Heat Exchanger Thermohydraulic Fundamentals

The energy conservation equation for an exchanger having an arbitrary flow arrangement is

$$q = C_h\,(t_{h,i} - t_{h,o}) = C_c\,(t_{c,o} - t_{c,i})$$

And the heat transfer rate equation is

$$q = UA\Delta t_m = \Delta t_m / R_o$$

Δt_m is the true mean temperature difference (MTD), which depends upon the exchanger flow arrangement and the degree of fluid mixing within each fluid stream

The inverse of the overall thermal conductance UA is referred to as the overall thermal resistance R_o

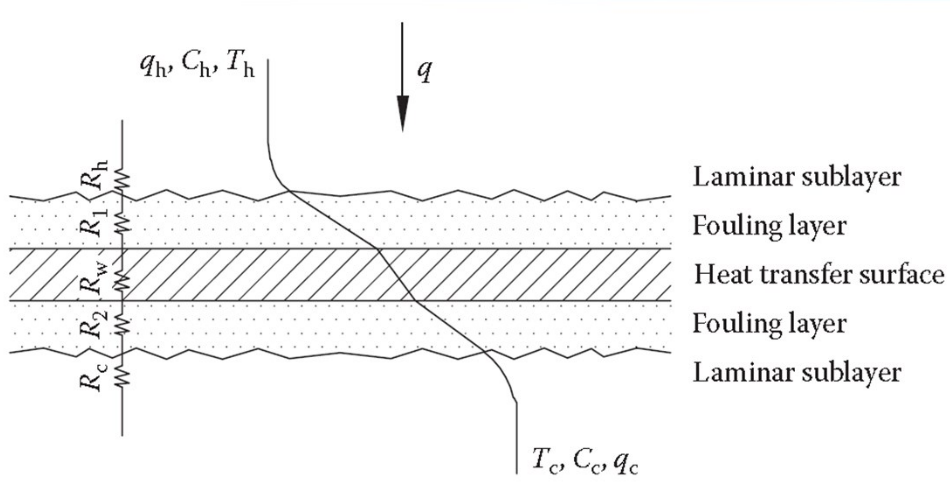
$$R_o = R_h + R_1 + R_w + R_2 + R_c$$

or

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_h} + \frac{R_{f,h}}{(\eta_o A)_h} + R_w + \frac{1}{(\eta_o hA)_c} + \frac{R_{f,c}}{(\eta_o A)_c}$$

η_o is the overall surface effectiveness of an extended surface

This is related to the fin efficiency η_f and the ratio of fin surface area A_f to total surface area A as follows:



$$\eta_o = 1 - \frac{A_f}{A}(1 - \eta_f)$$

Note that η_o is the unity for an all prime surface exchanger without fins.

Since $UA = U_h A_h = U_c A_c$

$$\frac{1}{U_o} = \frac{1}{h_o} + R_{f,o} + \frac{d \ln(d/d_i)}{2k_w} + \frac{R_{f,i} d}{d_i} + \frac{d}{h_i d_i}$$

The wall temperature in a heat exchanger is essential to determine the localized hot spots, freeze points, thermal stresses, local fouling characteristics, or boiling and condensing coefficients.

Based on the thermal circuit, when R_w is negligible, $T_{w,h} = T_{w,c} = T_w$ is computed as

$$T_w = \frac{T_h + T_c\left[(R_h + R_1)/(R_c + R_2)\right]}{1 + \left[(R_h + R_1)/(R_c + R_2)\right]}$$

Heat Transfer Analysis Methods

Energy Balance Equation

The first law of thermodynamics must be satisfied in any heat exchanger design procedure at both the macro and micro levels.

