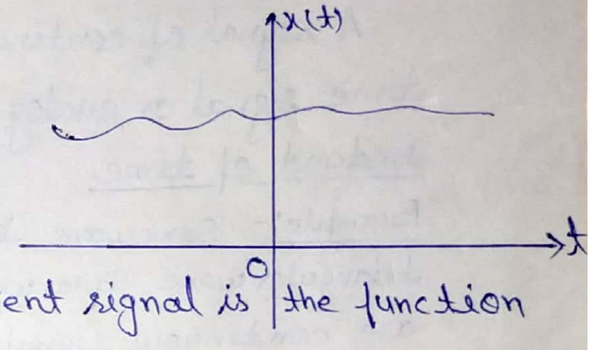
Introduction to Signals and Systems

1. Definition of Signal :⇒

- A signal may be a function of time, temperature, position, pressure, distance etc.
- Mathematically, A function of one or more independent variables which contains some information is called a signal.
- Some signals are used in our daily life ~~are~~ such as –

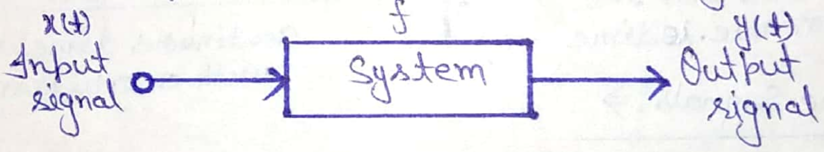
  - Music signal
  - Speech signal
  - Picture signal
  - Video signal
  - etc.



- In electrical sense, the voltage or current signal is the function of time as an independent variable.
- Many signals that are naturally generated signals. However few signals are also generated systematically.

2. System :⇒

- A system is defined as the entity that operates on one or more signals to accomplish a function, to produce new signals.



- Mathematically, the functional relationship between input and output, such as

$$y(t) = f[x(t)]$$

- (This means a functional blocks (f) produces an ouput in response to an input signal)
- Some systems are used in our daily life as –

  - Filters
  - Tuners
  - Amplifiers (IF, video, Audio,)
  - Communication channels
  - T.V. set, etc.

- This subject is also part of engineering such as signal processing and control system.

- In this modern age of microelectronics, signal and system play very important roles.
- This subject diverse application in area of science and technology such as circuit design, seismology, communication, biomedical engg. energy generation and distribution, speech processing etc.
- It is neccessery to every engineer must have good knowlegde of this subject.

3. Classification of Signals:⇒

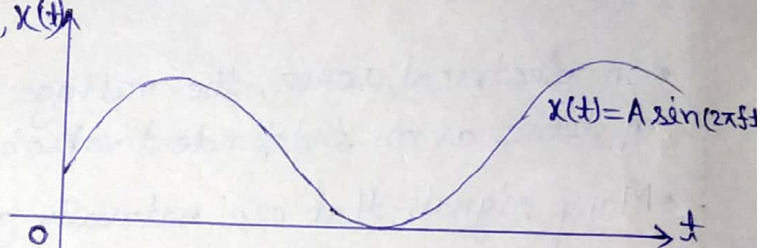
Based upon their nature and characteristics in the time domain, the signals may be broadly classified as
(i) Continuous Time Signals
(ii) Discrete Time Signals

Continuous Time Signals:⇒

A signal of continuous amplitude and time is known as a continuous time signal or analog signal, i.e. it have some values at every ~~time~~ instant of time.

Example:- Sinewave, Cosinewave, triangularwave, Squarewave etc. are continuous signals

- A continuous-time signal is represented by $x(t)$.



Continuous time signal

Represents the shape of the signal
$x(t)$
Shows that the independent variable. i.e. time

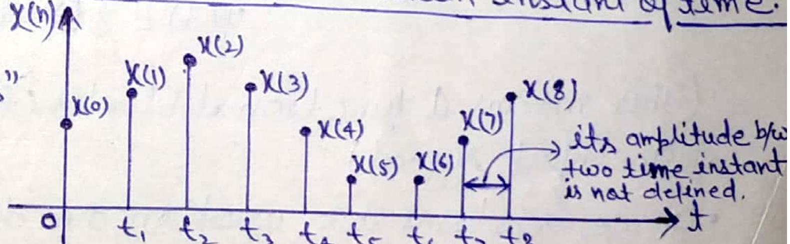
Continuous time signal with an arbitrary shape

Discrete Time Signals:⇒

A signal is represented only at certain time-instants is known as discrete time signal. i.e. it have values at certain instant of time.

