Lecture - 14

Prob. 1 : Obtain DFT of a sequence

$$x(n) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0 \right)$$

Using decimation in frequency FFT algorithm.

Soln. : The total flow graph is shown in Fig. G-18.

Here g (n) is output of first stage.

h (n) is output of second stage.

The values of twiddle factor are as follows :

$$W_8^0 = e^0 = 1$$

$$W_8^1 = e^{-\frac{j\pi}{4}} = 0.707 - j\,0.707$$

$$W_8^2 = e^{\frac{-j\pi}{2}} = -j$$

$$W_8^3 = -0.707 - j\,0.707$$

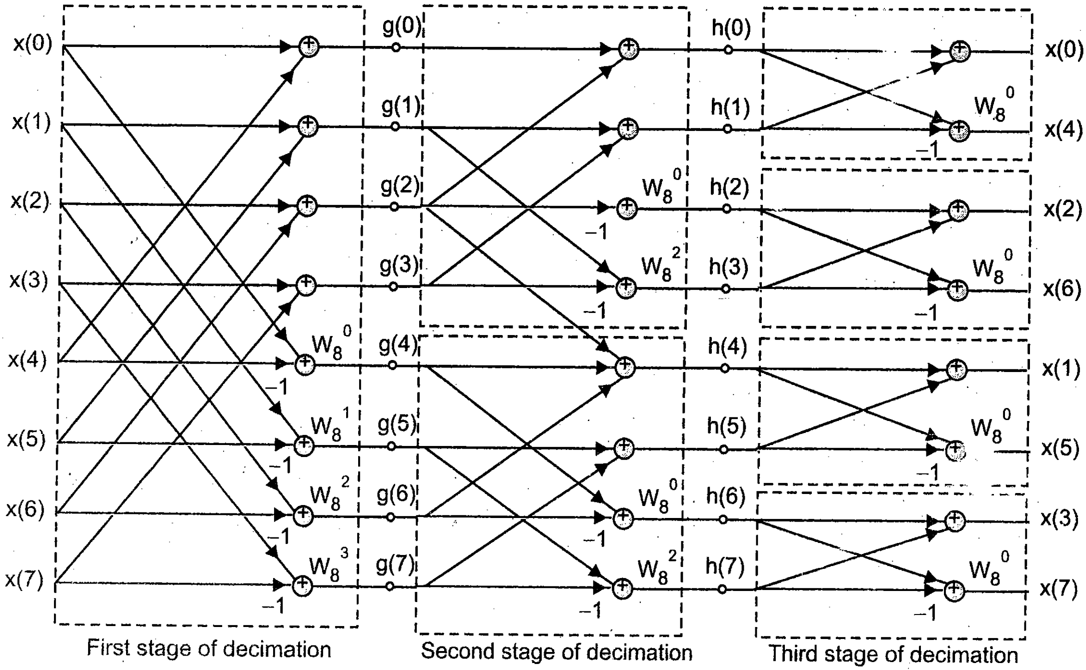

Fig. G-18

Output of stage - 1 :

$$g(0) = x(0)+x(4) = \frac{1}{2}+0 = 0.5$$

$$g(1) = x(1)+x(5) = \frac{1}{2}+0 = 0.5$$

$$g(2) = x(2)+x(6) = \frac{1}{2}+0 = 0.5$$

$$g(3) = x(3)+x(7) = \frac{1}{2}+0 = 0.5$$

$$g(4) = [x(0)-x(4)]W_8^0 = \left[\frac{1}{2}-0\right]1 = 0.5$$

$$g(5) = [x(1)-x(5)]W_8^1 = \left[\frac{1}{2}-0\right](0.707-j\,0.707)$$

$$= 0.3535-j\,0.3535$$

$$g(6) = [x(2)-x(6)]W_8^2 = \left[\frac{1}{2}-0\right](-j) = -j\,0.5$$

$$g(7) = [x(3)-x(7)]W_8^3 = \left[\frac{1}{2}-0\right](-0.707-j\,0.707)$$

$$= -0.3535-j\,0.3535$$

Output of stage - 2 :

