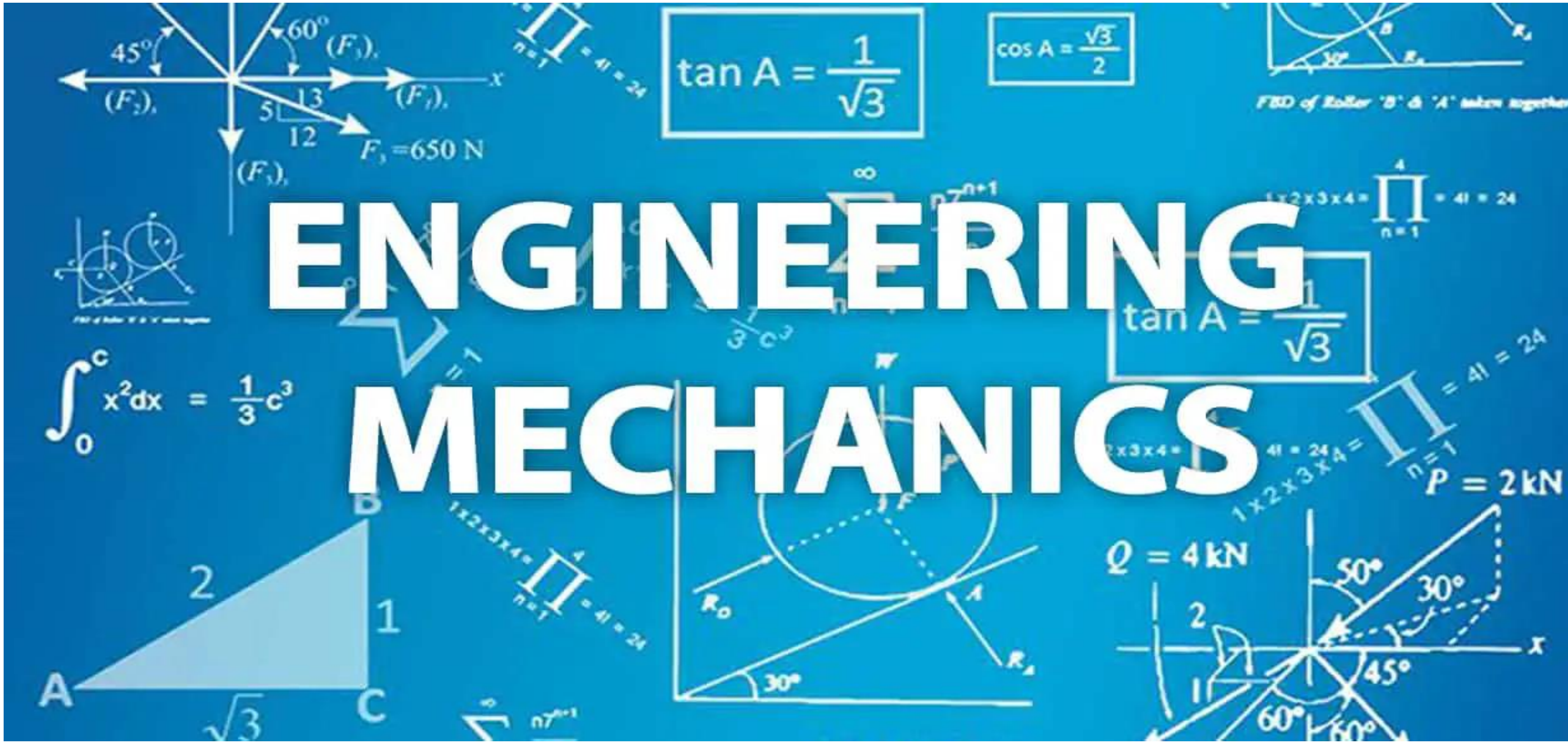


Asst. Prof. Yastuti Rao Guatam
Mechanical Engineering Department
UIET, CSJM Univ. Kanpur

Course Code: ESC-S201
Course Name: Engineering Mechanics

Course Details:

General Coplanar force systems : Basis concepts, Law of motions, principle of transmissibility of forces, Transfer of a force to parallel position, Resultant of a force system, simplest resultant of two dimensional concurrent & non concurrent force systems, free body diagrams, equilibrium & its equations, applications.

Trusses & Cables : Introductions, simple truss & solutions of simple truss, method of joints & method of sections.

Friction : Introduction , Laws of coulomb friction, equilibrium of bodies involving dry friction, belt friction, applications.

Centre of gravity , centroid, Moment of Inertia : Centroid of plane, curve, area ,volume & composite bodies, moment of inertia of plane area, parallel axis theorem, perpendicular axis theorem, principal moment inertia, mass moment of inertia of circular ring, disc, cylinder, sphere and cone about their axis of symmetry.

Beams: Introductions, shear force and bending moment , differential equations for equilibrium, shear force & bending moments diagrams for statically determinate beams.

Kinematics of rigid body: Introduction, plane motion of rigid bodies, velocity & acceleration under translation & rotational motion, Relative velocity, projectile motion.

Kinetics of rigid bodies: Introduction, force, mass & acceleration, work & energy, impulse & momentum, D'Alembert principles & dynamic equilibrium. Virtual work.

Resultants of Force Systems
&
Equilibrium of Force Systems

Laws of mechanics

- Newton’s first law
- Newton’s second law
- Newton’s third law
- Newton’s law of gravitation
- Law of parallelogram
- Principle of Transmissibility

Newton's first law: A body continues in its state of rest, or in uniform motion in a straight line, unless acted upon by a external force.

Newton's second law : A body acted upon by a force moves in such a manner that the time rate of change of momentum equals the force.

Newton's third law : If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction.

Newton's law of gravitation

- Force of attraction between any two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them .

$$F = G\frac{m_1 m_2}{d^2}$$

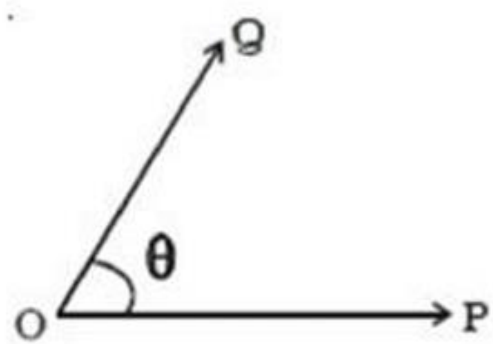
G is constant of gravitation

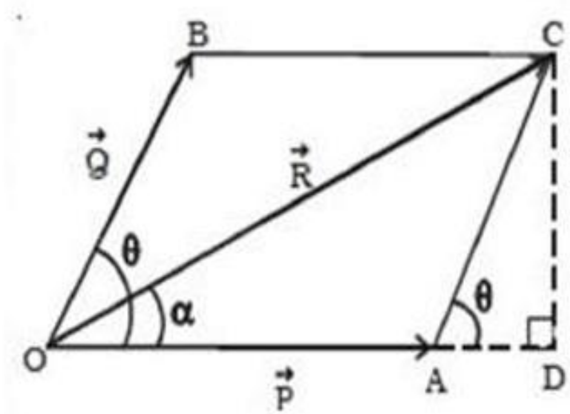
m_1 and m_2 mass of bodies

PARALLELOGRAM LAW OF FORCES

"If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their

point of intersection."





Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

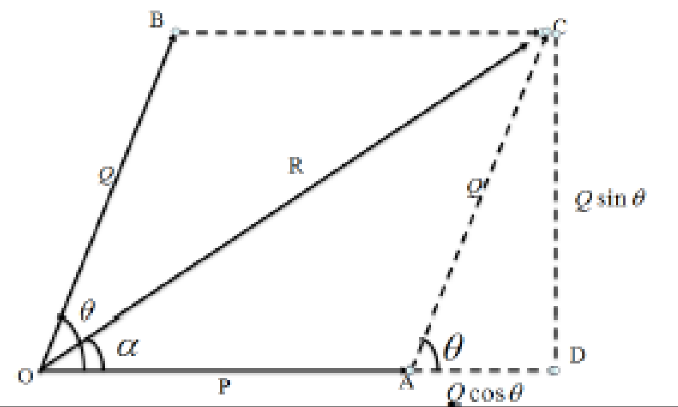
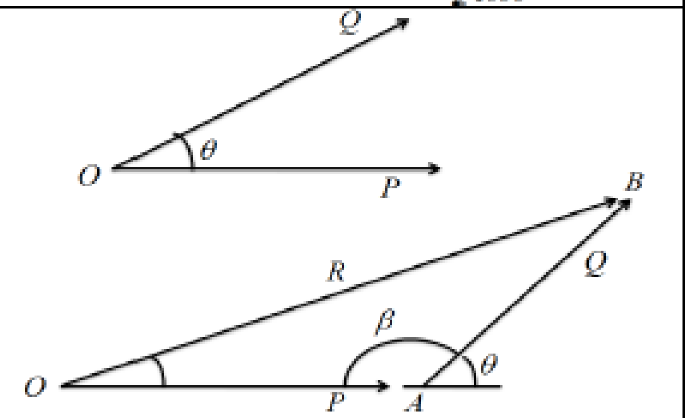
and

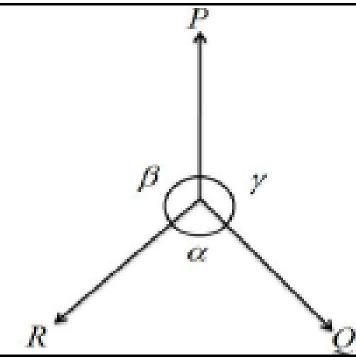
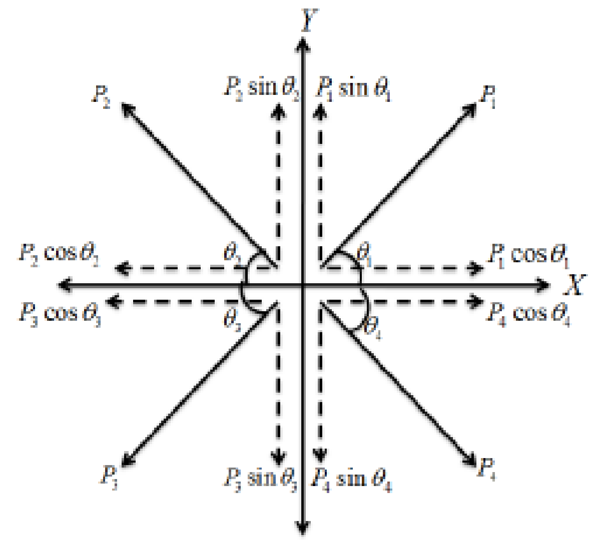
$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

[1]	**Parallelogram law of forces** $R=\sqrt{P^2+Q^2+2PQ\cos\theta}$ $\tan\alpha=\dfrac{Q\sin\theta}{P+Q\cos\theta}$ Where, R = Resultant force θ = angle between P and Q α = angle between P and R	
[2]	**Triangle law of forces** $R=\sqrt{P\ \ +Q^2-2PQ\cos\beta}$ Where, $180^{\circ}-\theta$ R =Resultant force θ = angle between P and Q α = angle between P and R $\alpha=\sin^{-1}\left(\dfrac{Q}{R}\sin\beta\right)$	

[3]	**Lami's theorem** $\dfrac{P}{\sin\alpha}=\dfrac{Q}{\sin\beta}\quad\dfrac{R}{\sin\gamma}$ Where, P, Q, R are given forces α = angle between Q and R β = angle between P and R γ = angle between P and Q	
[4]	**Resolution of concurrent forces** $\sum H = P_1\cos\theta_1 + P_2\cos\theta_2 + P_3\cos\theta_3 + P_4\cos\theta_4$ $\sum V = P_1\sin\theta_1 + P_2\sin\theta_2 + P_3\sin\theta_3 + P_4\sin\theta_4$ $R=\sqrt{(\quad H)^2\quad(\sum V)}$ $\tan\theta = \left\lvert\dfrac{\sum V}{\sum H}\right\rvert$ Where, P_1, P_2, P_3, P_4 are given forces $\theta_1, \theta_2, \theta_3, \theta_4$ are angle of accordingly P_1, P_2, P_3, P_4 forces from X-axes R = Resultant of all forces θ = angle of resultant with horizontal	

SYSTEM OF FORCES

When several forces of different magnitude and direction act upon a body, they constitute a system of forces.

If all the forces in a system lie in a single plane, it is called a **coplanar** force system.

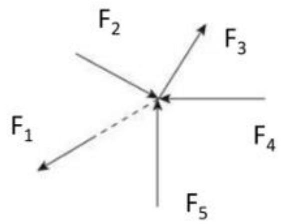
If the line of action of all the forces in a system pass through a single point it is called a **concurrent force system**.

In a system of **parallel forces** all the forces are parallel to each other.

If the line of action of all forces lie along a single line then it is called a **collinear force system**

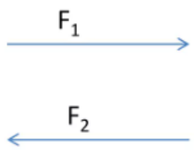
Force System	*Characteristic*	*Examples*
Collinear forces	Line of action of all the forces act along the same line.	Forces on a rope in a tug of war
Coplanar parallel forces	All forces are parallel to each other and lie in a single plane.	System of forces acting on a beam subjected to vertical loads (including reactions)
Coplanar like parallel forces	All forces are parallel to each other, lie in a single plane and are acting in the same direction.	Weight of a stationary train on a rail when the track is straight
Coplanar concurrent forces	Line of action of all forces pass through a single point and forces lie in the same plane.	Forces on a rod resting against a wall
Coplanar non-concurrent forces	All forces do not meet at a point, but lie in a single plane.	Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity
Non-coplanar parallel forces	All the forces are parallel to each other, but not in the same plane.	The weight of benches in a class room
Non-coplanar concurrent forces	All forces do not lie in the same plane, but their lines of action pass through a single point.	A tripod carrying a camera
Non-coplanar non-concurrent forces	All forces do not lie in the same plane and their lines of action do not pass through a single point.	Forces acting on a moving bus

•**Coplanar forces- Coplanar forces** means the **forces** in a plane.
Or a system in which all the **forces** lie in the same plane, it is known as coplanar **force** system.

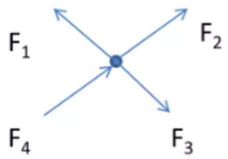


•**Coplanar Collinear forces** - **Collinear forces** are those **forces** which have a common line of action, i.e. the line of action of the **forces** lie along a single straight line either they are push or pull in nature.
Examples: two people standing at the opposite ends of a rope and pulling on it.