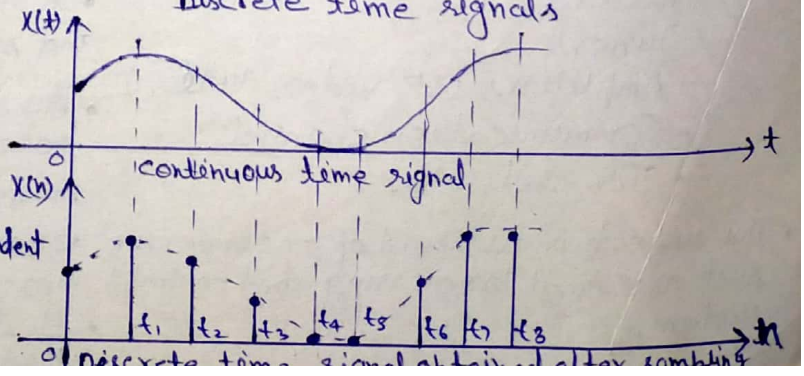
- The discrete time signal is obtained by "sampling process" of the analog signals

- A discrete time signal is represented by $x(n)$.



Discrete time signals

Represented the shape of the signal
$x(n)$
Show that the independent variable.



- The discrete time signal can also be visualised as a sequence of samples taken at uniform intervals and denoted as $x(n)$.

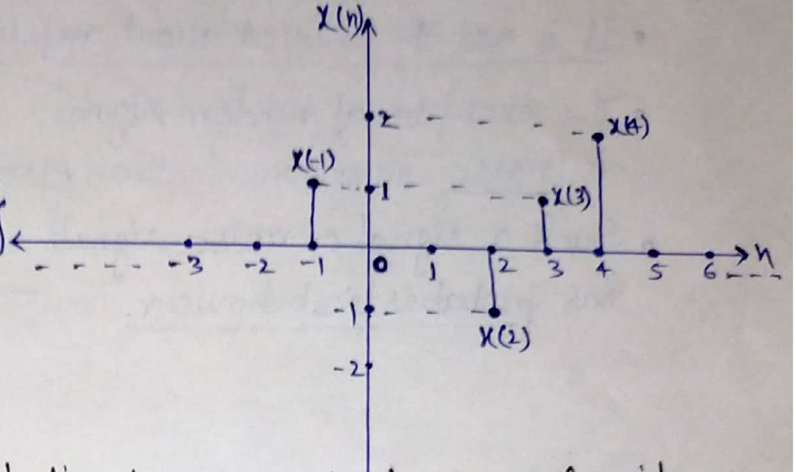
Example:-

Mathematically it is denoted as

$$x(n) = \{\ldots 0, 0, 1, 2, 0, -1, 1, 2, 0, 0, \ldots\}$$

↑ at $n=0$

(Sequence of Samples)



⇒ Both continuous-time and discrete-time signals may classify as—

(i) Deterministic and Non-deterministic signals

(ii) Periodic and Aperiodic signals

(iii) Even and odd signals

(iv) Energy and Power signals.

Deterministic Signal :⇒

A signal which can be described by a mathematical expression, or well defined rule is known as deterministic signal.

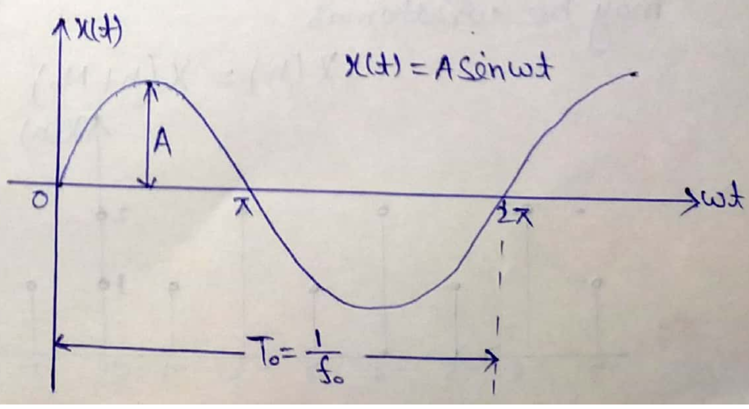
- It can be represented mathematically as