$$h(0) = g(0)+g(2) = 0.5 + 0.5 = 1$$

$$h(1) = [g(1)+g(3)] = (0.5+0.5) = 1$$

$$h(2) = [g(0)-g(2)]W_8^0 = (0.5-0.5)(+1) = 0$$

$$h(3) = [g(1)-g(3)]W_8^2 = (0.5-0.5)(-j) = 0$$

$$h(4) = g(4)+g(6) = 0.5-j\,0.5$$

$$h(5) = g(5)+g(7) = 0.3535-j\,0.3535-0.3535-j\,0.3535$$

$$\therefore \quad h(5) = -j\,0.707$$

$$h(6) = [g(4)-g(6)]W_8^0 = [0.5+j\,0.5]\,1 = 0.5+j\,0.5$$

$$h(7) = [g(5)-g(7)]W_8^2 = [0.3535-j\,0.3535+0.3535+j\,0.3535](-j)$$

$$\therefore \quad h(7) = -j\,0.707$$

Final output :

$$X(0) = h(0)+h(1) = 1+1 = 2$$

$$X(1) = h(4)+h(5) = 0.5-j\,0.5-j\,0.707 = 0.5-j\,1.207$$

$$X(2) = h(2)+h(3) = 0+0 = 0$$

$$X(3) = [h(6)+h(7)]W_8^0 = [0.5+j\,0.5-j\,0.707]\cdot 1 = 0.5-j\,0.207$$

$$X(4) = [h(0)-h(1)]W_8^0 = [1-1]\cdot 1 = 0$$

$$X(5) = [h(4)-h(5)]W_8^0 = [(0.5-j\,0.5)+j\,0.707]\cdot 1$$

$$= 0.5+j\,0.207$$

$$X(6) = [h(2)-h(3)]W_8^0 = 0$$

$$X(7) = [h(6)-h(7)]W_8^0 = [0.5+j\,0.5+j\,0.707]$$

$$= 0.5+j\,1.21$$

$$\therefore \quad X(k) = \{X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)\}$$

$$\therefore \quad X(k) = \{2, 0.5-j\,1.207, 0, 0.5-j\,0.207, 0, 0.5+j\,0.207, 0, 0.5+j\,1.21\}$$

Prob. 2 : Using DIFFFT Find

(i) DFT $X_1(k)$ of following sequence $x_1(n) = \{1, 2, -1, 2, 4, 2, -1, 2\}$

(ii) If $x_2(n) = x_1(-n)$ without performing FFT find $X_2(k)$.

Soln. : The total flow graph is shown in Fig. G-19(a).

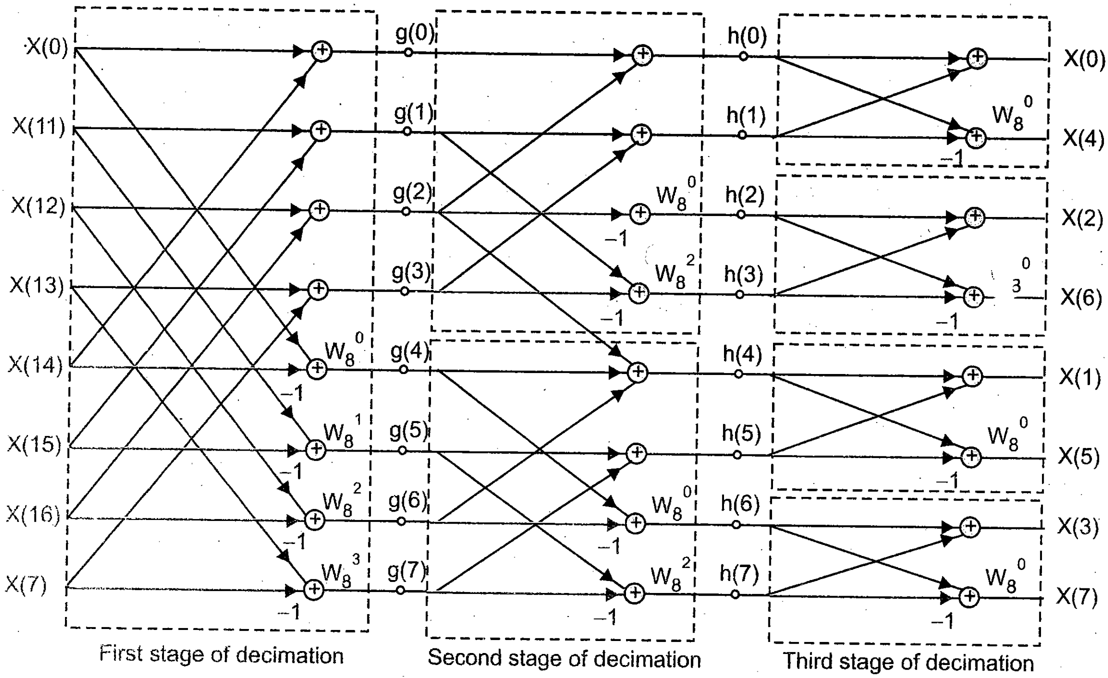

Fig. G-19(a)

