow \quad \downarrow$   
 $A, B \quad A, C = A, B, C$

is called lossless if



$\underline{R_1 \cap R_2} \rightarrow \underline{R_1}$   
 or

Common  $R_1 \cap R_2 \rightarrow R_2$

out  
between  
 $R_1, R_2$

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When a schema is decomposed into  $R_1$  &  $R_2$  for the purpose of Normalization. It must satisfy the following conditions

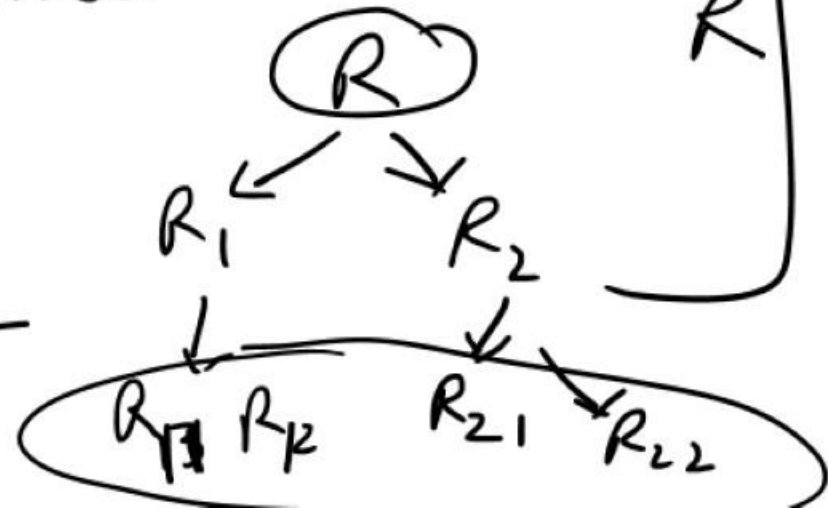
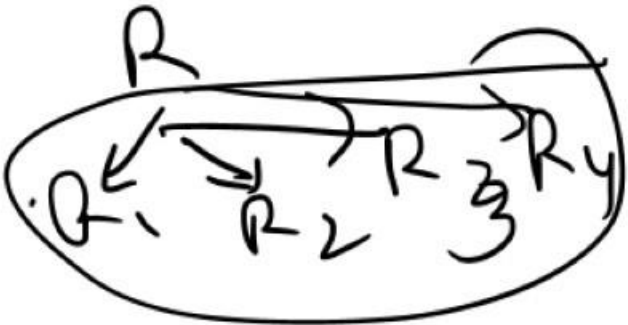
(i) Preservation of att's — mandatory  
 if decompose  $R$  into  $R_1, R_2$   
 then  $R_1 \cup R_2 = R$

(ii) The decomposition must be lossless

$$R_1 \cap R_2 \rightarrow R_1$$

$$\text{or}$$

$$R_1 \cap R_2 \rightarrow R_2$$



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## Desirable conditions

- (i) All FD's holding on  $R$  should preferably be preserved in the decomposition
- (ii) The subschemas obtained after decomposition must be in the desired Normal form.

## Dependency Preserving decomposition

Suppose  $\ast(R_1, R_2, R_3, \dots, R_n)$  is a decomposition of  $R$  s.t.  $(R_1 \cup R_2 \cup R_3 \cup \dots \cup R_n) = R$ . Suppose  $F$  is the set of FD's holding on  $R$ .

Let  $F' = \bigcup_{i=1}^{i=n} F_i$  if  $\{F'\}^+ = F^+$  then the

decomposition  $\ast(R_1, R_2, R_3, \dots, R_n)$  is called dependency

Preserving else it is not. We want it as desirable but not mandatory.

ex  $R(A, B, c)$  lossless

$F = \{A \rightarrow B, B \rightarrow c\}$

$R_1(A, B)$   
 $A \rightarrow B$   
 $R_2(B, c)$   
 $B \rightarrow c$

So dependency is preserved

not dependency preserving

ex  $R(A, B, c)$  CK = A

$F = \{A \rightarrow B, B \rightarrow c\}$  lost

$R_1(A, B)$   
 $A \rightarrow B$   
 $CK = A$   
lossless  
 $R_2(A, c)$   
 $CK = A$   
 $A \rightarrow c$   
 $B \rightarrow c$  lost