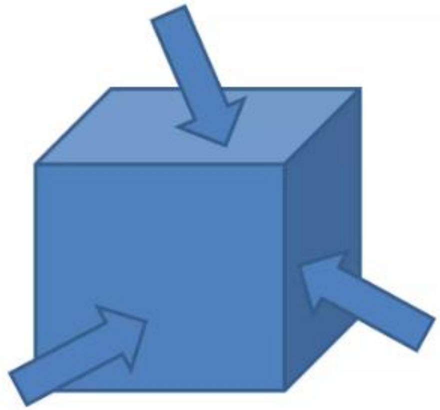
•**Coplanar Parallel forces**- Parallel forces are those forces which are in the same plane but never intersect by each other and they may be same or opposite in direction.



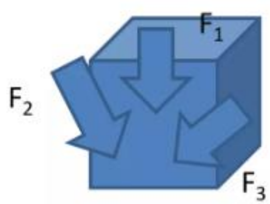
•**Coplanar Concurrent Forces-** Concurrent forces are those forces which are acting at a same point and at a same plane, also they may be pull or push in nature.



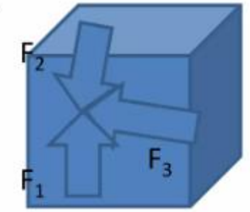
Non-coplanar forces-Non-coplanar forces are those forces which are not acting from a same plane.



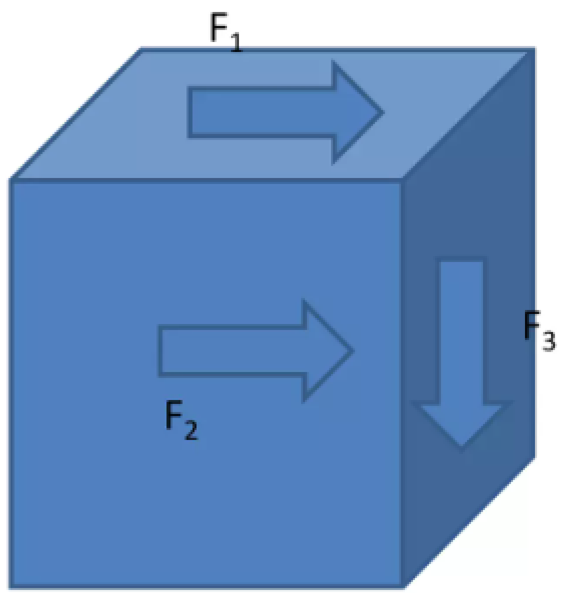
•Non coplanar non- concurrent forces- Non- Coplanar non-concurrent forces are those forces which are not acting at a same point and not at a same plane, also they may be pull or push in nature.



•Non-coplanar concurrent forces- Non- Coplanar concurrent forces are those forces which are acting at a same point but not from a same plane, also they may be pull or push in nature.



•Non-coplanar parallel forces- Non-coplanar Parallel forces are those forces which are not in the same plane and never intersect by each other, they may be same or opposite in direction.



Equilibrium:

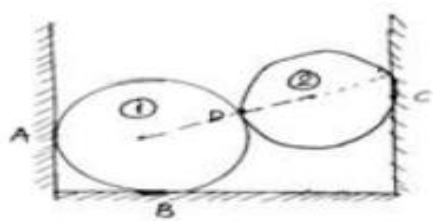
Equilibrium is the status of the body when it is subjected to a system of forces. We know that for a system of forces acting on a body the resultant can be determined.

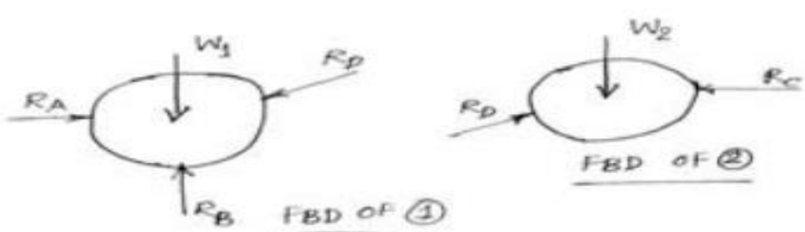
By Newton‟s 2nd Law of Motion the body then should move in the direction of the resultant with some acceleration. If the resultant force is equal to zero it implies that the net effect of the system of forces is zero this represents the state of equilibrium. For a system of coplanar concurrent forces for the resultant to be zero hence

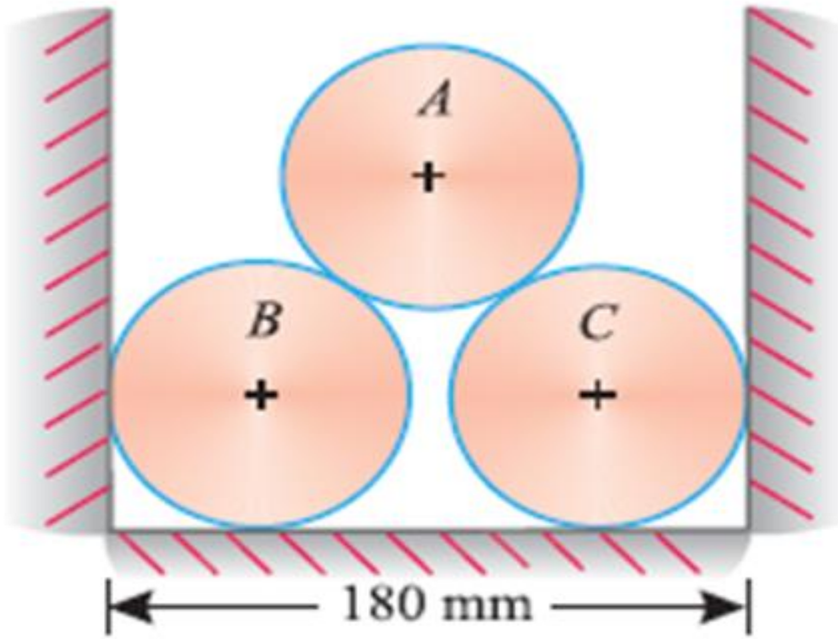
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

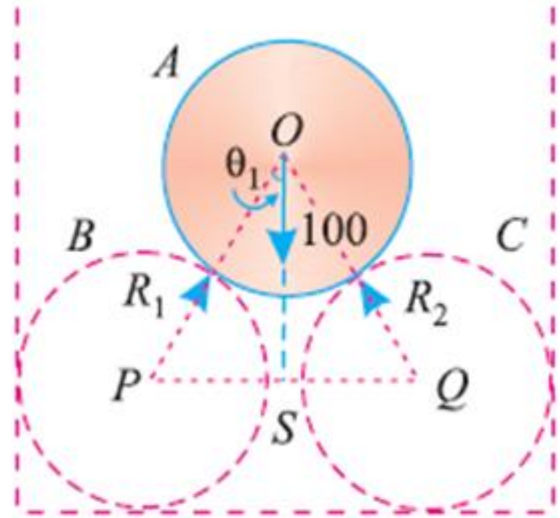
Free Body Diagram:

Free body diagram is nothing but a sketch which shows the various forces acting on the body. The forces acting on the body could be in form of weight, reactive forces contact forces etc. An example for Free Body Diagram is shown below.