The overall energy balance for any two-fluid heat exchanger is given by

$$m_h c_{p,h} (t_{h,i} - t_{h,o}) = m_c c_{p,c} (t_{c,o} - t_{c,i})$$

Heat Transfer

For any flow arrangement, heat transfer for two fluid streams is given by

$$q = C_h (t_{h,i} - t_{h,o}) = C_c (t_{c,o} - t_{c,i})$$

and the expression for maximum possible heat transfer rate q_{max} is

$$q_{max} = C_{min} (t_{h,i} - t_{c,i})$$

The maximum possible heat transfer rate would be obtained in a counter-flow heat exchanger with very large surface area and zero longitudinal wall heat conduction, and the actual operating conditions are the same as the theoretical conditions.

Basic Methods to Calculate Thermal Effectiveness

There are four design methods to calculate the thermal effectiveness of heat exchangers:

1. ε-NTU method
2. S-NTU$_t$ method
3. LMTD method
4. ψ-S method

ε-NTU Method

The ε-NTU method for the heat exchanger analysis was in 1942 by London and Seban.

In this method, the total heat transfer rate from the hot fluid to the cold fluid in the exchanger is expressed as

$$q = C_{min}(t_{h,i} - t_{c,i})$$

where ε is the heat exchanger effectiveness. It is non-dimensional and for a direct transfer type heat exchanger, in general, it is dependent on NTU, C*, and the flow arrangement:

$$\varepsilon = \phi\,(NTU, C^*, \text{flow arrangement})$$

These three non-dimensional parameters,

Heat capacity rate ratio, C^*: This is simply the ratio of the smaller to larger heat capacity rate for the two fluid streams so that $C^* \leq 1$.

$$C^* = \frac{C_{min}}{C_{max}} = \frac{(mc_p)_{min}}{(mc_p)_{max}}$$

In a two-fluid heat exchanger, one of the streams will usually undergo a greater temperature change than the other.

The first stream is said to be the "weak" stream, having a lower thermal capacity rate (C_{min}), and the other with higher thermal capacity rate (C_{max}) is the "strong" stream.

Number of transfer units, NTU: NTU designates the non-dimensional "heat transfer size" or "thermal size" of the exchanger.

It is defined as a ratio of the overall conductance to the smaller heat capacity rate:

$$\text{NTU} = \frac{UA}{C_{min}} = \frac{1}{C_{min}} \int_A U\,dA$$

The number of heat transfer units on the hot and cold sides of the exchanger may be defined as follows:

$$\mathrm{NUT_h} = \frac{(\eta_o hA)_h}{C_h} \qquad \mathrm{NTU_c} = \frac{(\eta_o hA)_c}{C_c}$$

Heat exchanger effectiveness, ε: Heat exchanger effectiveness, ε, is defined as the ratio of the actual heat transfer rate, q, to the thermodynamically possible maximum heat transfer rate (q_{max}) by the second law of thermodynamics:

$$\varepsilon = \frac{q}{q_{max}}$$

The value of ε ranges between 0 and 1.

$$\varepsilon = \frac{C_h(t_{h,i} - t_{h,o})}{C_{min}(t_{h,i} - t_{c,i})} = \frac{C_c(t_{c,o} - t_{c,i})}{C_{min}(t_{h,i} - t_{c,i})}$$

For $C^* = 1$, $\varepsilon_h = \varepsilon_c$

Dependence of ε on NTU: At low NTU, the exchanger effectiveness is generally low. With increasing values of NTU, the exchanger effectiveness generally increases, after reaching a maximum value, the effectiveness decreases with increasing NTU.

S-NTU$_t$ Method

This method represents a variant of the ε-NTU method.

In the shell and tube exchangers, in order to avoid possible errors, an alternative is to present the temperature effectiveness, S, of the fluid side under consideration as a function of NTU and heat capacity rate of that side to that of the other side, R.

General S-NTU$_t$ functional relationship: Similar to the exchanger effectiveness ε, the thermal effectiveness S is a function of NTU$_t$, R, and flow arrangement:

$$S = (NTU_t\ , R, \text{flow arrangement})$$

In this method, the total heat transfer rate from the hot fluid to the cold fluid is expressed by

$$q = SC_t(T_1 - t_1)$$

Thermal effectiveness, S:

For a shell and tube heat exchanger, the temperature effectiveness of the tube side fluid, S, is referred to as the "thermal effectiveness".