(a) $x(t) = A \sin(2\pi f t)$

where, A is the peak amplitude and f is the frequency of the signal

(b) $x(n) = \begin{cases} 2 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

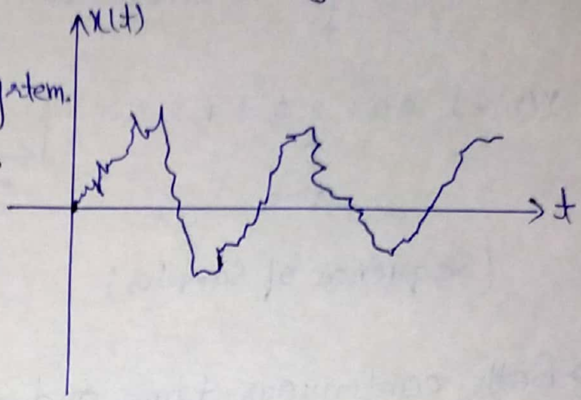
Hence, it is clear that the amplitude at any time instant can be predicted in advance.



Non-Deterministic Signal :⇒

A signal which can not be described by any mathematical expression is called non-deterministic OR random signals.

- It is not to predict about amplitude at any instant of time.
- The example of random signal is 'noise' in communication system.
- Such a signal or noise signals has probabiistic behaviour.



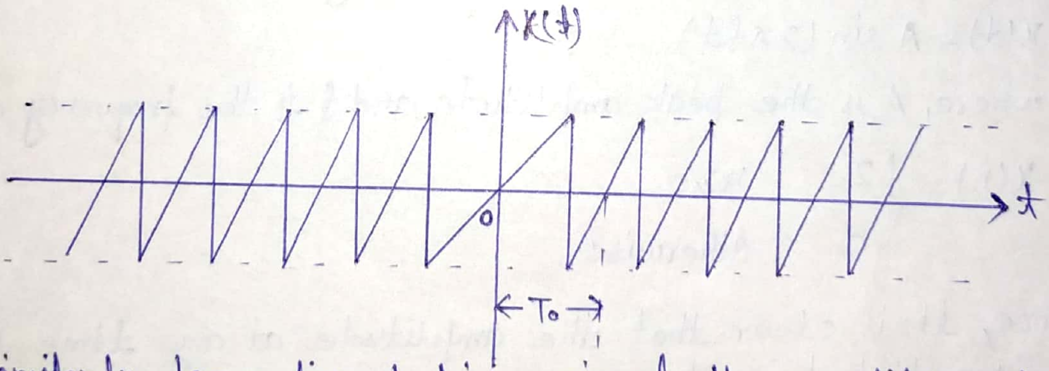
Periodic Signal :⇒

A signal which repets itself after a fixed time period is called a periodic signal. In other words, a signal is called periodi if it exhibits periodicity.

- It can be defined mathematically as follows:

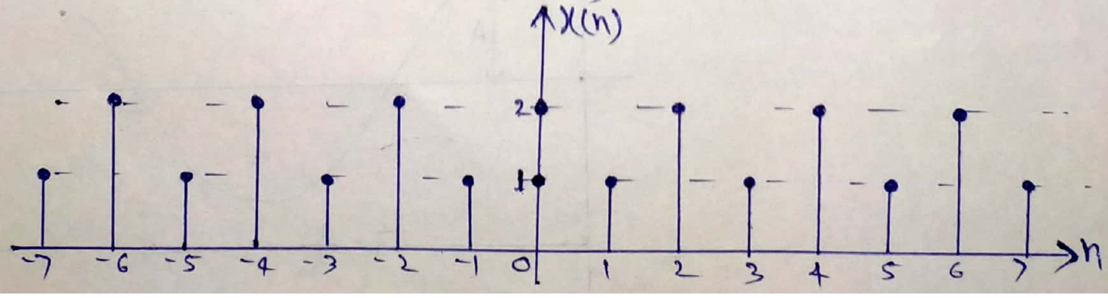
$$x(t) = x(t+T_0) \; ; \{-\infty < t < \infty\}$$

condition of periodicity



Similarly, for a discret-time signal, the condition of periodicity may be written as

$$x(n) = x(n+N_0) \; ; \; -\infty < n < +\infty$$



Aperiodic Signals :⇒

A signal is said to be aperiodic if it does not repeats at any instant.

