The procedure for design is one of trial and error. For example, as a first trial, heat losses might be neglected and trays calculated with fixed difference points, and after the size of the resulting column has been determined, the first estimate of heat losses for the two column sections can be made by the usual methods of heat-transfer calculations. The heat losses can then be apportioned among the trays and the number of trays redetermined with the appropriate difference points. This leads to a second approximation of the heat loss, and so forth.

As Fig. 9.34 shows, heat losses increase the internal-reflux ratio, and for a given condenser heat load, fewer trays for a given separation are required (recall that the higher the enriching-section difference point and the lower the stripping-section difference point, the fewer the trays). However, the reboiler must provide not only the heat removed in the condenser but also the heat losses. Consequently, for the same reboiler heat load as shown on Fig. 9.34 but with complete insulation against heat loss, all the heat would be removed in the condenser, all the stripping trays would use point J, and all the enriching trays would use point K as their respective difference points. It therefore follows that, for a given reboiler heat losses are eliminated. For this reason, fractionators are usually well insulated.

High-Purity Products and Tray Efficiencies

Methods of dealing with these problems are considered following the McCabe-Thiele method and are equally applicable to Ponchon-Savarit calculations.

MULTISTAGE TRAY TOWERS—METHOD OF McCABE AND THIELE

This method, although less rigorous than that of Ponchon and Savarit, is nevertheless most useful since it does not require detailed enthalpy data. If such data must be approximated from fragmentary information, much of the exactness of the Ponchon-Savarit method is lost in any case. Except where heat losses or heats of solution are unusually large, the McCabe-Thiele method will be found adequate for most purposes. It hinges upon the fact that, as an approximation, the operating lines on the xy diagram can be considered straight for each section of a fractionator between points of addition or withdrawal of streams.

Equimolal Overflow and Vaporization

Consider the enriching section of the fractionator of Fig. 9.17. In the absence of heat losses, which can be (and usually are) made very small by thermal insulation for reasons of economy if for no other, Eq. (9.61) can be written

$$\frac{L_n}{G_{n+1}} = 1 - \frac{H_{G_{n+1}} - H_{L_n}}{Q' - H_{L_n}}$$
(9.108)

where Q' includes the condenser heat load and the enthalpy of the distillate, per mole of distillate. The liquid enthalpy H_{L_n} is ordinarily small in comparison with Q' since the condenser heat load must include the latent heat of condensation of at least the reflux liquid. If then $H_{G_{n+1}} - H_{L_n}$ is substantially constant, L_n/G_{n+1} will be constant also for a given fractionation [48]. From Eq. (9.11)

$$H_{G_{n+1}} = [y_{n+1}C_{L,A}M_A + (1 - y_{n+1})C_{L,B}M_B](t_{n+1} - t_0) + y_{n+1}\lambda_AM_A + (1 - y_{n+1})\lambda_BM_B$$
(9.109)

where t_{n+1} is the temperature of the vapor from tray n + 1 and the λ 's are the latent heats of vaporization at this temperature. If the deviation from ideality of liquid solutions is not great, the first term in brackets of Eq. (9.109) is

$$y_{n+1}C_{L,A}M_A + (1 - y_{n+1})C_{L,B}M_B \approx C_L M_{av}$$
 (9.110)

From Eq. (9.10),

$$H_{L_{n}} = C_{L}(t_{n} - t_{0})M_{av} + \Delta H_{S}$$
(9.111)

and
$$H_{G_{n+1}} - H_{L_n} = C_L M_{av} (t_{n+1} - t_n) + y_{n+1} \lambda_A M_A + (1 - y_{n+1}) \lambda_B M_B - \Delta H_S$$
 (9.112)

For all but unusual cases, the only important terms of Eq. (9.112) are those containing the latent heats. The temperature change between adjacent trays is usually small, so that the sensible-heat term is insignificant. The heat of solution can in most cases be measured in terms of hundreds of kJ/kmol of solution, whereas the latent heats at ordinary pressures are usually in 10^4 kJ/kmol. Therefore, for all practical purposes,

$$H_{G_{a+1}} - H_{L_a} = (\lambda M)_{av} \tag{9.113}$$

where the last term is the weighted average of the molal latent heats. For many pairs of substances, the molal latent heats are nearly identical, so that averaging is unnecessary. If their inequality is the only barrier to application of these simplifying assumptions, it is possible to assign a fictitious molecular weight to one of the components so that the molal latent heats are then forced to be the same (if this is done, the entire computation must be made with the fictitious molecular weight, including operating lines and equilibrium data). This is, however, rarely necessary.†

If it is sufficiently important, therefore, one can be persuaded that, for all but exceptional cases, the ratio L/G in the enriching section of the fractionator is essentially constant. The same reasoning can be applied to any section of a fractionator between points of addition or withdrawal of streams, although each section will have its own ratio.