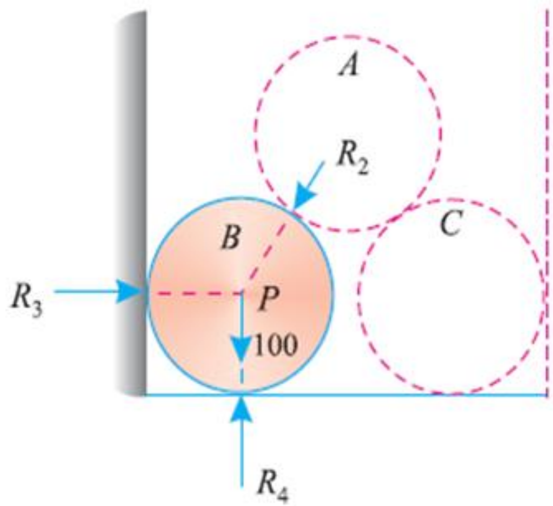








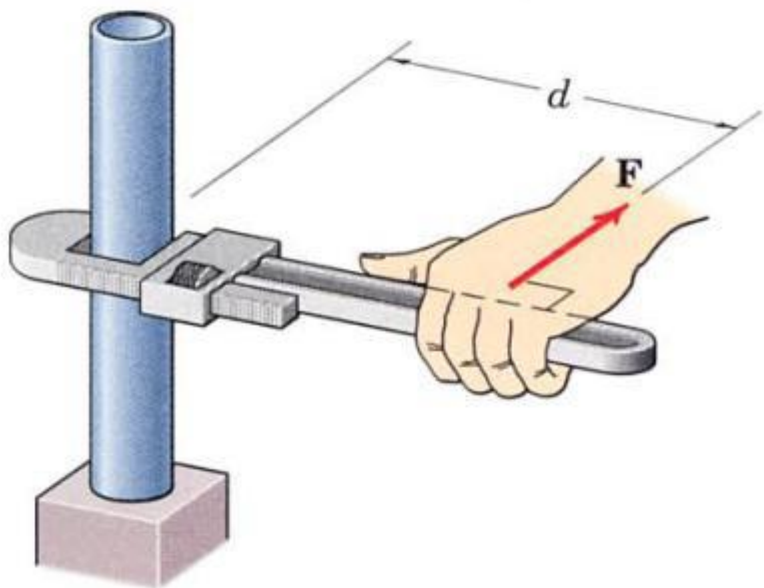
(*a*) Free body diagram



Moment

A force can tend to rotate a body about an axis which neither intersects nor is parallel to the line of action of the force.

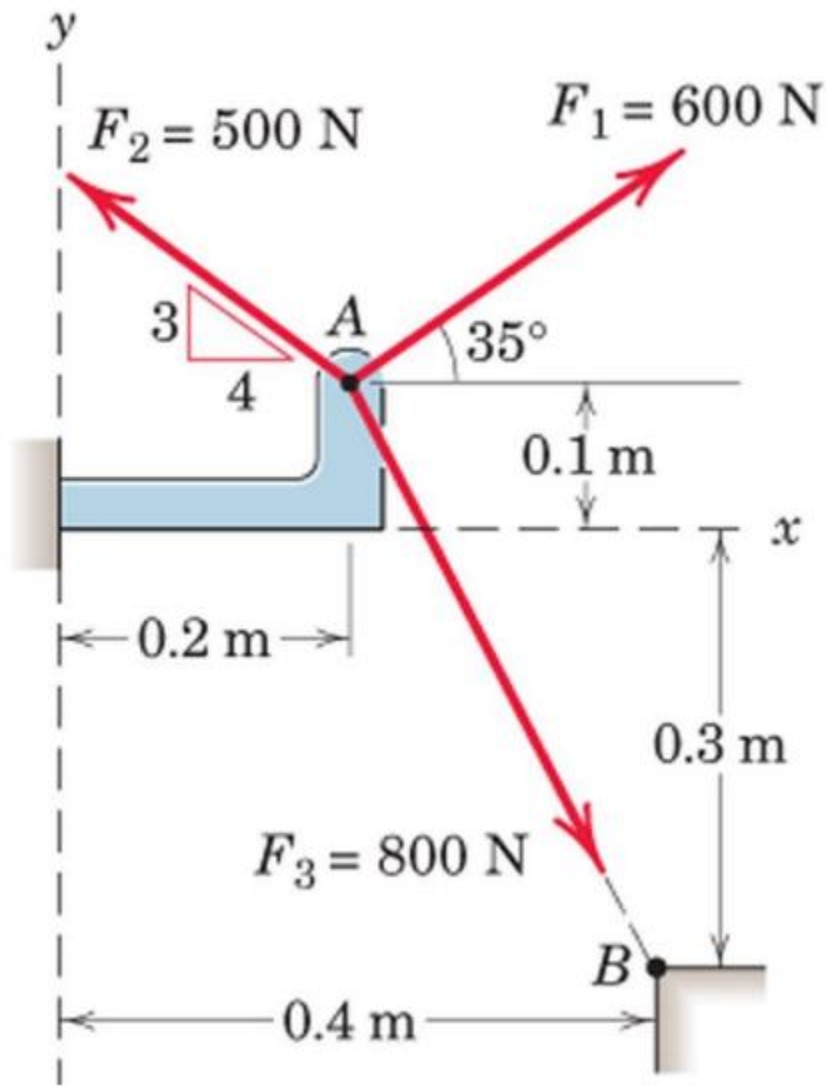
This rotational tendency is known as the moment M of a force.



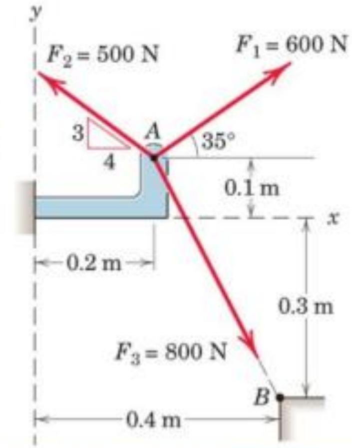
Components of Force

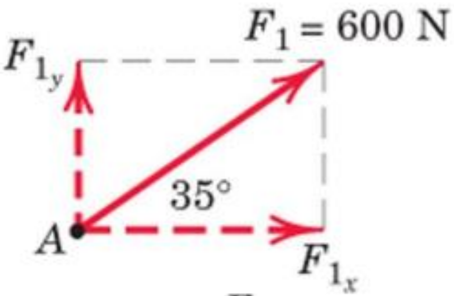
Example 1:

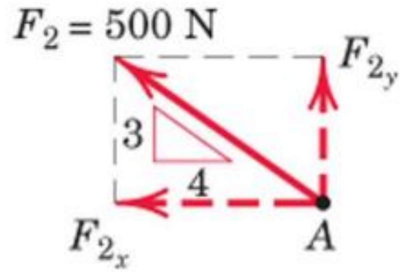
Determine the x and y scalar components of F_1, F_2, and F_3 acting at point A of the bracket

