Operations Research _AM301.

Definition: Operations (OR) research is a discipline that deals with the application of advanced analytical methods to help make better decisions.

This field of OR came into picture during World War II in Britain. The teams of scientists and mathematicians were formed to study the strategic and tactical problems involved in military operations*. But, later, the area was diversified to other streams like in management, Inventory problems, Waiting times, Budgeting in a company, Traffic control, to name a few.

Need: Why do we should study OR or why is OR required at all?

Operations research is widely used in various fields (mentioned above) for finding optimal solution ~~so as to~~ i.e, to maximize the profit or to minimize the cost or to yeild maximum production which ultimately results in the profit of organization.

We will start with Linear Programming Problems (LPP), which are probably, the most successful methods, for optimising the use of resources in manufacturing and service industry.

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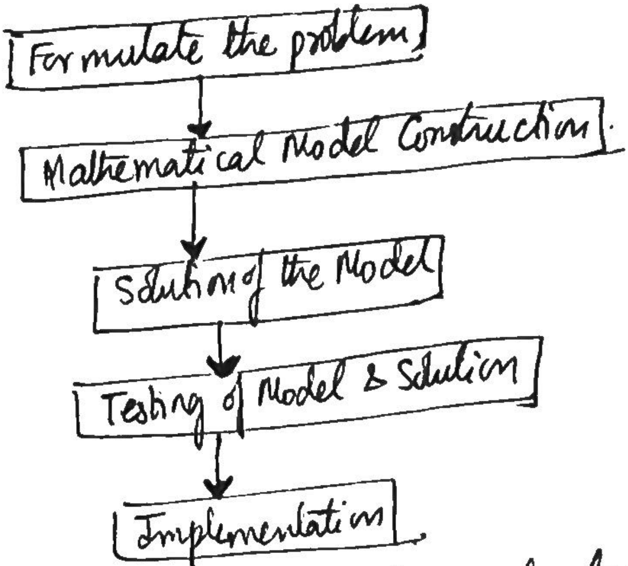
* The objective was to find most effective ultilization of limited military resources by use of quantitative techniques.

Flow chart shows the Phases in OR.



So, as is evident from the flow chart above a real life problem is first converted into mathematical model and then by applying mathematical tools or analytical methods the solution is found.

LPP has three parts:

1) Objective function: $z = cX$ (which is to be either maximized or minimized).

2) Constraints: $AX \leq b$ or $AX \geq b$ or mixture of these two.

3) Conditions on decision variables: $a, b \geq 0$
$X \geq 0$, $X \geq a$, $X \leq b$ or $a \leq X \leq b$.

Here $X_{n\times 1}$, $A_{m\times n}$, $c_{1\times n}$, $b_{m\times 1}$ matrices.

Alternatively: Let $x_1, x_2, \ldots x_n$ be decision variables and $c_1, c_2, \ldots c_n$ cost coefficients then $z = \sum_{i=1}^{n} c_i x_i$ is the objective function

Linear constraints:

$$a_{11}x_1 + \dots + a_{1n}x_n \,(\leq, =, \geq)\, b_1$$
$$a_{21}x_1 + \dots + a_{2n}x_n \,(\leq, =, \geq)\, b_2$$
$$\vdots$$
$$a_{m1}x_1 + \dots + a_{mn}x_n \,(\leq, =, \geq)\, b_m$$

OR

$$\sum_{j=1}^{n} a_{ij}x_j \,(\leq, =, \geq)\, b_i \qquad 0 \leq i \leq m$$

$x_j \geq 0 \quad 1 \leq j \leq n$ → non-negativity conditions

Formulation of LPP: For formulating LPP from the given real world problem following steps should be followed:

(1) Identify the decision variables.
(2) Limitations of resources (i.e, constraints)
(3) Objective function.
(4) Conditions on decision variables.

Lets see some problems on how to formulate the LPP:

Ex 1 A manufacturer produces two types of models M_1 and M_2. Each M_1 model requires 3 hours of grinding and 4 hrs of polishing, whereas, each model M_2 requires 5 hrs of grinding and 2 hrs of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 30 hrs a week and each polisher 50 hrs/week. Profit on M_1 model is Rs 30 and on model M_2 is Rs 20. Whatever the manufacturer produces in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make maximum profit in a week.