$A \rightarrow B$   
 $B \rightarrow c$   
 $A \rightarrow c$

$F' = \{A \rightarrow B, A \rightarrow c\}$

$[F']^+ = A \rightarrow B, A \rightarrow c$   
 ~~$A \rightarrow A$~~   
 $B \rightarrow B$   
 $c \rightarrow c$

$[F]^+ =$   
 $A \rightarrow B$   
 $B \rightarrow c$   
 $A \rightarrow c$   
 $A \rightarrow A$   
 $B \rightarrow B$   
 $c \rightarrow c$   
 ~~$A \rightarrow c$~~

$$[F']^+ \subsetneq F^+$$

## Normalization

1NF - first normal form.

A schema  $R$  is said to be in 1NF iff domain of each attribute of  $R$  is atomic  
i.e. each att must be simple, single valued att



## FULL FD

If a nonkey attr  $A \in R$  can only be determined by the FULL CK of  $R$  (and not part of CK) then that FD is called FULL FD

$R(A, B, C, D, E)$

CK<sub>1</sub> = {A, B}  
CK<sub>2</sub> = C

Key attr = A, B, C

nonkey = D, E

FD

$B \rightarrow \textcircled{D}$

Part of CK

not FULL FD

$\textcircled{D}$  is being determined by part of CK.

$\left. \begin{array}{l} \textcircled{AB} \rightarrow \textcircled{D} \\ \textcircled{C} \rightarrow \textcircled{D} \end{array} \right\} \text{ - FULL FD}$

## Partial FD

if a nonkey attr  $A \in R$  can be determined by part of CK of  $R$  then it is called Partial FD

---

## 2NF

- A schema  $R$  is said to be in 2NF if
    - it is in 1NF
    - no nonkey attr of  $R$  has Partial FD on its CK's.
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## Heath's theorem

If schema  $R(\alpha, \beta, \gamma)$  has FD  $\alpha \rightarrow \beta$  holding on it then it can be losslessly decomposed into  $R_1(\alpha, \beta)$ ,  $R_2(\alpha, \gamma)$ .

$$\underbrace{R_1(\alpha, \beta)}_{\alpha \rightarrow \beta}, \underbrace{R_2(\alpha, \gamma)}$$

CK -  $\alpha$

$$R_1 \cap R_2 \rightarrow R_1$$

$$\text{or } R_1 \cap R_2 \rightarrow R_2$$

comm

$$R_1 \cap R_2 = \alpha$$

$$\alpha \rightarrow R_1$$

✓ INF ✓, 2NF X because of Partial FD  
SPO (S#, Sname, Scity, Status, P#, Pname, Price, Qty)

F = { S# → Sname, Scity, ~~status~~  
 Scity → Status

→ P# → Pname, Price  
 ✓ (S#, P#) → Qty

CK = (S#, P#)

α → β

α γ → β γ

S# → Sname, Scity, Status  
 ↑                    ↑

(S# P#) → Sname, Scity, — — —, Qty  
 S# → Sname, Scity, Status — violation

S# → Sname, Scity, Status

P# → Pname, Price

(S#, P#) → Qty

(S#, P#) → Sname, Scity, Status, Qty

(S#, P#) → Sname, Scity, Status, Qty, Pname, Price

(S#, P#) → S#, P#

violation CK

# Anomalies

Insertion Anomaly

Deletion Anomaly

Update Anomaly

---

## decomposition

because SPO is not in 2NF, to have schema in 2NF we will have to decompose SPO

we will use Heath's theorem.

$SPO(S\#, Sname, Scity, Status, P\#, Pname, Price, Qty)$

FD -  $S\# \rightarrow Sname, Scity, Status$   
 $P\# \rightarrow Pname, Price.$

$R(\alpha, \beta, \gamma) \quad FD \quad \alpha \rightarrow \beta$   
 $R_1(\alpha, \beta) \quad R_2(\alpha, \gamma) \leftarrow$  lossless  
 $\alpha \rightarrow \beta$

$R_1(S\#, Sname, Scity, Status)$   
 $CK = S\#$   
 FD  $(S\# \rightarrow Sname, Scity, Status)$   
 key - S#  
 is R<sub>1</sub> 2NF → nonkey - Sname, Scity, Status  
 Yes