The values of twiddle factor are as follows :

$$W_8^0 = 1$$

$$W_8^1 = 0.707 - j\,0.707$$

$$W_8^2 = -j$$

$$W_8^3 = -0.707 - j\,0.707$$

Here, $x(0) = 1$, $x(1) = 2$, $x(2) = -1$, $x(3) = 2$, $x(4) = 4$, $x(5) = 2$, $x(6) = -1$, $x(7) = 2$

Output of stage - 1 :

$$g(0) = x(0) + x(4) = 1 + 4 = 5$$

$$g(1) = x(1) + x(5) = 2 + 2 = 4$$

$$g(2) = x(2) + x(6) = -1 - 1 = -2$$

$$g(3) = x(3)+x(7) = 2+2 = 4$$

$$g(4) = [x(0)-x(4)]W_8^1 = [1-4]\times 1 = -3$$

$$g(5) = [x(1)-x(5)]W_8^1 = (2-2)W_8^1 = 0$$

$$g(6) = [x(2)-x(6)]W_8^2 = (-1+1)W_8^2 = 0$$

$$g(7) = [x(3)-x(7)]W_8^3 = (2-2)W_8^3 = 0$$

Output of stage - 2 :

$$h(0) = g(0)+g(2) = 5-2 = 3$$

$$h(1) = g(1)+g(3) = 4+4 = 8$$

$$h(2) = [g(0)-g(2)]W_8^0 = 5+2 = 7$$

$$h(3) = [g(1)-g(3)]W_8^2 = (4-4)(-j) = 0$$

$$h(4) = g(4)+g(6) = -3$$

$$h(5) = g(5)+g(7) = 0+0 = 0$$

$$h(6) = [g(4)-g(6)]W_8^0 = -3$$

$$h(7) - [g(5)-g(7)]W_8^2 = 0$$

Final output :

$$X(0) = h(0)+h(1) = 3+8 = 11$$

$$X(1) = h(4)+h(5) = -3+0 = -3$$

$$X(2) = h(2)+h(3) = 7+0 = 7$$

$$X(3) = [h(6)+h(7)]W_8^0 = -3$$

$$X(4) = [h(0)-h(1)]W_8^0 = 3-8 = -5$$

$$X(5) = [h(4)-h(5)]W_8^0 = -3$$

$$X(6) = [h(2)-h(3)]W_8^0 = 7$$

$$X(7) = [h(6)-h(7)]W_8^0 = -3$$

$$\therefore \quad X(k) = \{X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)\}$$

$$\therefore \quad X(k) = \{11, -3, 7, -3, -5, -3, 7, -3\}$$

(ii) Given $x_2(n) = x_1(-n)$

According to time reversal property,

$$x((-n))_N \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X((-k))_N$$

Here $x((-n))_N$ represents circular folding of $x(n)$ and $X((-k))_N$ represents circular folding of $X(k)$. Thus $X_2(k)$ is obtained by circularly folding $X_1(k)$ as shown in Fig. G-19(b).

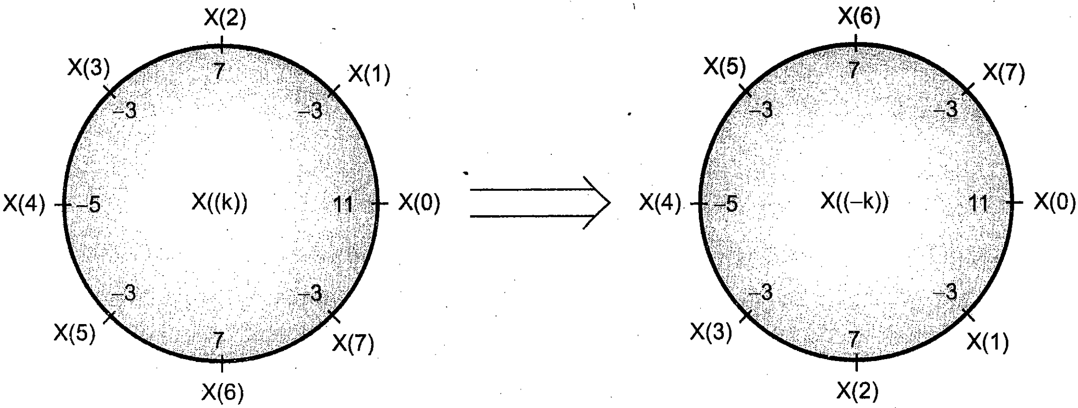

Fig. G-19(b)

Thus in this case $X_2(k)$ is same as $X_1(k)$.

