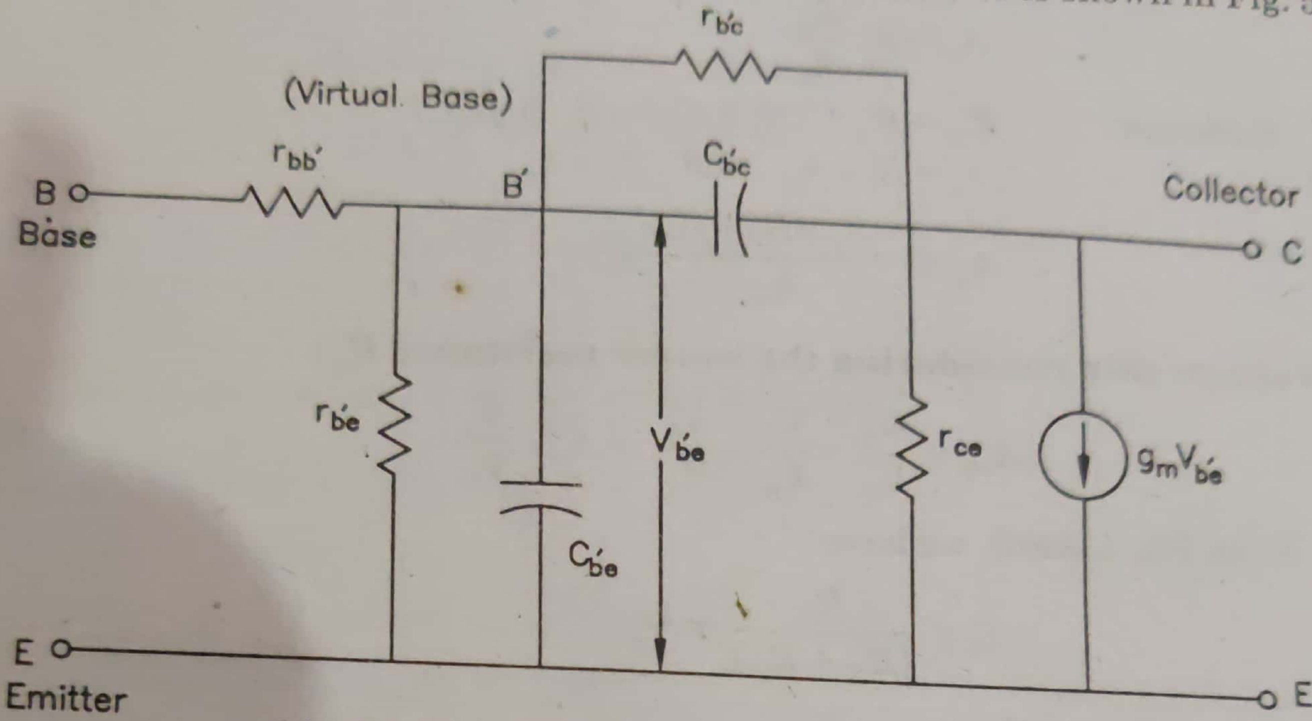
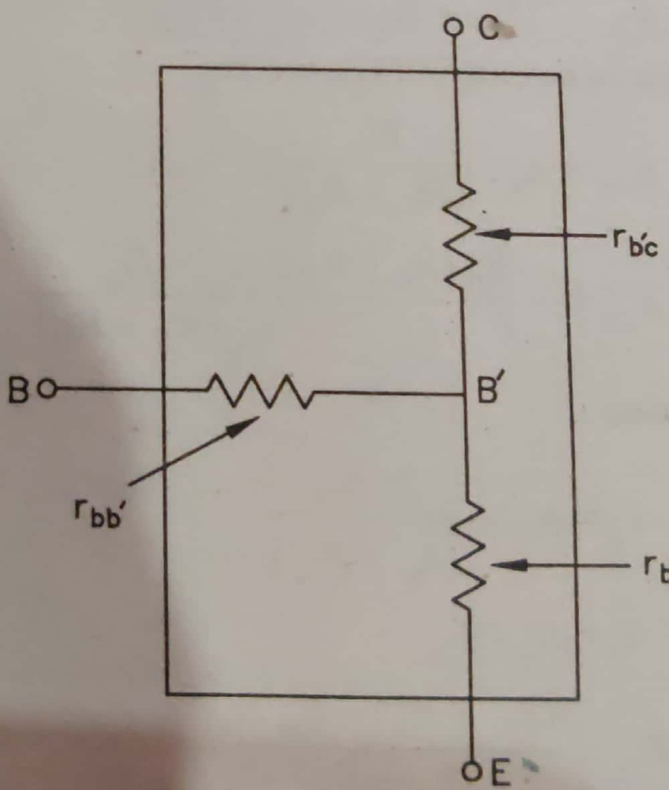
5.8 HIGH FREQUENCY π MODEL FOR A TRANSISTOR