Consider next two adjacent trays n and r, between which neither addition nor withdrawal of material from the tower occurs. A material balance provides

$$L_{r-1} + G_{n+1} = L_n + G_r \tag{9.114}$$

Since $L_{r-1}/G_r = L_n/G_{n+1}$, it follows that $L_n = L_{r-1}$ and $G_{n+1} = G_r$, which is the principle of equimolal overflow and vaporization. The rate of liquid flow from each tray in a section of the tower is constant on a molar basis, but since the average molecular weight changes with tray number, the weight rates of flow are different.

It should be noted that, as shown in the discussion of Fig. 9.12, if the H_{GV} and H_{LX} lines on the H_{XY} diagram are straight and parallel, then in the absence of heat loss the L/G ratio for a given tower section will be constant regardless of the relative size of H_{Lx} and Q' in Eq. (9.108).

The general assumptions involved in the foregoing are customarily called the usual simplifying assumptions.

† If the only enthalpy data available are the latent heats of vaporization of the pure components, an approximate Savarit-Ponchon diagram using these and straight H_L vs x and H_G vs y lines will be better than assuming equal latent heats.

Enriching Section; Total Condenser-Reflux at the Bubble Point

Consider a section of the fractionator entirely above the point of introduction of feed, shown schematically in Fig. 9.35*a*. The condenser removes all the latent heat from the overhead vapor but does not cool the resulting liquid further. The reflux and distillate product are therefore liquids at the bubble point, and $y_1 = x_D = x_0$. Since the liquid, $L \mod/h$, falling from each tray and the vapor, $G \mod/h$, rising from each tray are each constant if the usual simplifying assumptions pertain, subscripts are not needed to identify the source of these streams. The compositions, however, change. The trays shown are theoretical trays, so that the composition y_n of the vapor from the *n*th tray is in equilibrium with the liquid of composition x_n leaving the same tray. The point (x_n, y_n) , on x, y coordinates, therefore falls on the equilibrium curve.

A total material balance for the envelope in the figure is

$$G = L + D = D(R + 1)$$
 (9.115)

For component A,

$$Gy_{n+1} = Lx_n + Dx_D$$
 (9.116)

from which the enriching-section operating line is

$$y_{n+1} = \frac{L}{G} x_n + \frac{D}{G} x_D$$
(9.117)

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$
(9.118)

This is the equation of a straight line on x, y coordinates (Fig. 9.35b) of slope L/G = R/(R+1), and with a y intercept of $x_D/(R+1)$. Setting $x_D = x_D$



Figure 9.35 Enriching section.

shows $y_{n+1} = x_D$, so that the line passes through the point $y = x = x_D$ on the 45° diagonal. This point and the y intercept permit easy construction of the line. The concentration of liquids and vapors for each tray is shown in accordance with the principles of Chap. 5, and the usual staircase construction between operating line and equilibrium curve is seen to provide the theoretical tray-concentration variation. The construction obviously cannot be carried farther than point P.

In plotting the equilibrium curve of the figure, it is generally assumed that the pressure is constant throughout the tower. If necessary, the variation in pressure from tray to tray can be allowed for after determining the number of real trays, but this will require a trial-and-error procedure. It is ordinarily unnecessary except for operation under very low pressures.

Exhausting Section; Reboiled Vapor in Equilibrium with Residue

Consider next a section of the fractionator below the point of introducing the feed, shown schematically in Fig. 9.36*a*. The trays are again theoretical trays. The rates of flow \overline{L} and \overline{G} are each constant from tray to tray, but not necessarily equal to the values for the enriching section. A total material balance is

$$\overline{L} = \overline{G} + W \tag{9.119}$$

and, for component A,

$$\overline{L}x_m = \overline{G}y_{m+1} + Wx_W \tag{9.120}$$



Figure 9.36 Exhausting section.