Solution (Ex 1) Let x: no of units of M_1 produced
y: " " " M_2 "

No. of griders avaiable = 2.
No of hrs grinder can work = 30 hr/week
∴ No. of available grinder hrs/week = 2 × 30 = 60 hrs
Similarly, no of available polishing hrs/week = 3 × 50 = 150 hrs.
Lets write the given information in form of table:

	Processing time M_1	Processing time M_2	Available (hrs)
grinding	3	5	60
polishing	4	2	150
Profit (Rs)	30	20	

LPP
maximize $z = 30x + 20y$. (profit)
subject to
$3x + 5y \leq 60$ (grinding constraint)
$4x + 2y \leq 150$ (polishing constraint)
$x \geq 0, \; y \geq 0$ (decision variable restriction)

Ex 2 A Research Institute wants to develop a low cost, low calorie, high protien diet. For that 3 kinds of food ~~Food~~ F1, F2 and F3 are undertaken for consideration and the following information is collected about the foods:

Nutritional Element	F1 per unit of food	F2 per unit of food	F3 per unit of food
Calories	400	300	500
Protiens	300	450	550
Vitamin A	750	400	300
Vitamin B	500	800	600
Cost (Rs per unit on Rs)	50	100	120

The specifications about nutritional elements are maximum 3000 units of calories, minimum 6000 units of protien, minimum 6000 units of Vitamin A and minimum 5000 units of vitamin C. Formulate suitable LP model.

Solution (Ex 2): Let x_1 be no. of units of Food F1
x_2 " " " " " " F2
x_3 " " " " " " F3.

From table (given), the constraints equation are

LPP
$$400x_1 + 300x_2 + 500x_3 \leq 3000 \text{ (calories)}$$
$$300x_1 + 450x_2 + 550x_3 \geq 6000 \text{ (protien)}$$
$$750x_1 + 400x_2 + 300x_3 \geq 6000 \text{ (Vitamin A)}$$
$$500x_1 + 800x_2 + 600x_3 \geq 5000 \text{ (Vitamin B)}$$
$$x_1, x_2, x_3 \geq 0.$$

Objective function: minimize $z = 50x_1 + 100x_2 + 120x_3$ (total cost).

Ex 3 The owner of Goods shop is interested to determine how many advertisements to relase in the selected 3 magazines A, B and C. His main purpose is to advertise in such a way that total exposure to principal buyers of his goods is maximized. Percentage of readers for each magazine are known. Exposure in any particular magazine is number of advertisements released multiplied by number of principal buyers. The following data are available:

Particulars	A	B	C
Readers	1 lakh	0.6 lakh	0.4 lakh
Principal buyers	20%	15%	8%
cost/ad (Rs)	8000	6000	5000.

(magazines: A, B, C)

The budgeted amount is ~~the~~ atmost 1 lakh for advertisement. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each gets atleast 8 advertisements. Formulate LP model for this problem.

Solution (Ex 3) x_1 : No of advertisements in magazine A
x_2 : " " " " B
x_3 : " " " " C.

Exposure in a magazine = (no. of adv.) × no of principal buyers.

∴ Total Exposure = x_1(20% of 1 lakh) + (15% of 0.6 lakh) x_2
+ x_3(8% of 0.4 lakh)
$= 20000x_1 + 9000x_2 + 3200x_3$

$z = 20000x_1 + 9000x_2 + 3200x_3$

∴ LPP
maximize $z = 20000x_1 + 9000x_2 + 3200x_3$
subject to $8000x_1 + 6000x_2 + 5000x_3 \leq 1,00,000$ (budgeting constraint)

$0 \leq x_1 \leq 15, \quad x_2 \geq 8, x_3 \geq 8$ (Variable constraint)

Ex 4. Evening shift resident doctors in a government hospital work five consecutive days and have 2 consecutive days off. Their five days of work can start on any day of the week and the schedule rotates indefinitely. The hospital requires the following minimum number of doctors working:

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
35	55	60	50	60	50	45

No more than 40 doctors can start their five days on the same day. Formulate LPP to minimize number of doctors employed by hospital.

Solution (Ex 4): Let x_i = number of doctors who start their duty on ith day of week.

$x_1 \to$ Mon, Tues, Wed, Thurs, Fri

$x_2 \to$ Tues, Wed, Thurs, Fri, Sat

⋮

$x_7 \to$ Sun, Mon, Tues, Wed, Thurs

∴ No. of Doctors working on Sunday → x_3, x_4, x_5, x_6, x_7.

" " " " Monday → x_4, x_5, x_6, x_7, x_1

Similarly you can compute for other days.

∴ minimize $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$ (total no. of doctors).

LPP:

subject to

$x_3 + x_4 + x_5 + x_6 + x_7 \geq 35$ (Sunday)

$x_4 + x_5 + x_6 + x_7 + x_1 \geq 55$ (Monday)

$x_5 + x_6 + x_7 + x_1 + x_2 \geq 60$ (Tuesday)

$x_6 + x_7 + x_1 + x_2 + x_3 \geq 50$ (Wednesday)

$x_7 + x_1 + x_2 + x_3 + x_4 \geq 60$ (Thursday)

$x_1 + x_2 + x_3 + x_4 + x_5 \geq 50$ (Friday)

$x_2 + x_3 + x_4 + x_5 + x_6 \geq 45$ (Saturday).