At high frequencies, the *capacitive effects* of the transistor junctions and the *delay* in response of the transistor caused by the process of diffusion of carriers should be taken into account in determining the high frequency model of a transistor.

A *hybrid* π (*Pie*) or *Giacoletto model* for a transistor is shown in Fig. 5.21.



(*a*) Hybrid π model for a transistor in the *CE* connection



(*b*) Diagram showing virtual base *B*′ and ohmic base-spreading resistance $r_{bb'}$

Fig. 5.21.

The components of hybrid π model are as follows :

1. Base spreading resistance (r_{bb}'). It represents the base spreading resistance and account for the voltage drop in the path between the base contact and active base region. Its value decreases with the increase in current value. Its value is in range of 40 Ω and 400 Ω.

2. AC resistance ($r_{b'e}$). This is the resistance between the virtual base B' and the emitter terminal E. It represents the ac resistance of a forward biased emitter-base junction. Typical value is 1 kΩ.

Input resistance from base to emitter with the output shorted is simply $r_{bb}' + r_{b'e}$ and this is same as h_{ie}. Hence

$$h_{ie} = r_{bb}' + r_{b'e} \quad \text{...(51)}$$

3. Resistance ($r_{b'c}$). This is the resistance between the virtual base B' and the collector terminal C. It represents the effect of feedback between the emitter base junction and collector base junction due to an early effect. It has a large value (typical value = 4 MΩ).

4. Diffusion capacitance ($C_{b'e}$). This is the capacitance of the normally forward biased base-emitter junction. It has a typical value of 100 pF.

5. Transistor capacitance ($C_{b'c}$). This is the capacitance of the normally reverse biased collector-base junction. It has a typical value of 3 pF.

6. Resistance (r_{ce}). It represents output resistance with a typical value of 80 kΩ. Since $r_{ce} >> R_L$, if a load R_L is connected r_{ce} can be neglected.

7. Controlled current source ($g_m V_{b'e}$). It represents the coupling between the junctions. Its value is proportional to the base current (i_b). (g_m is the transconductance of the transistor). The transconductance represents the small change in collector current about the operating point produced by the small changes in base emitter voltage.

5.8.1 Relation between Hybrid-π and *h*-parameter

***(a)* Transconductance g_m** : Transconductance is the ratio of change in collector current due to a small change in base-emitter voltage of a transistor

$$\therefore \quad g_m = \frac{\delta I_c}{\delta V_{b'e}}$$

It is possible to derive from the above expression the final form, which is

$$\boxed{g_m = \frac{|I_c|}{V_T}} \quad \text{...(52)}$$

where $|I_c|$ = magnitude of collector current

and $V_T = \frac{T}{11600}$ = 0.026 volts at room temperature

Hence, $g_m = \frac{I_c \text{ (in mA)}}{26}$

***(b)* Input Resistance ($r_{b'e}$) and input conductance ($g_{b'e}$)** : It is clear from Fig. 5.21 that the voltage drop across resistance $r_{b'e}$ is given by the relation

$$V_{b'e} = i_b \cdot r_{b'e} \quad \text{...(53)}$$

where i_b is the base current.