These provide the equation of the exhausting-section operating line,

$$y_{m+1} = \frac{\overline{L}}{\overline{G}} x_m - \frac{W}{\overline{G}} x_W$$
(9.121)

$$y_{m+1} = \frac{L}{\overline{L} - \overline{W}} x_m - \frac{W}{\overline{L} - W} x_W$$
(9.122)

This is a straight line of slope $\overline{L}/\overline{G} = \overline{L}(\overline{L} - W)$, and since when $x_m = x_W$, $y_{m+1} = x_W$, it passes through $x = y = x_W$ on the 45° diagonal (Fig. 9.36b). If the reboiled vapor y_W is in equilibrium with the residue x_W , the first step of the staircase construction represents the reboiler. The steps can be carried no farther than point T.

Introduction of Feed

It is convenient before proceeding further to establish how the introduction of the feed influences the change in slope of the operating lines as we pass from the enriching to the exhausting sections of the fractionator.

Consider the section of the column at the tray where the feed is introduced (Fig. 9.37). The quantities of the liquid and vapor streams change abruptly at this tray, since the feed may consist of liquid, vapor, or a mixture of both. If, for example, the feed is a saturated liquid, \overline{L} will exceed L by the amount of the added feed liquid. To establish the general relationship, an overall material balance about this section is

$$F + L + \overline{G} = G + \overline{L} \tag{9.123}$$

and an enthalpy balance,

$$FH_F + LH_{L_{f-1}} + \bar{G}H_{G_{f+1}} = GH_{G_f} + \bar{L}H_{L_f}$$
(9.124)

The vapors and liquids inside the tower are all saturated, and the molal enthalpies of all saturated vapors at this section are essentially identical since the temperature and composition changes over one tray are small. The same is true of the molal enthalpies of the saturated liquids, so that $H_{G_{l}} = H_{G_{l+1}}$ and



 $H_{L_{l-1}} = H_{L_l}$. Equation (9.124) then becomes

$$(\widetilde{L} - L)H_L = (\widetilde{G} - G)H_G + FH_F$$
(9.125)

Combining this with Eq. (9.123) gives

$$\frac{\overline{L} - L}{F} = \frac{H_G - H_F}{(H_G - H_F)} = q$$
(9.126)

The quantity q is thus seen to be the heat required to convert 1 mol of feed from its condition H_F to a saturated vapor, divided by the molal latent heat $H_G - H_L$. The feed may be introduced under any of a variety of thermal conditions ranging from a liquid well below its bubble point to a superheated vapor, for each of which the value of q will be different. Typical circumstances are listed in Table 9.1, with the corresponding range of values of q. Combining Eqs. (9.123) and (9.126), we get

$$\overline{G} - G = F(q - 1)$$
 (9.127)

which provides a convenient method for determining \overline{G} .

The point of intersection of the two operating lines will help locate the exhausting-section operating line. This can be established as follows. Rewriting Eqs. (9.117) and (9.121) without the tray subscripts, we have

$$yG = Lx + Dx_D \tag{9.128}$$

$$y\overline{G} = \overline{L}x - Wx_{W} \tag{9.129}$$

Subtracting gives

$$(\bar{G} - G)y = (\bar{L} - L)x - (Wx_W + Dx_D)$$
 (9.130)

Further, by an overall material balance,

$$Fz_F = Dx_D + Wx_W \tag{9.76}$$

Substituting this and Eqs. (9.126) and (9.127) in (9.130) gives

$$y = \frac{q}{q-1}x - \frac{z_F}{q-1} \int_{Q_{1}}^{Q_{1}} \int_{Q_{1}}^{Q_{1}} (9.131)$$

This, the locus of intersection of operating lines (the q line), is a straight line of slope q/(q-1), and since $y = z_F$ when $x = z_F$, it passes through the point $x = y = z_F$ on the 45° diagonal. The range of the values of the slope q/(q-1) is listed in Table 9.1, and the graphical interpretation for typical cases is shown in Fig. 9.38. Here the operating-line intersection is shown for a particular case of feed as a mixture of liquid and vapor. It is clear that, for a given feed condition, fixing the reflux ratio at the top of the column automatically establishes the liquid/vapor ratio in the exhausting section and the reboiler heat load as well.