1.5 Computation of Inverse DFT (IDFT) using FFT Algorithms :

We have studied Radix-2 DIT and DIF FFT algorithms to compute the DFT, $X(k)$. The same algorithm can be used to obtain input sequence $x(n)$ from its DFT. That means to compute IDFT. Recall the definition of IDFT,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}, \quad n = 0, 1, \ldots N-1 \qquad \text{..(1)}$$

Just for comparison we will write the definition of DFT.

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{+kn}, \quad k = 0, 1, \ldots N-1 \qquad \text{...(2)}$$

Thus IDFT differs from DFT by,

(a) Multiplication by $\frac{1}{N}$ factor.

(b) Negative sign of imaginary part osf (W_N)

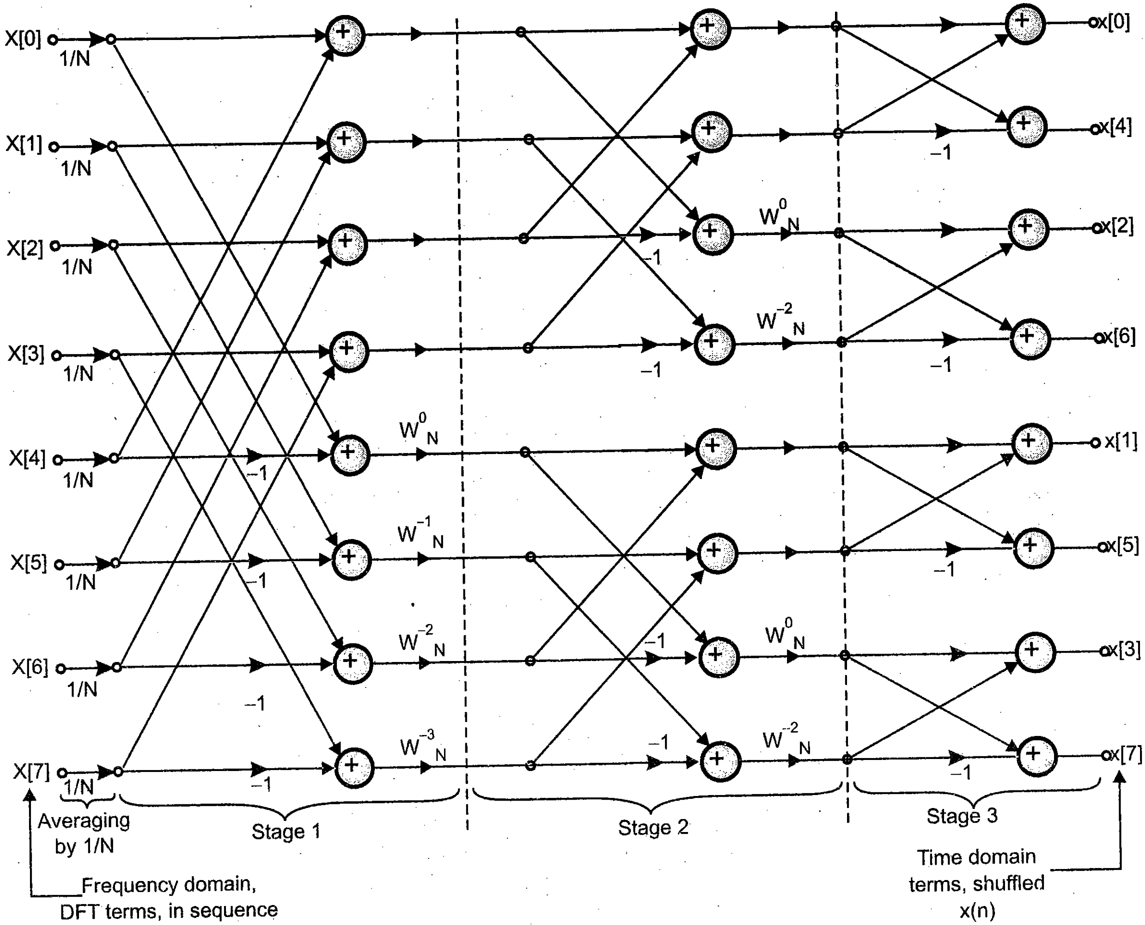

Fig. G-20 : Computation of IDFT using FFT

Thus we can use the same algorithm to compute IDFT; but we have to change the sign of twiddle factor and for DIF FFT algorithm we have to multiply input sequence $X(k)$ by $\frac{1}{N}$. The total flow graph is shown in Fig. G-20.