- it do not satisfy the condition of periodicity

  i.e. $x(t) \neq x(t+T_0)$

- Some times aperiodic signal are said to have a period $T_0 = \infty$. e.g. shows a decaying exponential signal, i.e.

$$x(t) = e^{-\alpha t}$$

- Mathematically,

$$x(t) = e^{-\alpha t}$$

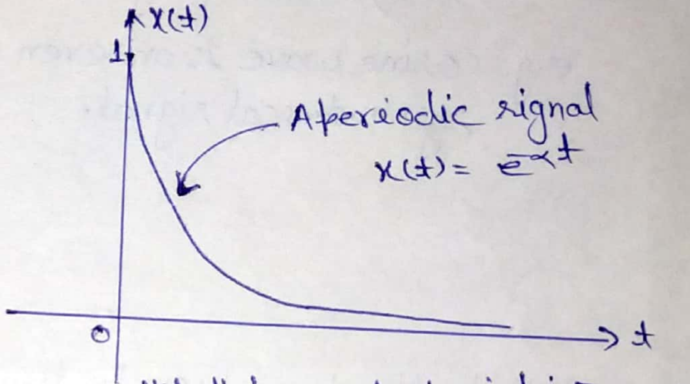
Now, $x(t+T_0) = e^{-\alpha(t+T_0)}$

$= e^{-\alpha(t+\infty)}$

or, $x(t+T_0) = e^{-\alpha t} \cdot e^{-\infty}$

$= e^{-\alpha t} \cdot 0$

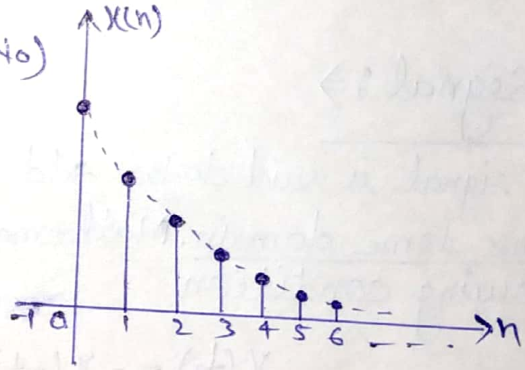
$x(t+T_0) = 0$ (Which is not equal to $x(t)$)



Note that one cycle period is $T_0 = \infty$

⇒ Similarly, for a discret-time signal aperiodic signal can be expressed as, mathematically

$$x(n) \neq x(n+N_0)$$



Even Signals :⇒

A signal is said to be even signal which exhibits symmetry in the time domain.

- Mathematically, an even signal must satisfy the following ~~satis~~ condition

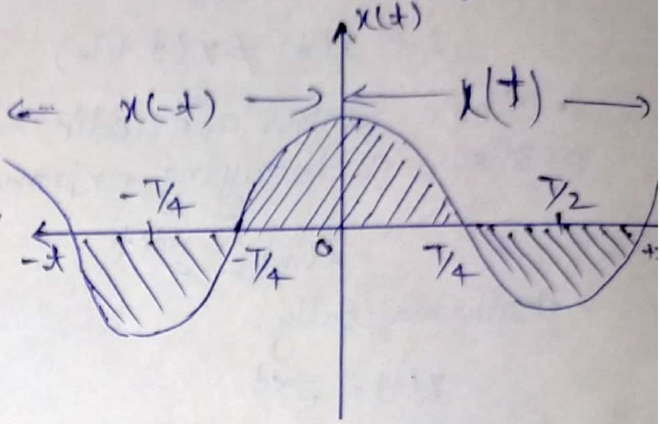
$$x(t) = x(-t)$$

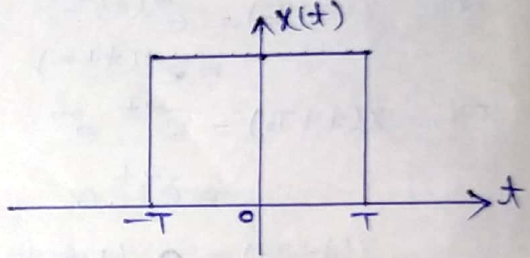
where,

$x(t)$ = value of signal for +ive t.