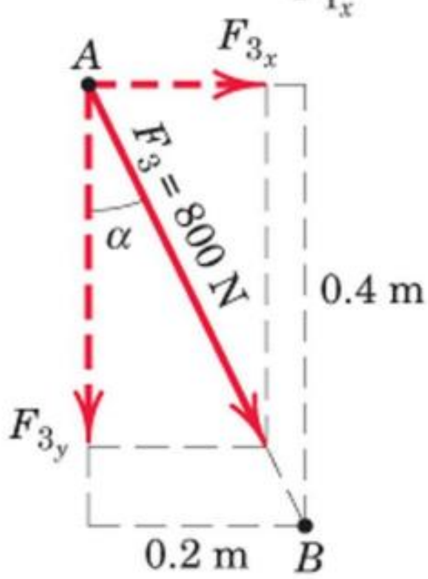


Components of Force









$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

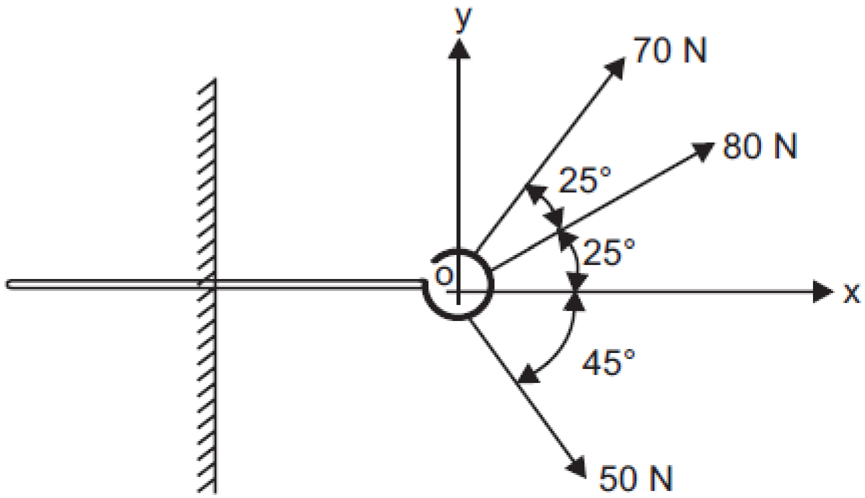
$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

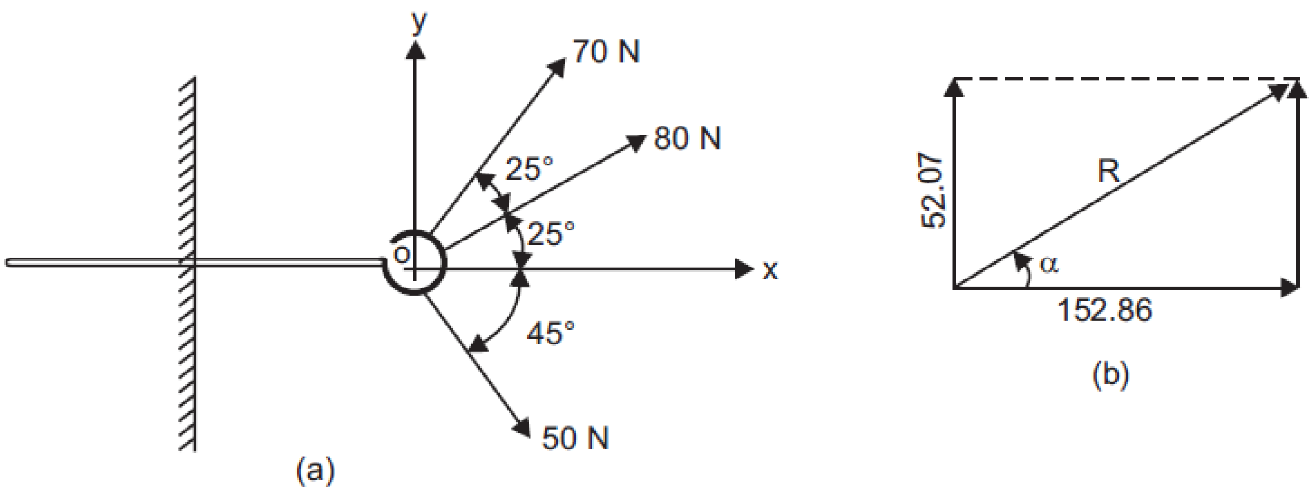
Determine the resultant of the three forces acting on a hook as shown in Fig.



(a)

Fig. 2.13

Determine the resultant of the three forces acting on a hook as shown in Fig.



Force	x component	y component
70 N	45.00	53.62
80 N	72.50	33.81
50 N	35.36	–35.36

$R_x = \Sigma F_x = 152.86,$ $\qquad R_y = \Sigma F_y = 52.07$

$$R = \sqrt{152.86^2 + 52.07^2}$$

$$\mathbf{R = 161.48\ N.}$$

$$\alpha = \tan^{-1}\frac{52.07}{152.86}$$

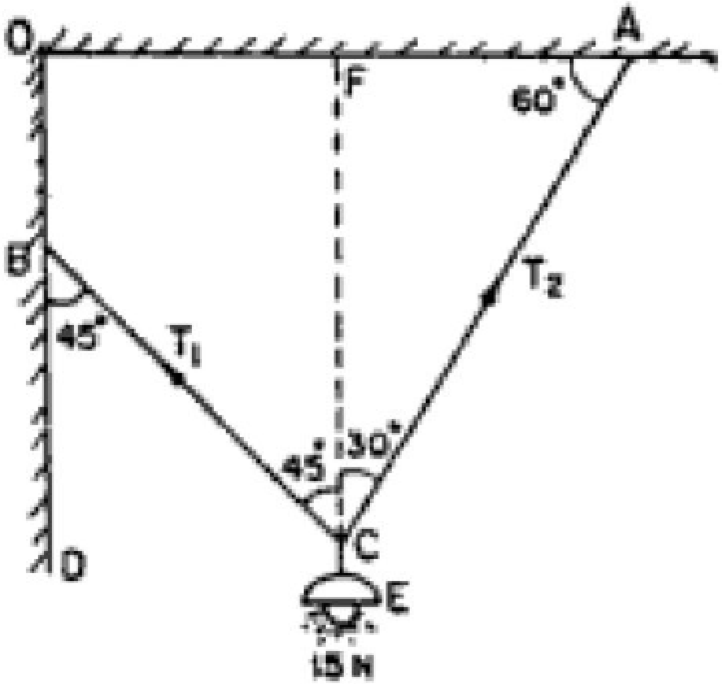
$$\boldsymbol{\alpha = 18.81°}$$

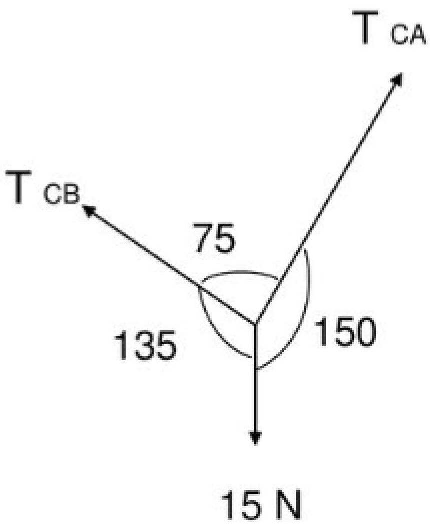
Lami's Theorom

Lami's Theorm Statement

When three concurrent forces are acting on a body simultaneously, be in equilibrium then any force is directly proportional to the SINE of the angle subtended by other two forces.