$0 \leq x_i \leq 40 \quad 1 \leq i \leq 7$.

Ex 5: ABC manufactures 3 products A, B and C which use 3 raw materials X, Y and Z. The raw materials required for each unit of product are

Raw material	Product A	B	C
X	3	4	1
Y	2	4	2
Z	1	1	1

Currently manufactures have 2000 units of X, 3000 units of Y and 2000 units of Z in stock. The next

• delivery of raw materials will be in a weeks time. The labour requirement per item of product is

Product	A	B	C
Hours	3	2	3

Total number of labour hours available is 400. If current orders for the products have to be met, if the company is not to run the risk of losing customers. These orders at the moment stand at 10 units of A, 10 units of B and 40 units of C. Labour is hired mainly on a weekly basis and paid at the rate of Rs 20/hr. Only a small proportion of the work force is employed on a permanent basis. Other variable costs totals Rs 20 for product A, Rs 30 for product B and Rs 40 for product C. ~~Fixed costs are estimated at ... Rs~~ a week. The products are sold at the following price

Product	A	B	C
Unit Price (Rs)	130	140	170

Formulate the above situation as LPP to determine production levels for the 3 products A, B & C

Solution (Ex 5) $x_1 \to$ production level of A

$x_2 \to$ " " " B

$x_3 \to$ " " " C

(no of labour hrs)

Cost of A $20 \times 3 + 20 = 80$. Profit of A $= 130 - 80 = 50$

Cost of B $= 20 \times 2 + 30 = 70$ " " B $= 140 - 70 = 70$

Cost of C $= 20 \times 3 + 40 = 100$ " " C $= 170 - 100 = 70$

∴ Total profit → Maximize $z = 50x_1 + 70x_2 + 70x_3$

Subject to

$3x_1 + 4x_2 + x_3 \leq 2000$ (X constraint) (1^{st} Table)

$2x_1 + 4x_2 + 2x_3 \leq 3000$ (Y ")

$x_1 + x_2 + x_3 \leq 2000$ (Z ")

$30x_1 + 20x_2 + 3x_3 \leq 400$ (labour constraints)

$x_1 \geq 10$, $x_2 \geq 10$, $x_3 \geq 40 \rightarrow$ current order constraints

Ex A Mutual Fund company has Rs 25 lakhs available for investment in government bonds, blue chip stocks, speculative stocks and short term bank deposits. The annual expected return and risk factor are:

Type of investment	Annual expected return (%)	Risk factor (0 to 100)
Government bonds	12	08
Blue chip stocks	19	40
speculative stocks	23	45
short term deposits	10	04

Mutual fund is required to keep atleast Rs 4 lakhs in short term deposits and not exceed an average risk factor of 50. Speculative stocks must be atmost 20% of total amount invested. How should the mutual fund invest the funds so as to maximize the total expected annual return? Formulate LPP

Solution: Let $x_1 \rightarrow$ Amount of funds invested in govt bonds,

$x_2 \rightarrow$ Amount of funds invested in Blue chip stocks

$x_3 \rightarrow$ " " " " speculative "

x_4 " " " " " Short term deposits

Now, $x_1 + x_2 + x_3 + x_4 \leq 2{,}50{,}000$ (Funds available).

$x_4 \geq 4{,}00{,}000$ (atleast 4 lakhs to be invested in short term deposit) average risk factor ≤ 50

i.e, $\frac{8x_1+40x_2+48x_3+4x_4}{x_1+x_2+x_3+x_4} \leq 50$

or, $42x_1+40x_2+2x_3+46x_4 \geq 0$ or, $21x_1+20x_2+x_3+23x_4 \geq 0$.

Constraint for speculative stock

$x_3 \leq 20\% (x_1+x_2+x_3+x_4)$

or, $20x_1+20x_2+80x_3+20x_4 \geq 0$

or, $x_1+x_2-4x_3+x_4 \geq 0$

∴ LPP is

maximize $z = 0.12x_1+0.19x_2+0.23x_3+0.10x_4$

subject to:

$x_1+x_2+x_3+x_4 \leq 2,50,000$

$21x_1+5x_2+x_3+23x_4 \geq 0$

LPP

$x_1+x_2-4x_3+x_4 \geq 0$

$x_4 \geq 4,00,000$

$x_1, x_2, x_3, x_4 \geq 0$

Note: Here average is taken to get 2nd constraint at unlike previous problems.

Hence, depending on the problem different approaches are taken for formulating LPP.