$x(-t)$ = value of signal for −ive t.

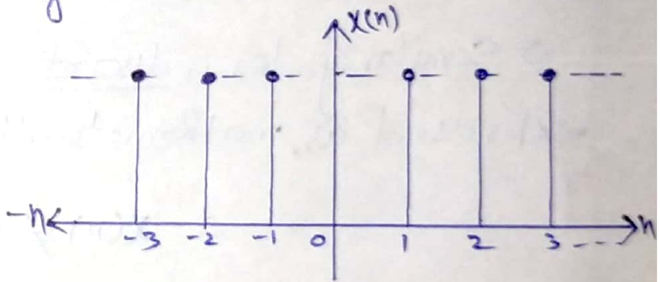
e.g. cosine wave is an even or symmetrical signal.





⇒ Similarly, for a discrete-time even signal satisfy the following condition

$$x(n) = x(-n)$$

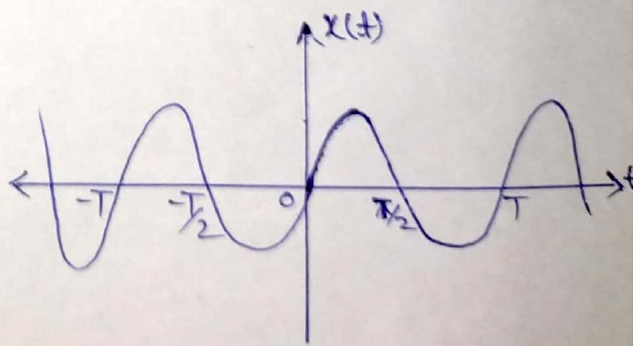


Odd Signal :⇒

A signal is said to be odd signal which exhibits anti-symmetry in the time domain. Mathematically, an odd signal satisfy the following condition

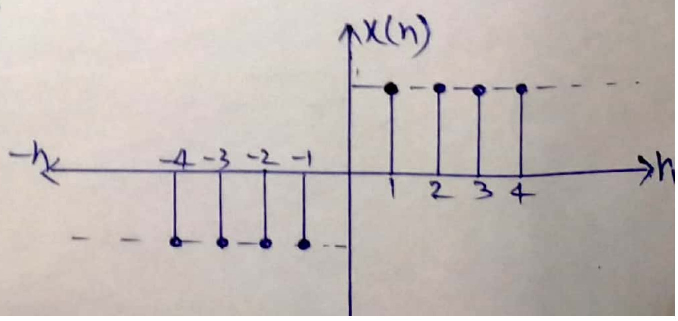
$$x(t) = -x(-t)$$

e.g sine wave is an odd or antisymmetrical signal.



⇒ Similarly, for a discrete-time odd signal satisfy the following condition

$$x(n) = -x(-n)$$



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Energy and Power Signal :⇒

Signal may also be classified as energy and power signals. i.e. some signals which can neither be classified as energy signals nor power signals.

- A signal having a finite energy and zero average power is called as an energy signal.
  Hence, $x(t)$ is an energy signal if :

$$0 < E < \infty \quad \text{and} \quad P = 0$$

- It has been observed almost all non-periodic signals are energy signals

⇒
- A signal having a finit average power and infinite energy is called as a power signal.
  Hence, $x(t)$ is a power signal if :

$$0 < P < \infty \quad \text{and} \quad E = \infty$$

- It has been observed almost all periodic signals are power signals.

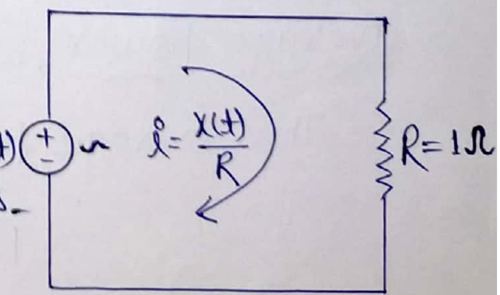
➡ However, if the signal does not satisfy any of the above two conditions, then it is neither an energy signal nor a power signal.