Prob. 1 : Using FFT and IFFT, find the output of system if input $x(n)$ and impulse response $h(n)$ are given by,

$x(n) = \{2, 2, 4\}$

$h(n) = \{1, 1\}$

Soln. : If the given sequences are very long then we have to divide such sequences into smaller segments. In this case it is not necessary to divide the sequences.

Given $x(n) = \{2, 2, 4\}$ and $h(n) = \{1, 1\}$

Here L = Number of samples in $x(n) = 3$

M = Number of samples in $h(n) = 2$

$\therefore \quad L + M - 1 = 3 + 2 - 1 = 4$

That means we have to make length of x (n) and h (n) equal to 4.

$$\therefore \quad x(n) = \{2, 2, 4, 0\} \text{ and } h(n) = \{1, 1, 0, 0\}$$

First we will obtain DFT of x (n) using DITFFT algorithms. We can also use DIFFFT algorithm.

The calculations are shown in Fig. G-21(a)

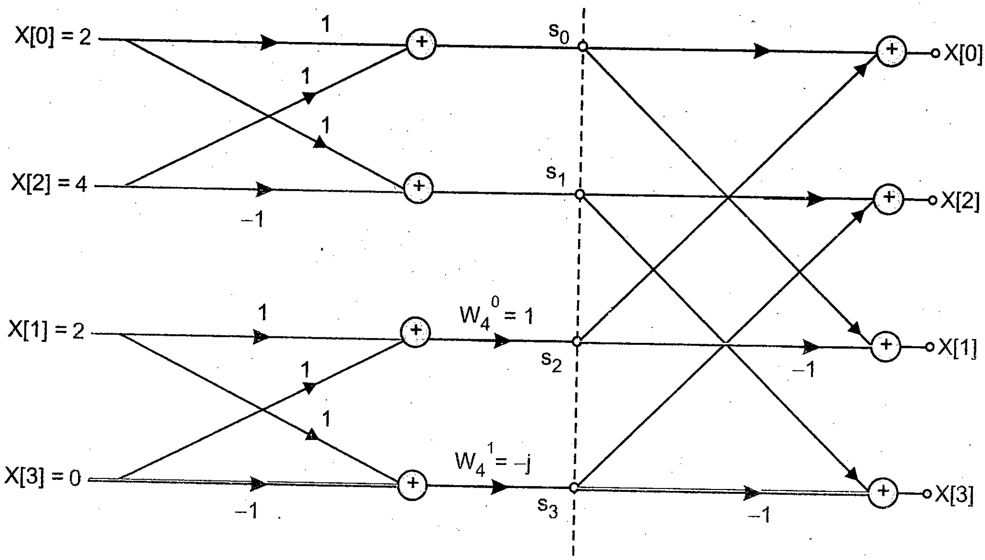

Fig. G-21(a)

Here $s_0 = x(0) + x(2) = 2 + 4 = 6$

$$s_1 = x(0) - x(2) = 2 - 4 = -2$$

$$s_2 = [x(1) + x(3)]\, W_4^0 = 2 + 0 = 2$$

$$s_3 = [x(1) - x(3)]\, W_4^1 = (2 - 0)(-j) = -j2$$

The final output is,

$$X(0) = s_0 + s_2 = 6 + 2 = 8$$

$$X(1) = s_1 + s_3 = -2 - j2$$

$$X(2) = s_0 - s_2 = 6 - 2 = 4$$

$$X(3) = s_1 - s_3 = -2 + j2$$

$$\therefore \quad X(k) = \{8, -2 - j2, 4, -2 + j2\}$$

Now we will obtain DFT of h (n) using DITFFT as shown in Fig. G-21(b).