$$\frac{P}{\text{Sin } \alpha} = \frac{Q}{\text{Sin } \beta} \qquad \frac{R}{\text{Sin } \gamma}$$





F B D

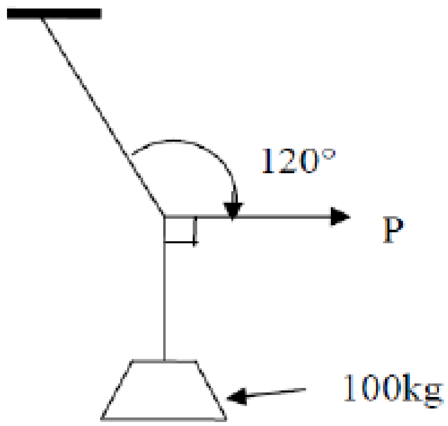
Lami's theorem

$$\frac{T_{CB}}{\text{Sin } 150} = \frac{T_{CA}}{\text{Sin } 135} = \frac{15}{\text{Sin } 75}$$

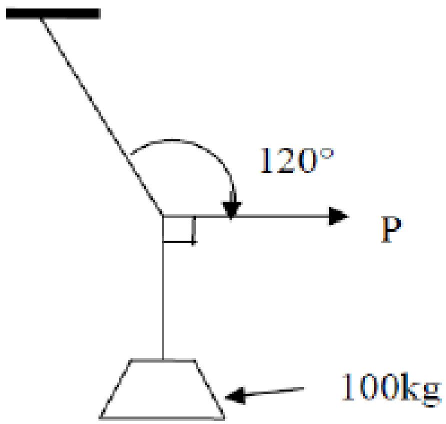
$T_{CB} = 7.76$ N

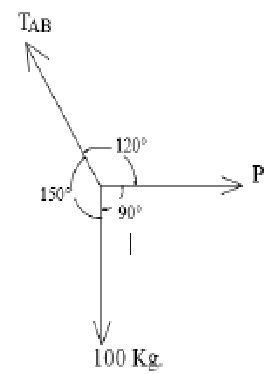
$T_{CA} = 10.98$ N

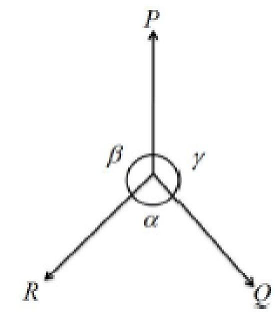
Find the magnitude of the force P, required to keep the 100 kg mass in the position by strings as shown in the Figure



Find the magnitude of the force P, required to keep the 100 kg mass in the position by strings as shown in the Figure







Free Body Diagram will be as show in fig. and there are three coplanar concurrent forces which are in equilibrium so we can apply the lami's theorem.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} \quad \frac{R}{\sin\gamma}$$

$$\therefore \frac{P}{\text{Sin } 150} = \frac{TAB}{\text{Sin } 90} = \frac{100}{\text{Sin } 120}$$

$$P = 566.38 \text{ N}$$

$$T_{AB} = 1132.76 \text{ N}$$

METHOD OF RESOLUTION FOR THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*,$\sum H$).

2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.*, $\sum V$).

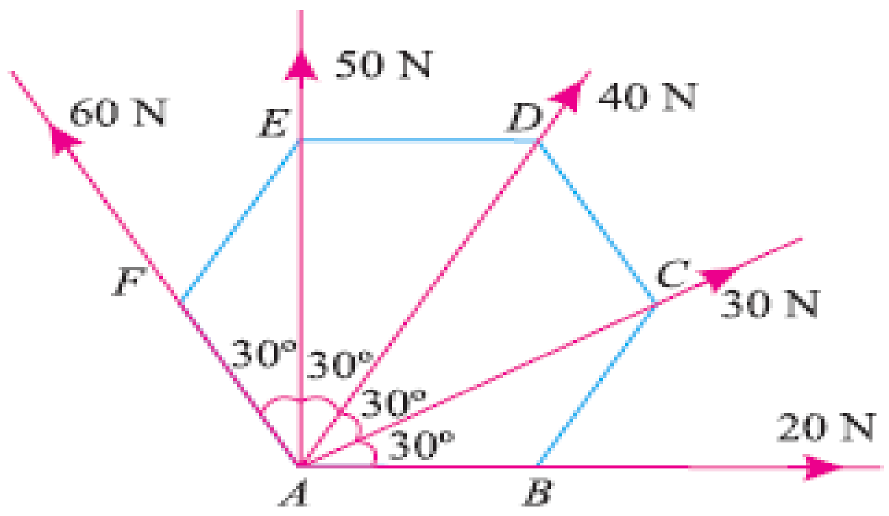
3. The resultant R of the given forces will be given by the equation:

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan \theta = \frac{\sum V}{\sum H}$$

Q. The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.



Conti…

Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along AB),

$$\sum H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$

$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60\,(-0.5) \text{ N}$$

$$= 36.0 \text{ N} \quad \ldots(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to AB),

$$\sum V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$

$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$

$$= 151.6 \text{ N} \quad \ldots(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

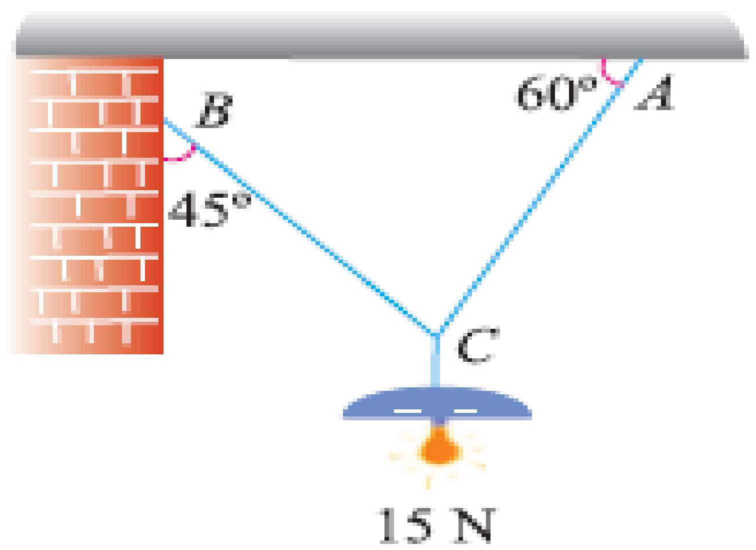
Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, AB).

We know that

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Q. An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.



SOLUTION:

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.