" It is defined as the ratio of the temperature rise (drop) of the tube side fluid (regardless of whether it is hot or cold fluid) to the difference of inlet temperature of the two fluids.

According to this definition, S is given by

$$S = \frac{t_2 - t_1}{T_1 - t_1} \quad \text{(S is referred to tube side)}$$

where

t_1 and t_2 refer to tube side inlet and outlet temperatures,

T_1 and T_2 refer to shell side inlet and outlet temperatures

The thermal effectiveness S and the exchanger effectiveness ε are related as

$$S = \frac{C_{min}}{C_t}\varepsilon = \varepsilon \quad \text{for } C_t = C_{min}$$

$= \varepsilon C^*$ For $C_t = C_{max}$

Note that S is always less than or equal to ε.

The thermal effectiveness of the shell side fluid can be determined from the tube side values by the relationship given by

$$S_s = S\frac{C_t}{C_s} = SR$$

For $R^* = 1$ $S_s = S$ (tube side)

Heat capacity ratio, R: For a shell and tube exchanger, R is the ratio of the capacity rate of the tube fluid to the shell fluid.

This definition gives in terms of temperature drop (rise) of the shell fluid to the temperature rise (drop) of the tube fluid:

$$R = \frac{C_t}{C_s} = \frac{T_1 - T_2}{t_2 - t_1}$$

The value of R ranges from zero to infinity, zero being for pure vapor condensation and infinity being for pure liquid evaporation.

$$R = \frac{C_t}{C_s} = C^* \quad \text{for } C_t = C_{min}$$

$$= \frac{1}{C^*} \quad \text{for } C_t = C_{max}$$

Thus *R* is always greater than or equal to *C**.

Number of transfer units, NTU_t: For a shell and tube exchanger, the number of transfer units NTU_t is defined as a ratio of the overall conductance to the tube side fluid heat capacity rate:

$$NTU_t = \frac{UA}{C_t}$$

Thus, NTU_t is related to NTU based on C_{min} by

$$NTU_t = NTU\frac{C_{min}}{C_t} = NTU \qquad \text{for } C_t = C_{min}$$

$$= NTU\,C^* \quad \text{for } C_t = C_{max}$$

Thus NTU_t is always less than or equal to NTU.

Log Mean Temperature Difference Correction Factor Method

The maximum driving force for heat transfer is always the log mean temperature difference (LMTD) when two fluid streams are in countercurrent flow.

Most heat exchangers to be designed have different flow patterns from true countercurrent flow.

The true MTD of such flow arrangements will differ from the logarithmic MTD by a certain factor dependent on the flow pattern and the terminal temperatures.

This factor is usually designated as the log MTD correction factor, *F*. The factor *F* may be defined as the ratio of the true MTD to the logarithmic MTD.

The heat transfer rate equation incorporating *F* is given by

$$q = UA\Delta t_m = UAF\Delta t_{lm}$$

The expression for LMTD for a counter flow exchanger is given by

$$\text{LMTD} = \Delta t_{\text{lm}} = \frac{\Delta t_1 - \Delta t_2}{\ln(\Delta t_1/\Delta t_2)}$$

where $\Delta t_1 = t_{h,i} - t_{c,o} = T_1 - t_2$ and $\Delta t_2 = t_{h,o} - t_{c,i} = T_2 - t_1$ for all flow arrangements except for parallel flow.

For parallel flow $\Delta t_1 = t_{h,i} - t_{c,i} (= T_1 - t_1)$ and $\Delta t_2 = t_{h,o} - t_{c,o} (= T_2 - t_2)$.

Therefore, LMTD can be represented in terms of the terminal temperatures, that is, greater terminal temperature difference

(GTTD or GTD) and smaller terminal temperature difference (STTD or STD) for both pure parallel- and counter flow arrangements.