Feed condition	G _F , mol/(area)(time)	L _F , mol/(area)(time)	H _{GF} , energy/mol	H _{LF} , energy/mol	H _F , energy /mol	$q = \frac{H_G - H_F}{H_G - H_L}$	$\frac{q}{q-1}$
Liquid below bubble point	0	F		H _F	$H_F < H_L$	>1.0	> i.C
Saturated liquid	0	F		H _F	H _L	1.0	œ
Mixture of liquid and	G _F					$\frac{L_F}{F}$	-
vaport	$F = G_F + L_F$	L _F	H _G	H _L	$H_G > H_F > H_L$	1.0 > q > 0	$\frac{L_F}{L_F - F}$
Saturated vapor	F	0	H _F		H _G	0	0
Superheated vapor	F	0	H _F		$H_F > H_G$	<0	$1.0 > \frac{q}{q-1} > 0$

Table 9.1 Thermal conditions for the feed

 \dagger In this case the intersection of the q line with the equilibrium curve gives the compositions of the equilibrium liquid and vapor which constitute the feed. The q line is the flash-vaporization operating line for the feed.

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Figure 9.38 Location of q line for typical feed conditions.

Location of the Feed Tray

The q line is useful in simplifying the graphical location of the exhausting line, but the point of intersection of the two operating lines does not necessarily establish the demarcation between the enriching and exhausting sections of the tower. Rather it is the introduction of feed which governs the change from one operating line to the other and establishes the demarcation, and at least in the design of a new column some latitude in the introduction of the feed is available.

Consider the separation shown partially in Fig. 9.39, for example. For a given feed, z_F and the q line are fixed. For particular overhead and residue products, x_D and x_W are fixed. If the reflux ratio is specified, the location of the enriching line DG is fixed, and the exhausting line KC must pass through the q line at E. If the feed is introduced upon the seventh tray from the top (Fig. 9.39a), line DG is used for trays 1 through 6, and, beginning with the seventh tray, the line KC must be used. If, on the other hand, the feed is introduced upon the fourth from the top (Fig. 9.39b), line KC is used for all trays below the fourth. Clearly a transition from one operating line to the other must be made somewhere between points C and D, but anywhere within these limits will serve. The least total number of trays will result if the steps on the diagram are kept as large as possible or if the transition is made at the first opportunity after passing the operating-line intersection, as shown in Fig. 9.39c. In the design of a new column, this is the practice to be followed.

In the adaptation of an existing column to a new separation, the point of introducing the feed is limited to the location of existing nozzles in the column



Figure 9.39 Location of feed tray.

wall. The slope of the operating lines (or reflux ratio) and the product compositions to be realized must then be determined by trial and error, in order to obtain numbers of theoretical trays in the two sections of the column consistent with the number of real trays in each section and the expected tray efficiency.

Total Reflux, or Infinite Reflux Ratio

As the reflux ratio R = L/D is increased, the ratio L/G increases, until ultimately, when $R = \infty$, L/G = 1 and the operating lines of both sections of the column coincide with the 45° diagonal as in Fig. 9.40. In practice this can be



Figure 9.40 Total reflux and minimum trays.

realized by returning all the overhead product back to the column as reflux (total reflux) and reboiling all the residue product, whereupon the forward flow of fresh feed must be reduced to zero. Alternatively such a condition can be interpreted as requiring infinite reboiler heat and condenser cooling capacity for a given rate of feed.

As the operating lines move farther away from the equilibrium curve with increased reflux ratio, the number of theoretical trays required to produce a given separation becomes less, until at total reflux the number of trays is the minimum N_m .

If the relative volatility is constant or nearly so, the analytical expression of Fenske, Eq. (9.85), can be used.