The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$$\therefore \quad \angle ACB = 180° - (45° + 60°) = 75°$$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98\text{ N} \quad \text{Ans.}$$

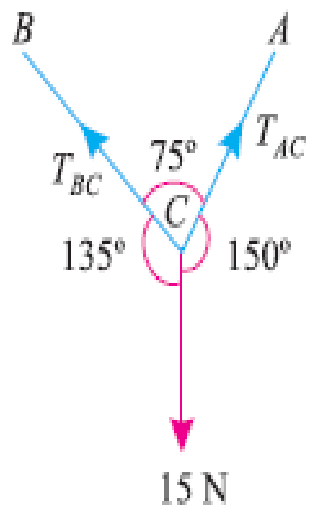
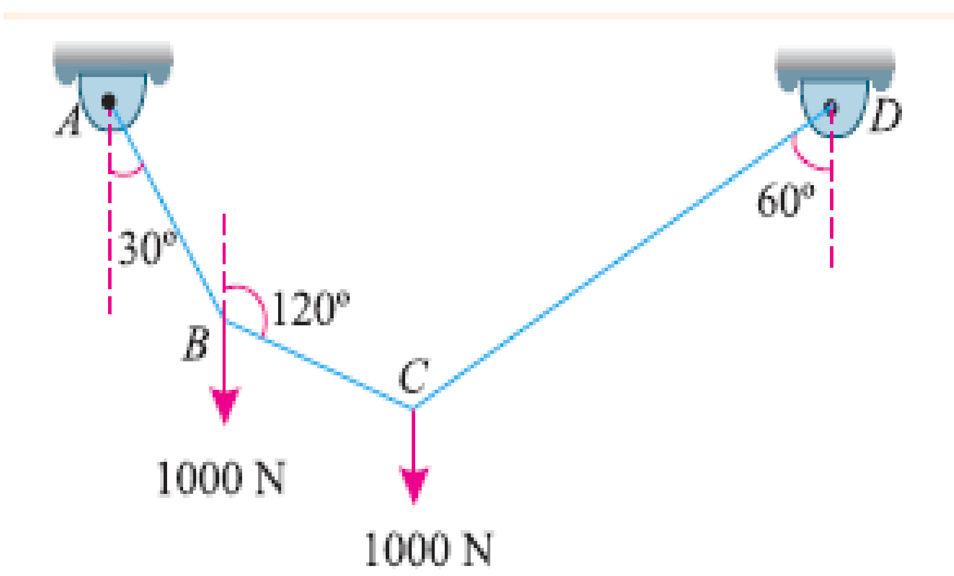


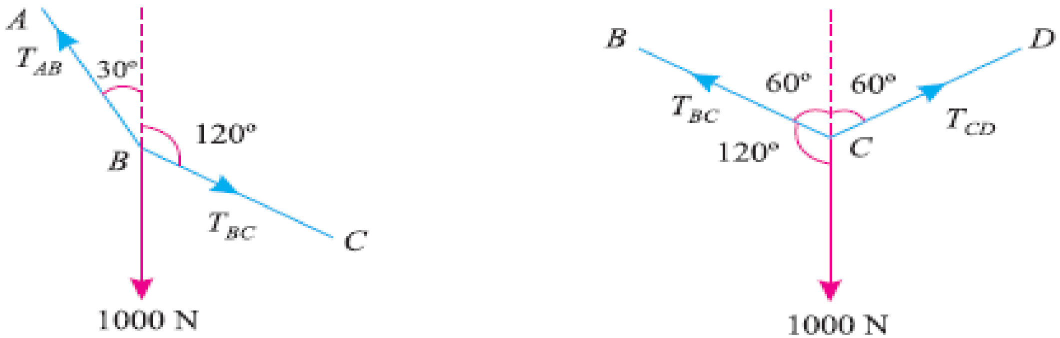
Fig. 5.4.

A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Fig. A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Fig.



SOLUTION:

For the sake of convenience, let us split up the string $ABCD$ into two parts. The system of forces at joints B and is shown in Fig. 5.6 (a) and (b).



(a) Joint B (b) Joint C

Fig. 5.6.

Let T_{AB} = Tension in the portion AB of the string,

T_{BC} = Tension in the portion BC of the string, and

T_{CD} = Tension in the portion CD of the string.

Applying Lami's equation at joint B,

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{1000}{\sin 150^\circ}$$

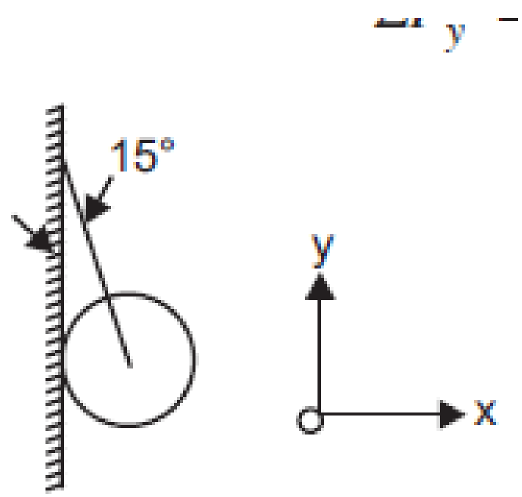
$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 30^\circ} = \frac{1000}{\sin 30^\circ} \qquad \text{...}[\because \sin(180^\circ - \theta) = \sin\theta]$$

$$\therefore \qquad T_{AB} = \frac{1000 \sin 60^\circ}{\sin 30^\circ} = \frac{1000 \times 0.866}{0.5} = 1732 \text{ N} \quad \textbf{Ans.}$$

and

$$T_{BC} = \frac{1000 \sin 30^\circ}{\sin 30^\circ} = 1000 \text{ N} \quad \textbf{Ans.}$$

A sphere of weight 100 N is tied to a smooth wall by a string as shown in Fig. 2.33(a). Find the tension T in the string and reaction R of the wall.



(a)

$$\frac{T}{\sin 90°} = \frac{R}{\sin(180-15)} = \frac{100}{\sin(90+15)}$$

$\therefore$

$$T = \mathbf{103.53\ N.}$$

$$R = \mathbf{26.79\ N.}$$

The above problem may be solved using equations of equilibrium also.

Taking horizontal direction as x axis and vertical direction as y axis,

$$\Sigma F_y = 0 \text{ gives}$$

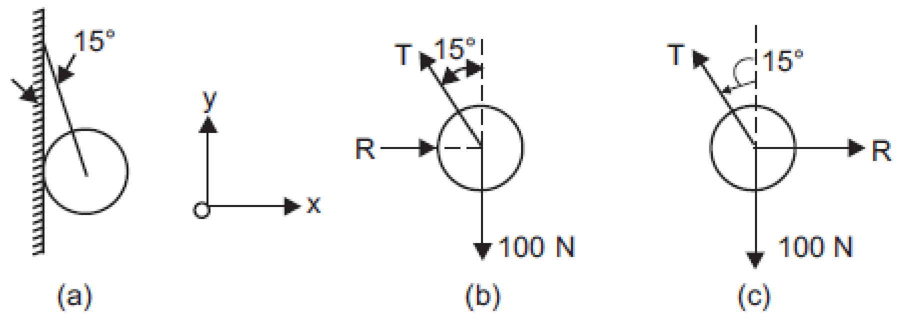


Fig. 2.33

$$T \cos 15° - 100 = 0$$

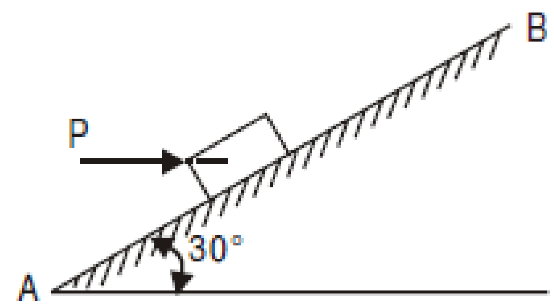
$$T = \mathbf{103.53\ N.}$$

$$\Sigma F_x = 0 \text{ gives}$$

$$R - \text{T} \sin 15° = 0$$

$$R = \mathbf{26.79\ N.}$$

Determine the horizontal force P to be applied to a block of weight 1500 N to hold it in position on a smooth inclined plane AB which makes an angle of 30° with the horizontal [Fig