Accordingly, LMTD is given by

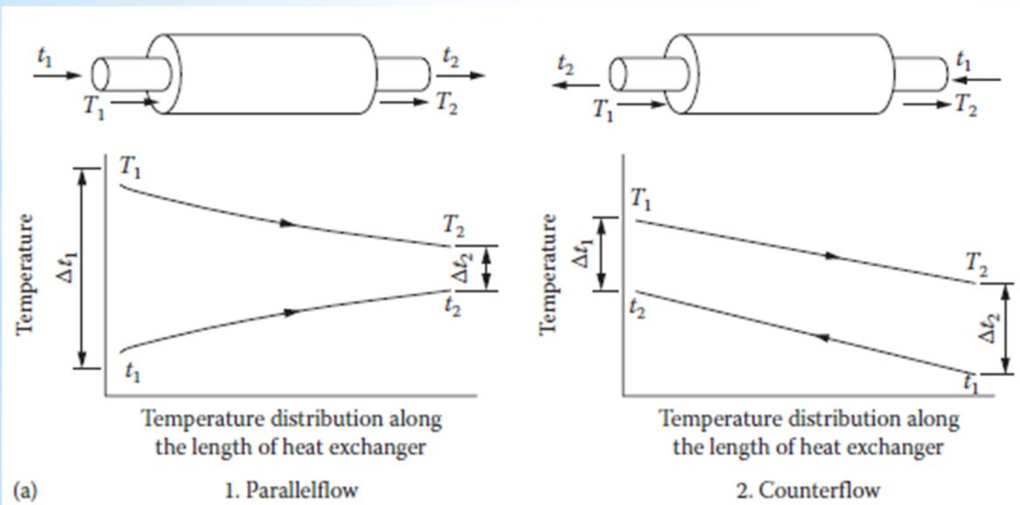
$$\text{LMTD} = \Delta t_{\text{lm}} = \frac{\text{GTTD} - \text{STTD}}{\ln(\text{GTTD}/\text{STTD})}$$

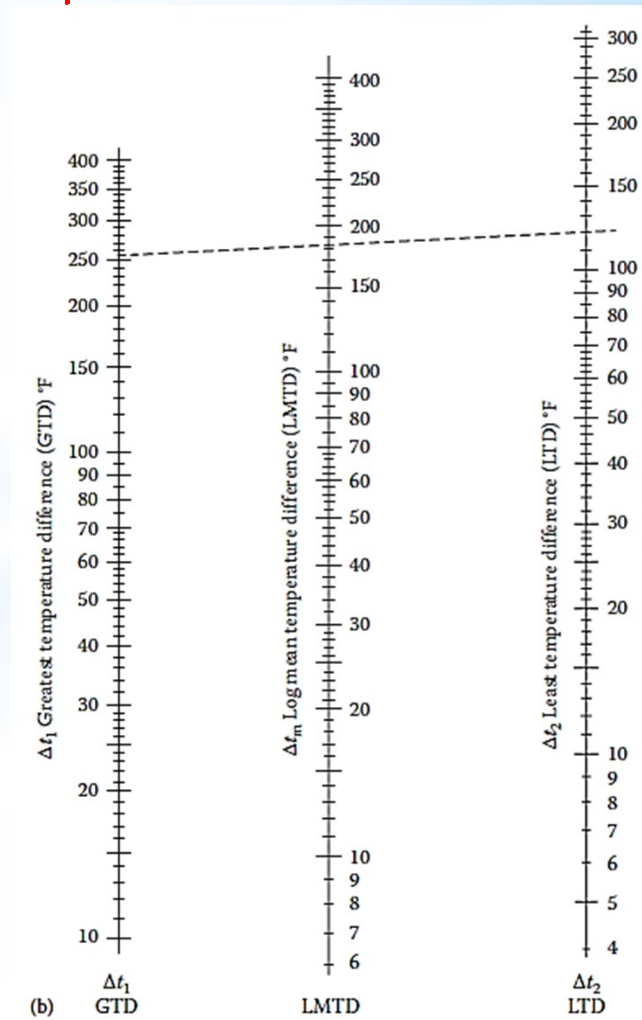
LMTD Correction Factor, F

LMTD calculated from the terminal temperature differences are shown in Figure.