Minimum Reflux Ratio

The minimum reflux ratio R_m is the maximum ratio which will require an infinite number of trays for the separation desired, and it corresponds to the minimum reboiler heat and condenser cooling capacity for the separation. Refer to Fig. 9.41*a*. As the reflux ratio is decreased, the slope of the enriching



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operating line becomes less, and the number of trays required increases. Operating line MN, which passes through the point of intersection of the q line and the equilibrium curve, corresponds to the minimum reflux ratio, and an infinite number of trays would be required to reach point N from either end of the tower. In some cases, as in Fig. 9.41b, the minimum-reflux operating line will be tangent to the equilibrium curve in the enriching section, as at point P, while a line through K would clearly represent too small a reflux ratio. It has been pointed out that all systems show concave-upward xy diagrams near the critical condition of the more volatile component [66]. Because of the interdependence of the liquid/vapor ratios in the two sections of the column, a tangent operating line in the exhausting section may also set the minimum reflux ratio, as in Fig. 9.41c.

When the equilibrium curve is always concave downward, the minimum reflux ratio can conveniently be calculated analytically [63]. The required relationship can be developed by solving Eqs. (9.118) and (9.131) simultaneously to obtain the coordinates (x_a, y_a) of the point of intersection of the enriching operating line and the q line. When the tray-number designation in Eq. (9.118) is dropped, these are

$$x_a = \frac{x_D(q-1) + z_F(R+1)}{R+q} \qquad y_a = \frac{Rz_F + qx_D}{R+q}$$
(9.132)

At the minimum reflux ratio R_m , these coordinates are equilibrium values since they occur on the equilibrium curve. Substituting them into the definition of α , Eq. (9.2), gives

$$\frac{R_m z_F + q x_D}{R_m (1 - z_F) + q(1 - x_D)} = \frac{a[x_D(q - 1) + z_F(R_m + 1)]}{(R_m + 1)(1 - z_F) + (q - 1)(1 - x_D)}$$
(9.133)

This conveniently can be solved for R_m for any value of q. Thus, for example:

$$R_m = \begin{cases} \frac{1}{\alpha - 1} \left[\frac{x_D}{x_F} - \frac{\alpha(1 - x_D)}{1 - x_F} \right] & q = 1 \text{ (feed liquid at the bubble point)} \quad (9.134) \end{cases}$$

$$\left[\frac{1}{\alpha-1}\left(\frac{\alpha x_D}{y_F}-\frac{1-x_D}{1-y_F}\right)-1\right] = 0 \text{ (feed vapor at the dew point)}$$
(9.135)

In each case, the α is that prevailing at the intersection of the q line and the equilibrium curve.

Optimum Reflux Ratio

The discussion given under the Ponchon-Savarit method applies.

Illustration 9.10 Redesign the methanol-water fractionator of Illustration 9.8, using the simplifying assumptions of the McCabe-Thiele method. The circumstances are:

- Feed. 5000 kg/h, 216.8 kmol/h, $z_F = 0.360$ mole fraction methanol, av mol wt = 23.1, temperature entering the fractionator = $58.3^{\circ}C$
- Distillate. 2620 kg/h, 84.4 kmol/h, $x_D = 0.915$ mole fraction methanol, liquid at the bubble point

Residue. 2380 kg/h, 132.4 kmol/h, $x_W = 0.00565$ mole fraction methanol Reflux ratio = 1.5 times the minimum

SOLUTION Refer to Fig. 9.42. From the *txy* diagram, the bubble point of the feed at 0.360 mole fraction methanol is 76.0°C, and its dew point is 89.7°C. The latent heats of vaporization at 76.0°C are $\lambda_A = 1046.7$ kJ/kg for methanol and $\lambda_B = 2284$ kJ/kg for water. Specific heat of liquid methanol = 2721, of liquid water = 4187, of feed solution = 3852 J/kg K. If heats of



Figure 9.42 Solution to Illustration 9.10. Minimum reflux ratio and minimum trays.

solution are ignored, as is customary with the McCabe-Thiele method, the enthalpy of the feed at 76.0°C (the bubble point) referred to 58.3°C (the feed temperature) is

3.852(23.1)(76.0 - 58.3) = 1575 kJ/kmol

The enthalpy of the saturated vapor at 89.7°C referred to liquids at 58.3°C is

$$0.36[2.721(32.04)(89.7 - 58.3) + 1046.7(32.04)] + (1 - 0.36)[4.187(18.02)(89.7 - 58.3) + 2284(18.02)] = 40 915 kJ/kmol$$

$$q = \frac{\text{heat to convert to saturated vapor}}{\text{heat of vaporization}} = \frac{40 915 - 0}{40 915 - 1575} = 1.04$$