It may also be noted that if the collector to emitter voltage (V_{CE}) is equal to zero, then there is no current through resistance r_{ce}. In this case the whole current will pass through the controlled current generator. The value of collector current,

$$i_c = g_m \cdot V_{b'e} \qquad ...(54)$$

Substituting the value of $V_{b'e}$ ($= i_b \cdot r_{b'e}$) in the above equation, the collector current

$$i_c = g_m \cdot i_b \cdot r_{b'e}$$

$$\frac{i_c}{i_b} = g_m \cdot r_{b'e} \qquad ...(55)$$

This is called *small signal short circuit current gain*, ($= h_{fe}$). Thus ac current gain

$$h_{fe} = \frac{i_c}{i_b} = g_m \cdot r_{b'e}$$

Hence, Input resistance

$$\boxed{r_{b'e} = \frac{h_{fe}}{g_m}} \qquad ...(56)$$

and Input conductance

$$g_{b'e} = \frac{1}{r_{b'e}} = \frac{g_m}{h_{fe}} \qquad ...(57)$$

(c) Base-spreading Resistance ($r_{bb'}$) : We know that h_{ie} is the input resistance when the output terminals *i.e.* collector and emitter are shorted. Under this condition $r_{b'c}$ comes in parallel with $r_{b'e}$. But as $r_{b'c} >> r_{b'e}$.

$\therefore \qquad r_{b'e} \parallel r_{b'c} \simeq r_{b'e}$

The value of base spreading resistance may be obtained by using the relation,

$$\boxed{r_{bb'} = h_{ie} - r_{b'e}} \qquad ...(58)$$

(d) Output conductance (g_{ce}) and output resistance (r_{ce}) : The value of output conductance may be obtained by the relation,

$$g_{ce} = h_{oe} - (1 + h_{fe})\, g_{be} \qquad ...(59)$$

where h_{oe} = Output conductance with input as short-circuited

$g_{b'c}$ = Feedback conductance

$= h_{re} \cdot g_{b'e}$

The parameter h_{re} is called reverse voltage gain and is usually specified by the manufacturer.

In actual practice the value of $h_{fe} >> 1$. In what case the output conductance

$$g_{ce} = h_{oe} - h_{fe} \cdot g_{b'e} \qquad ...(60)$$

Substituting the value of $g_{b'c}(=h_{re} \cdot g_{b'c})$ in the above equation, the value of output conductance

$$g_{ce} = h_{oe} - h_{fe} \cdot h_{re} \cdot g_{b'c}$$

$$\boxed{g_{ce} = h_{oe} - h_{re} \cdot g_m} \quad ...(61)$$

∴ Value of output resistance

$$r_{ce} = \frac{1}{g_{ce}} = \frac{1}{h_{oe} - (1+h_{fe})\, g_{b'c}}$$

$$\boxed{= \frac{1}{(h_{oe} - h_{re} g_m)}} \quad ...(62)$$

(e) Feedback resistance ($r_{b'c}$) : As reverse voltage gain in h parameter model is given as :

$$h_{re} = \frac{V_{b'e}}{V_{ce}} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}} \quad ...(63)$$

or $\quad h_{re} \cdot r_{b'e} + h_{re} \cdot r_{b'c} = r_{b'e}$

or $\quad r_{b'e}(1 - h_{re}) = h_{re} \cdot r_{b'c}$

Since $\quad h_{re} << 1$

∴ $\quad r_{b'e} = h_{re} \cdot r_{b'c}$

or Feedback resistance $r_{b'c} = \dfrac{r_{b'e}}{h_{re}}$ ...(64)

Table 5.2 shows the relationship between low frequency h-parameters and high frequency parameters.

Table 5.2.

(i)	$g_m = \dfrac{\lvert I_c \rvert}{V_T}$
where	$V_T = \dfrac{T}{11{,}600}$ with T in °K. At room temperature (300 °K),
	$V_T = 0.026$ V
	$g_m = \dfrac{I_c \text{ (in mA)}}{26}$
(ii)	$r_{b'e} = \dfrac{h_{fe}}{g_m} = \dfrac{h_{fe} V_T}{\lvert I_c \rvert}$ or $g_{b'e} = \dfrac{g_m}{h_{fe}}$
(iii)	$r_{bb'} = h_{ie} - r_{b'e}$
(iv)	$r_{b'c} = \dfrac{1}{g_{b'c}} = \dfrac{r_{b'e}}{h_{re}}$
(v)	$g_{ce} = \dfrac{1}{r_{ce}} = h_{oe} - (1+h_{fe})\, g_{b'c}$
(vi)	$r_{b'c} = \dfrac{1}{h_{re}} r_{b'e}$

5.16 CASCODE AMPLIFIER

Cascode amplifier is a composite amplifier pair with a large bandwidth, consists of a *CE* stage followed by a *CB* stage directly coupled to each other and combines some of the features of both the amplifiers.