From its definition, *F* is expressed by

$$F = \frac{\Delta t_m}{\Delta t_{lm}}$$





In situations where the heat release curves are nonlinear, the approach just described is not applicable and a "weighted" temperature difference must be determined.

It can be shown that, in general, *F* is dependent upon the thermal effectiveness S, the heat capacity rate ratio *R*, and the flow arrangement.

Therefore, *F* is represented by

$$F = (S, R, NTU_t, \text{flow arrangement})$$

and the expression for F in terms of P, R, and NTU is given by

$$\mathbf{F} = \frac{1}{(R-1)NTU} \ln\left[\frac{1-S}{1-SR}\right] \quad \text{for } R \neq 1$$

$$= \frac{S}{(1-S)NTU} \quad \text{for } R = 1$$

$$\mathbf{F} = \frac{1}{(1-C^*)NTU} \ln\left[\frac{1-\varepsilon C^*}{1-\varepsilon}\right] \quad \text{for } C^* \neq 1$$

$$= \frac{\varepsilon}{(1-\varepsilon)NTU} \quad \text{for } C^* = 1$$

The factor F is dimensionless. The value of F is unity for a true counter flow exchanger, and thus independent of S and R.

For other arrangements, F is generally less than unity, and can be explicitly presented as a function of P, R, and NTU_t.

The value of F close to unity does not mean a highly efficient heat exchanger, but it means a close approach to the counter flow behavior for the comparable operating conditions of flow rates and inlet fluid temperatures.

As a rule of thumb, the F value selected is 0.80 and higher.

ψ-S Method

The ψ-S method was originally proposed by Smith and modified by Mueller.

In this method, a new term ψ is introduced, which is expressed as the ratio of the true MTD to the inlet temperature difference of the two fluids:

$$\psi = \frac{\Delta t_m}{t_{h,i} - t_{c,i}} = \frac{\Delta t_m}{T_1 - t_1}$$

and ψ is related to ε and NTU and S and NTU_t as

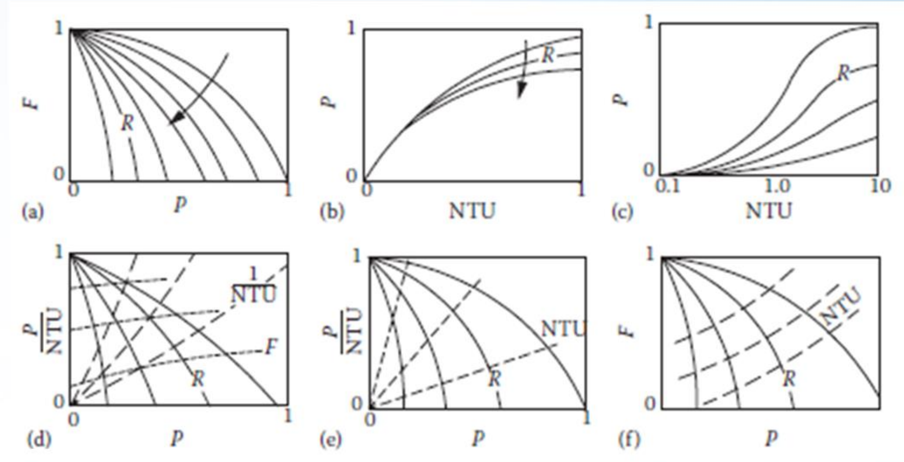
$$\psi = \frac{\varepsilon}{NTU} = \frac{S}{NTU_t}$$