(Essentially the same value is obtained if the heat of solution is considered.)

$$\frac{q}{q-1} = \frac{1.04}{1.04-1} = 26$$

In Fig. 9.42, x_D , x_W , and z_F are located on the 45° diagonal, and the q line is drawn with slope = 26. The operating line for minimum reflux ratio in this case passes through the intersection of the q line and equilibrium curve, as shown.

$$\frac{x_O}{R_m + 1} = \frac{0.915}{R_m + 1} = 0.57$$

$$R_m = 0.605 \text{ mole reflux/mole distillate}$$

The minimum number of theoretical trays is determined using the 45° diagonal as operating line (Fig. 9.42). Theoretical stages to the number of 4.9, including the reboiler, are determined.

For $R = 1.5R_m = 1.5(0.605) = 0.908$ mol reflux/mol distillate, and for equimolal overflow and vaporization:

Eq. (9.49):

$$L = L_0 = RD = 0.908(84.4) = 76.5 \text{ kmol/h}$$

Eq. (9.115):

$$G = D(R + 1) = 85.4(0.908 + 1) = 160.9 \text{ kmol/h}$$

Eq. (9.126):

$$\overline{L} = qF + L = 1.04(216.8) + 76.5 = 302.5 \text{ kmol/h}$$

Eq. (9.127):

$$\overline{G} = F(q-1) + G = 216.8(1.04 - 1) + 160.9 = 169.7 \text{ kmol/h}$$

 $\frac{x_D}{R+1} = \frac{0.915}{0.908+1} = 0.480$

Refer to Fig. 9.43. The y intercept 0.480 and enriching and exhausting operating lines are plotted. Steps are drawn to determine the number of theoretical trays, as shown. The exhausting operating line is used immediately after crossing the operating-line intersection, and the feed is therefore to be introduced on the fifth theoretical tray from the top. A total of 8.8 theoretical trays, including the reboiler, is required, and the tower must therefore contain 7.8 theoretical trays. An integral number can be obtained by very slight adjustment of the reflux ratio, but since a tray efficiency must still be applied to obtain the number of real trays, this need not be done.

When the residue composition is very small, it will be necessary to enlarge the scale of the lower left-hand part of the diagram in order to obtain the number of trays. In some cases the graphical determination may still be difficult because of the closeness of the exhausting line and the equilibrium curve. Logarithmic coordinates can then be used to maintain a satisfactory separation of the lines, as in the insert of Fig. 9.43. On such a graph, for very low values of x, the equilibrium curve will be substantially given by $y^* = \alpha x$, which is a straight line of unit slope. The exhausting operating line will, however, be curved and must be plotted from its equation. In this example, the equation is [Eq. (9.121)]

$$y = \frac{302.5}{169.7}x - \frac{132.4}{169.7}0.00565 = 1.785x - 0.0044$$

The steps representing the theoretical stages are made in the usual manner on these coordinates, continued down from the last made on the arithmetic coordinates (see also page 422).

The diameter of the tower and the tray design are established through the methods of Chap. 6. Note the substantially different liquid loads in the enriching and exhausting sections. A column of constant diameter for all sections is usually desired for simplicity in construction and lower cost. If the discrepancy between liquid or vapor quantities in the two sections is considerable, however, and particularly if expensive alloy or nonferrous metal is used, different diameters for the two sections may be warranted.



Figure 9.43 Solution to Illustration 9.10. R = 0.908 mol reflux/mol distillate product.

Computations for other reflux ratios are easily and quickly made once the diagram and equations have been set up for one value of R. These provide data for determining the most economical R. The following table lists the important quantities for this separation at various values of R:

R	L^{-1}	G	\bar{L}	Ĝ	No. theoretical stages
$R_m = 0.605$	51.0	135.4	277	144.1	80
0.70	59.0	143.4	285	152.1	11.5
0.80	67.5	151.9	294	160.6	10
0.908	76.5	160.9	303	169.7	8.8
1.029	86.7	171.3	313	180	8.3
2.0	168.8	253	395	262	6.5
4.0	338	422	564	431	5.5
8	80	8	80	8	$4.9 = N_m + 1$

Included in the table are the data for the McCabe-Thiele method at R = 1.029, the reflux ratio