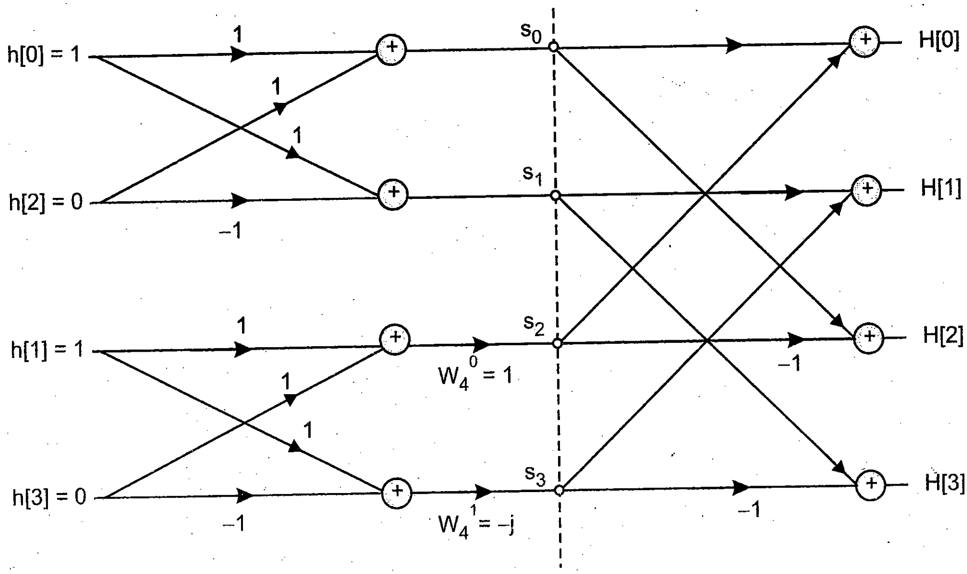

Fig. G-21(b)

$$s_0 = h(0)+h(2) = 1 + 0 = 1$$

$$s_1 = h(0)-h(2) = 1-0 = 1$$

$$s_2 = [h(1)+h(3)]\,W_4^0 = 1 + 0 = 1$$

$$s_3 = [h(1)-h(3)]\,W_4^1 = (1-0)-j = -j$$

The final output is,

$$H(0) = s_0+s_2 = 1 + 1 = 2$$

$$H(1) = s_1+s_3 = 1-j$$

$$H(2) = s_0-s_2 = 1-1 = 0$$

$$H(3) = s_1-s_3 = 1+j$$

$$\therefore \quad H(k) = \{2,\ 1-j,\ 0,\ 1+j\}$$

Now we will multiply H (k) and X (k).

$$\text{Let} \quad Y(k) = X(k)\cdot H(k)$$

$$\therefore \quad Y(k) = \{8,\ -2-j2,\ 4,\ -2+j2\}\cdot\{2,\ 1-j,\ 0,\ 1+j\}$$

$$\therefore \quad Y(k) = \{16,\ -4,\ 0,\ -4\}$$

Now we will perform IFFT to obtain sequence y (n). For this we have to multiply each input by $\frac{1}{N}$ that means $\frac{1}{4}$ and we have to change the sign of imaginary part of twiddle factor. This computation is shown in Fig. G-21(c)

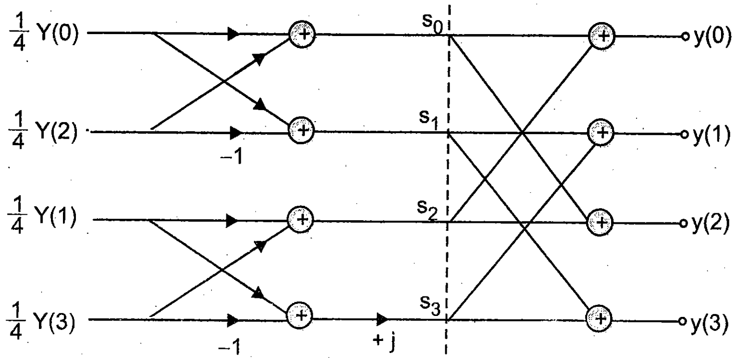

Fig. G-21(c)

$$s_0 = \frac{1}{4}Y(0)+\frac{1}{4}Y(2) = \frac{1}{4}(16)+\frac{1}{4}(0) = 4$$

$$s_1 = \frac{1}{4}Y(0)-\frac{1}{4}Y(2) = \frac{1}{4}(16)-\frac{1}{4}(0) = 4$$

$$s_2 = \left[\frac{1}{4}Y(1)+\frac{1}{4}Y(3)\right]\cdot 1 = \frac{1}{4}(-4)+\frac{1}{4}(-4) = -2$$

$$s_3 = \left[\frac{1}{4}Y(1)-\frac{1}{4}Y(3)\right](-j) = \left[\frac{1}{4}(-4)-\frac{1}{4}(-4)\right](-j) = 0$$