Signal Energy and Power :⇒

In fact, the signal energy does not indicate the actual energy of the signal because the signal energy also depends on the load resistor.

Here, energy is interpreted as the energy dissipated in a normalized load of a 1-Ω resistor.

Now, if a voltage $x(t)$ is applied across a resistor, the current through the resistor is



$$i = \frac{x(t)}{R}$$

hence, the instantaneous power is

$$p(t) = v(t) \cdot i(t)$$

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Therefore,

$$p(t) = x(t) \cdot \frac{x(t)}{R} = \frac{x^2(t)}{R}$$

⇒ We know that total energy dissipated is the integral of the instantaneous power. i.e. the energy dissipated is –

$$E = \int_{-\infty}^{\infty} \frac{x^2(t)}{R}\, dt$$

Now, if $R = 1\Omega$, we get

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt \quad \text{(for real signal)}$$

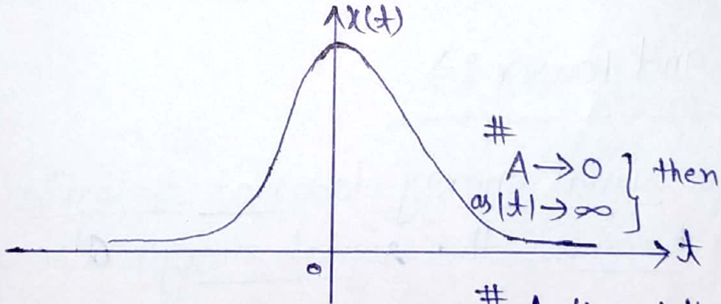
for a complex valued signal,

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

Similarly, the energy in discrete-time signal $x(n)$ is

$$E = \sum_{n=-\infty}^{+\infty} |x(n)|^2$$

⇒ from above eq. the signal energy must be finite. therefore the signal amplitude must go to zero as $|t| \to \infty$, otherwise the integral equation or sum equation will not correct.



$\left.\begin{array}{l} A \to 0 \\ \text{as } |t| \to \infty \end{array}\right\}$ then E is finite. (Non-periodic)

$\left.\begin{array}{l} A \not\to 0 \\ \text{as } |t| \to \infty \end{array}\right\}$ then E is infinite. (Periodic)

In such cases, we define another parameter called as average power, which is the time average of energy.

The average power P is given by;

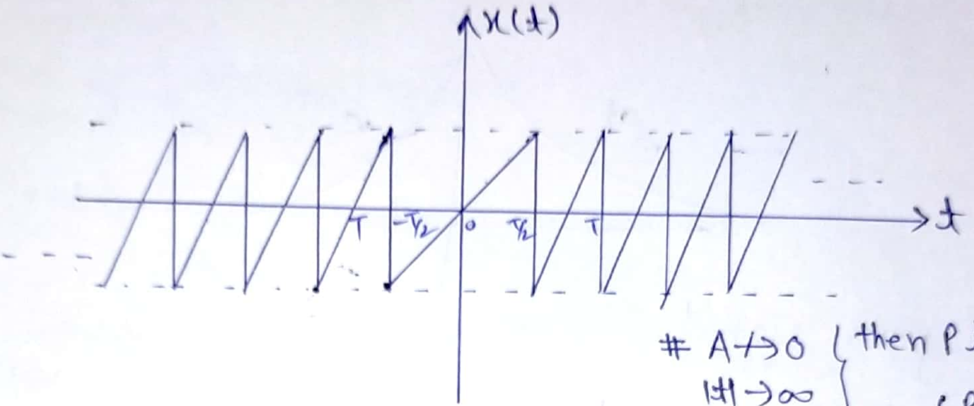
$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

Or,

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt \quad \text{(for complex valued signal)}$$

Similarly, for a discrete-time signal

$$P = \lim_{N \to \infty} \frac{1}{N} \sum_{n=-\frac{N}{2}}^{N/2} |x(n)|^2$$



$\#\ A \not\to 0 \\ |t| \to \infty \Big\}$ then P is finite (Periodic)