For high frequency applications, *CB* configuration has the most desirable characteristics. However, it suffers from low input impedance ($Z_i \approx h_{ib}$). The cascode configuration is designed to have the input impedance essentially that of *CE* amplifier, the current gain that of *CE* amplifier, the voltage gain that of *CB* amplifier and good isolation between the input and output. Fig. 5.29 shows a cascode amplifier.

This amplifier is used for *RF* applications and as a video amplifier.

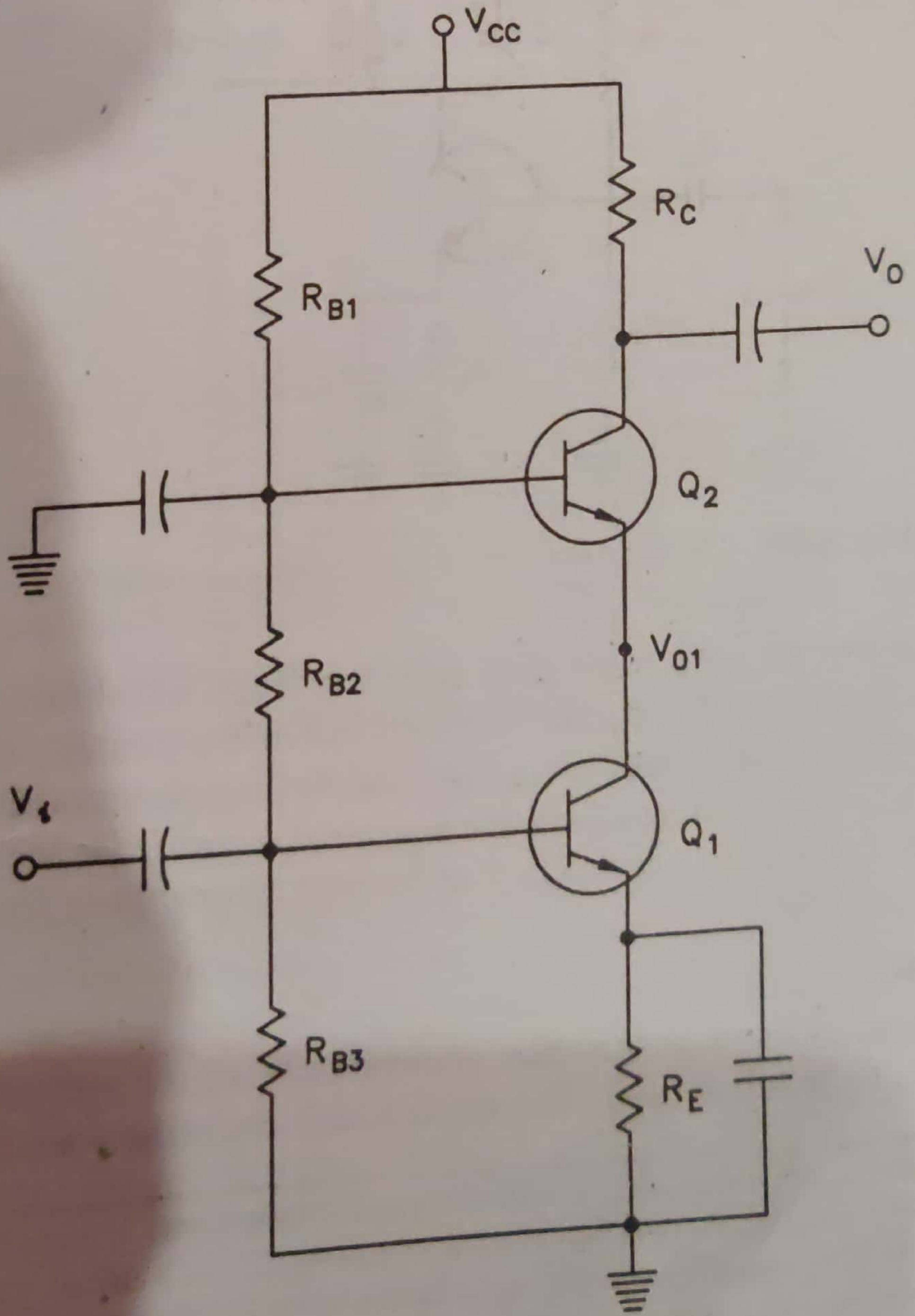


Fig. 5.29. Cascode amplifier.