The final output is,

$$y(0) = s_0+s_2 = 4-2 = 2$$

$$y(1) = s_1+s_3 = 4+0 = 4$$

$$y(2) = s_0-s_2 = 4+2 = 6$$

$$y(3) = s_1-s_3 = 4-0 = 4$$

$$\therefore\ y(n) = \{2, 4, 6, 4\}$$

Prob. 2 : Let x (n) be 8 point sequence. Its corresponding DFT X (k) is

$X(k) = \{(0.5), (2+j), (3+j2), (j), (3), (-j), (3-j2), (2-j)\}$

Find x (n) by performing IFFT.

Soln. : We will use inverse DIF FFT algorithm. The flow graph is shown in Fig. G-22.

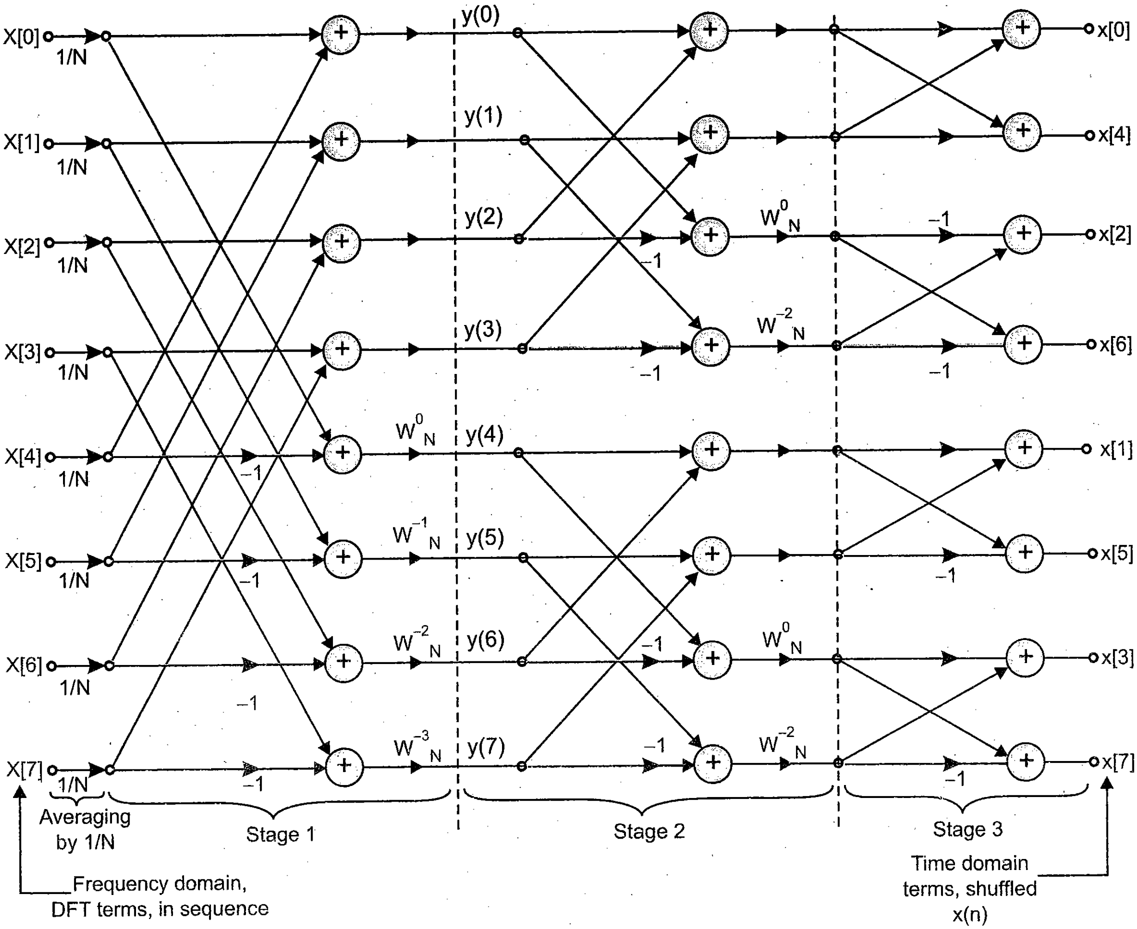

Fig. G-22

Let g (n) represent output of first stage and h (n) represent output of second stage. By changing sign of imaginary part of twiddle factor the values of twiddle factor are as follows :