$R_2(S\#, P\#, Pname, Price, Qty)$

FD  $\left\{ \begin{array}{l} P\# \rightarrow Pname, Price \\ (S\#, P\#) \rightarrow Qty \end{array} \right.$

CK = ?  $(S\#, P\#) \rightarrow Pname, Price, Qty$

$CK(S\#, P\#) \rightarrow R_2 \rightarrow S\#, P\#, Pname, Price, Qty$

$R_2(\underbrace{S\#}_1, \underbrace{P\#}_2, \underbrace{Pname}_3, \underbrace{Price}_4, \underbrace{Qty}_5)$

$FD = \{ \underbrace{P\# \rightarrow Pname, Price}_{\alpha} \}$   $CK = (\underline{S\#}, \underline{P\#})$

$\rightarrow \underbrace{(S\#, P\#) \rightarrow Qty}_{\beta}$

is  $R_2$  in 1NF yes

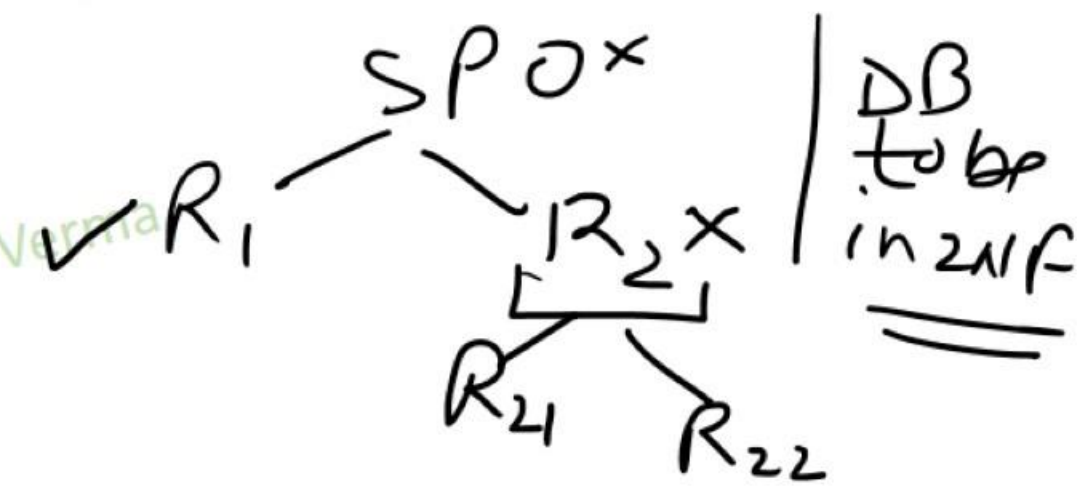
is  $R_2$  in 2NF — NO

Why

key att —  $S\#, P\#$   
 nonatt —  $Pname, Price, Qty$

is  $Pname$  fully FD on CK — NO  
 "  $Price$  " " " " — NO  
 "  $Qty$  " " " " — YES

$R_2$  is NOT in 2NF



$R_{21} (P\#, Pname, Price)$

violating 2NF.  
 $\rightarrow P\# \rightarrow Pname, Price$   
 $\alpha \quad \beta$

$R_{21}(\alpha, \beta)$   
 $R_{22}(\alpha, \gamma)$

FD =  $\{P\# \rightarrow Pname, Price\}$   
2NF — yes

CK —  $(P\#)$  at key  $(P\#)$  nonkey — ~~Pname~~ Price

$R_{22} (P\#, S\#, Qty)$

FD =  $\{(S\#, P\#) \rightarrow Qty\}$

$R_{22}$  CK —  $(S\#, P\#) \rightarrow Qty$

fully key —  $(S\#, P\#)$  at nonkey —  $(Qty)$

3NF yes

$(S\#, P\#) \rightarrow S\#, P\#, Qty$  CK

CK  $(S\#, P\#) \rightarrow R_{22}$



DB will have.

$R_1 \rightarrow (S\#, Sname, Scity, Status)$

$R_{21} \rightarrow (P\#, Pname, Price)$

$R_{22} \rightarrow (S\#, P\#, Qty)$

2NF

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