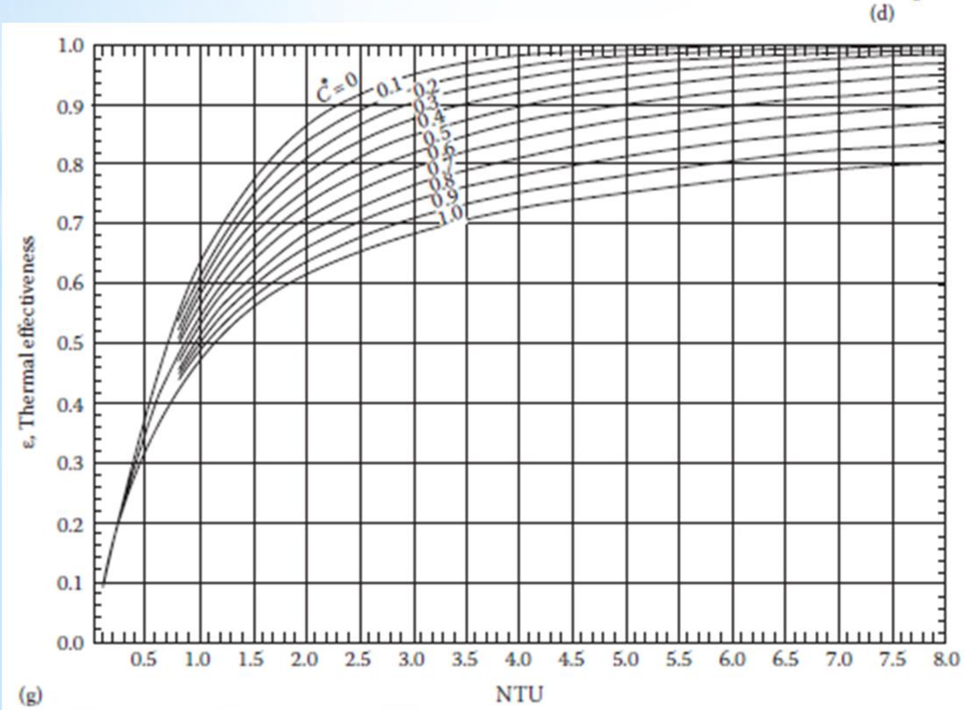
and the heat transfer rate is given by

$$q = UA\psi(t_{h,i} - t_{c,i}) = UA\psi(T_1 - t_1)$$

Since ψ represents the nondimensional Δt_m, there is no need to compute Δt_{lm} in this method.

THERMAL EFFECTIVENESS CHARTS





Thermal effectiveness charts.
(a) Bowman chart
(b) Kays and London chart
(c) TEMA chart5
(d) *F–S–R*-chart
(e) ψ chart
(f) *F-S-R*-NTU chart (From Turton, R. et al., *Trans. ASME J. Heat Transfer*, 106, 893, 1984)
(g) ε-NTU chart for unmixed–unmixed crossflow

Temperature Approach, Temperature Meet, And Temperature Cross

Temperature approach is the difference of the hotside and coldside fluid temperature at any point of a given exchanger.

In a counter flow exchanger or a multipass exchanger,

(1) if the cold fluid outlet temperature $t_{c,o}$ is less than the hot fluid outlet temperature $t_{h,o}$, then this condition is referred to as temperature approach;

(2) if $t_{c,o} = t_{h,o}$, this condition is referred to as temperature meet; and

(3) if $t_{c,o}$ is greater than $t_{h,o}$, the difference ($t_{c,o} - t_{h,o}$) is referred to as the temperature cross or temperature pinch. In this case, the temperature approach ($t_{h,o} - t_{c,o}$) is negative and loses its meaning.

Temperature cross indicates a negative driving force for heat transfer between the fluids. It requires either a large area for heat transfer or the fluid velocity to increase overall heat transfer coefficient.