$$W_8^0 = 1$$

$$W_8^1 = 0.707 + j\,0.707$$

$$W_8^2 = j$$

$$W_8^3 = -0.707 + j\,0.707$$

Output of first stage :

$$g(0) = \frac{1}{8}X(0)+\frac{1}{8}X(4) = \frac{1}{8}(0.5)+\frac{1}{8}(3) = 0.44$$

$$g(1) = \frac{1}{8}X(1)+\frac{1}{8}X(5) = \frac{1}{8}(2+j)+\frac{1}{8}(-j) = 0.25$$

$$g(2) = \frac{1}{8}X(2)+\frac{1}{8}X(6) = \frac{1}{8}(3+j2)+\frac{1}{8}(3-j2) = 0.75$$

$$g(3) = \frac{1}{8}X(3)+\frac{1}{8}X(7) = \frac{1}{8}(j)+\frac{1}{8}(2-j) = 0.25$$

$$g(4) = \left[\frac{1}{8}X(0)-\frac{1}{8}X(4)\right](+W_8^0) = \left[\frac{1}{8}(0.5)-\frac{1}{8}(3)\right]\cdot 1 = -0.31$$

$$g(5) = \left[\frac{1}{8}X(1)-\frac{1}{8}X(5)\right](W_8^1) = \left[\frac{1}{8}(2+j)-\frac{1}{8}(-j)\right](+0.707+j\,0.707)$$

$$= (0.25+j\,0.25)(+0.707+j\,0.707) = j\,0.35$$

$$g(6) = \left[\frac{1}{8}X(2)-\frac{1}{8}X(6)\right]W_8^2 = \left[\frac{1}{8}\cdot(3+j2)-\frac{1}{8}(3-j2)\right]j$$

$$= -0.5$$

$$g(7) = \left[\frac{1}{8}X(3)-\frac{1}{8}X(7)\right]W_8^3 = \left[\frac{1}{8}(j)-\frac{1}{8}(2-j)\right](-0.707+j\,0.707)$$

$$= \left(\frac{1}{4}j-\frac{1}{4}\right)(-0.707+j\,0.707) = -j\,0.35$$

Output of second stage :

$$h(0) = g(0)+g(2) = 0.44 + 0.75 = 1.19$$

$$h(1) = g(1)+g(3) = 0.25 + 0.25 = 0.5$$

$$h(2) = [g(0)-g(2)]W_8^0 = 0.44-0.75 = -0.31$$

$$h(3) = [g(1)-g(3)]W_8^2 = 0$$

$$h(4) = g(4)+g(6) = -0.31-0.5 = -0.81$$

$$h(5) = g(5)+g(7) = j\,0.35-j\,0.35 = 0$$

$$h(6) = [g(4)-g(6)]W_8^0 = -0.31+0.5 = 0.19$$

$$h(7) = [g(5)-g(7)]W_8^2 = [j\,0.35+j\,0.35]j = -0.7$$

Final output :

$$x(0) = h(0)+h(1) = 1.19 + 0.5 = 1.69$$

$$x(1) = h(4)+h(5) = -0.81$$

$$x(2) = h(2)+h(3) = -0.31$$

$$x(3) = [h(6)+h(7)]W_8^0 = (0.19-0.7)\cdot 1 = -0.51$$

$$x(4) = [h(0)-h(1)]\cdot W_8^0 = (1.19-0.5)\cdot 1 = 0.69$$

$$x(5) = [h(4)-h(5)]W_8^0 = -0.81$$

$$x(6) = [h(2)-h(3)]W_8^0 = -0.31$$

$$x(7) = [h(6)-h(7)]W_8^0 = (0.19+0.7)\cdot 1 = 0.89$$

Now sequence $x(n)$ is,

$$x(n) = \{x(0), x(1), x(2), x(3), x(4), x(5), x(6), x(7)\}$$

$$\therefore\ x(n) = \{1.69, -0.81, -0.31, -0.51, 0.69, -0.81, -0.31, 0.89\}$$