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Temperature Approach

$t_{h,i} \rightarrow t_{h,o}$

$t_{c,o} \leftarrow t_{c,i}$

$t_{c,o} < t_{h,o}$

Temperature Meet

$t_{h,i} \rightarrow t_{h,o}$

$t_{c,o} \leftarrow t_{c,i}$

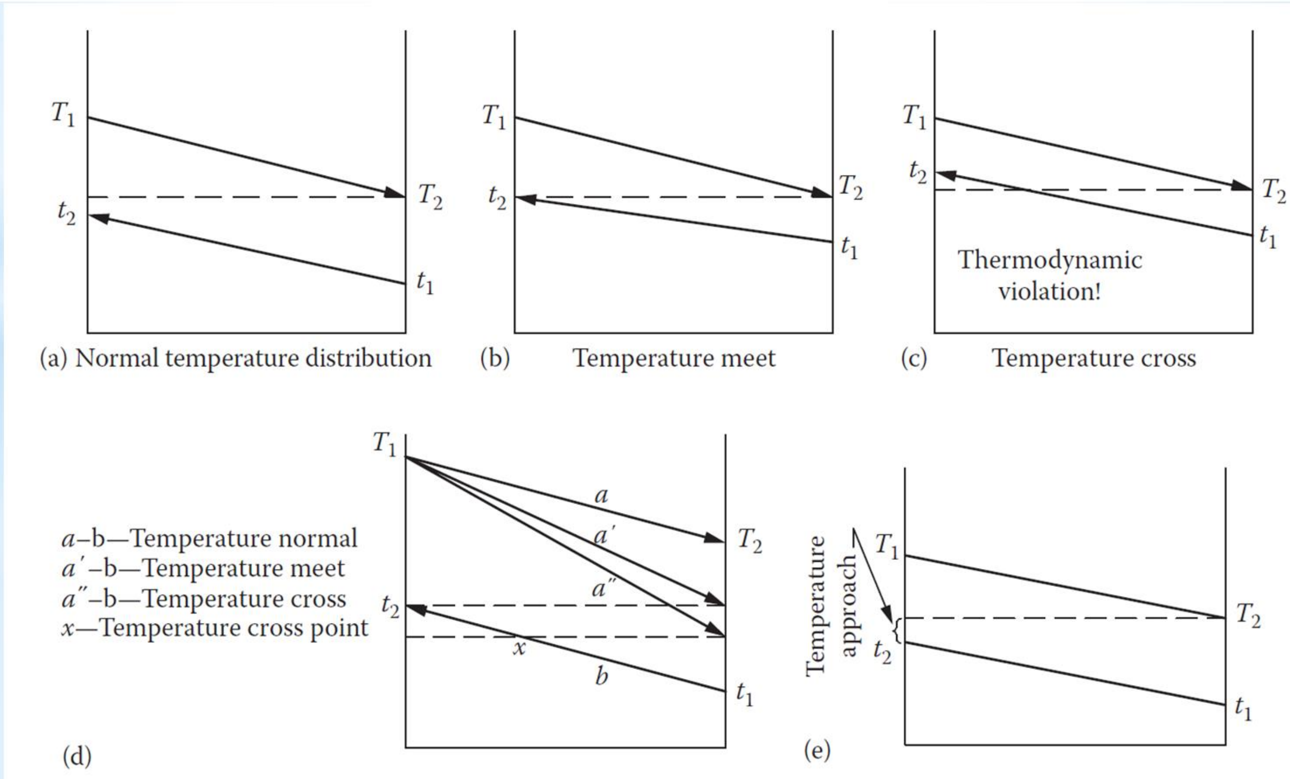
$t_{c,o} = t_{h,o}$

Temperature Cross

$t_{h,i} \rightarrow t_{h,o}$

$t_{c,o} \leftarrow t_{c,i}$

$t_{c,o} > t_{h,o}$



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Difference Between ΔT_{LM} for Cocurrent and Countercurrent Flow.