(a)

$$\Sigma F_y = 0, \text{ gives}$$

$$R \cos 30° - 1500 = 0$$

$$\mathbf{R = 1732.06 \text{ N.}}$$

$$\Sigma F_x = 0, \text{ gives}$$

$$P - R \sin 30° = 0$$

$$P = R \sin 30°$$

$$\mathbf{P = 866.03 \text{ N.}}$$

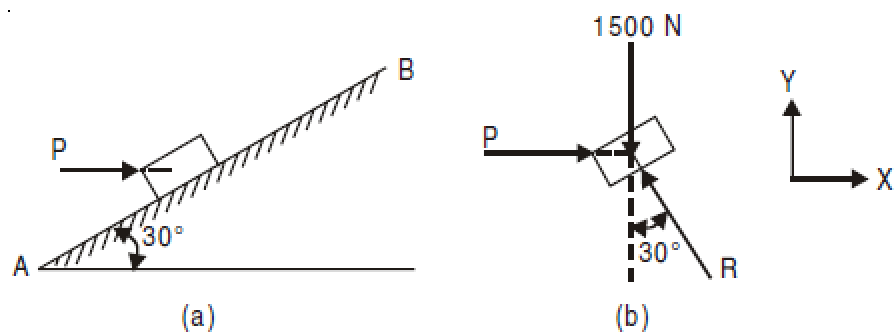


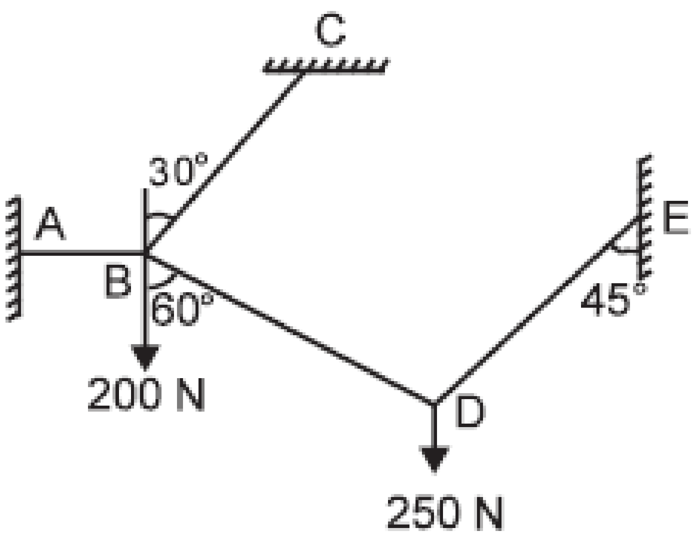
Fig. 2.34

Note: Since the body is in equilibrium under the action of only three forces the above problem can be solved using Lami's theorem as given below:

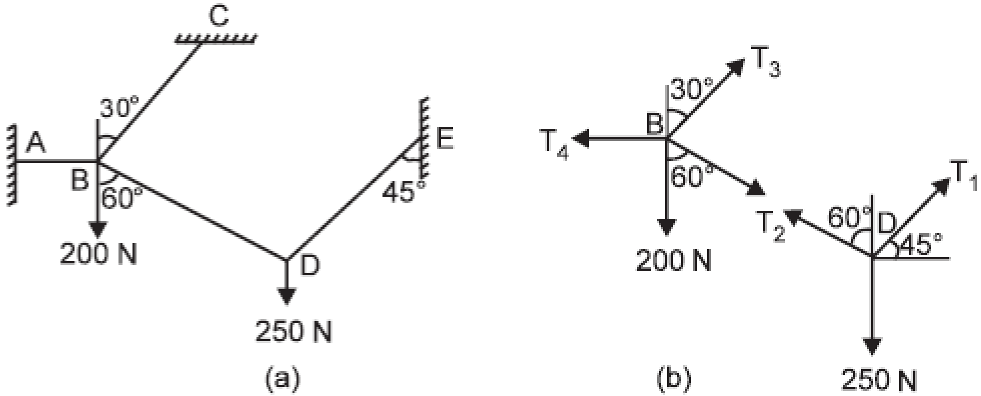
$$\frac{R}{\sin 90°} = \frac{P}{\sin(180 - 30)} = \frac{1500}{\sin(90 + 30)}$$

$$\mathbf{R = 1732.06 \text{ and } P = 866.03.}$$

A system of connected flexible cables shown in Fig. (a) is supporting two vertical forces 200 N and 250 N at points B and D. Determine the forces in various segments of the cable.



(a)



Applying Lami's theorem to the system of forces at point D,

$$\frac{T_1}{\sin 120°} = \frac{T_2}{\sin 135°} = \frac{250}{\sin 105°}$$

$\therefore$

$$\boldsymbol{T_1 = 224.14 \text{ N.}}$$

$$\boldsymbol{T_2 = 183.01 \text{ N.}}$$

Consider the system of forces acting at B.

$$\Sigma V = 0$$

$$T_3 \cos 30° - 200 - T_2 \cos 60° = 0$$

$$T_3 = \frac{200 + 183.01 \cos 60°}{\cos 30°}$$

$$\boldsymbol{T_3 = 336.60 \text{ N.}}$$

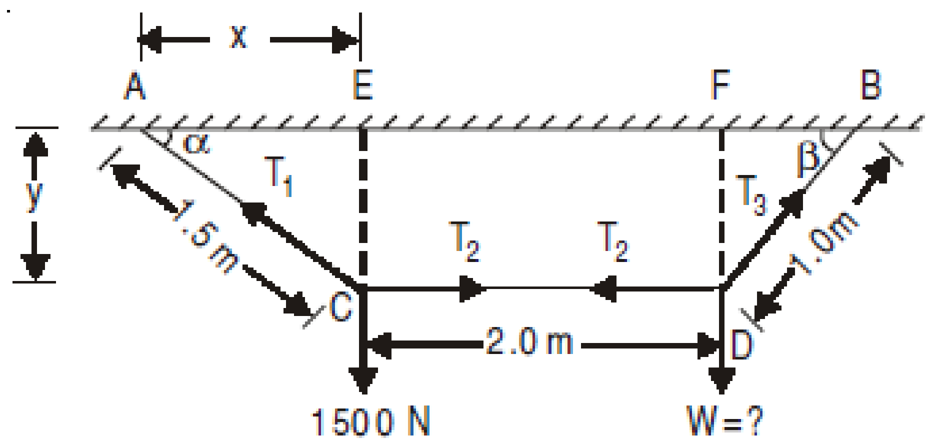
$$\Sigma H = 0$$

$$T_4 - T_2 \sin 60° - T_3 \sin 30° = 0$$

$$T_4 = 183.01 \times \sin 60° + 336.60 \sin 30°$$

$$\boldsymbol{T_4 = 326.79 \text{ N.}}$$

A rope AB, 4.5 m long is connected at two points A and B at the same level 4 m apart.
A load of 1500 N is suspended from a point C on the rope 1.5m from A as shown in Fig.
What load connected at a point D on the rope, 1 m from B will be necessary to keep the position CD level ?



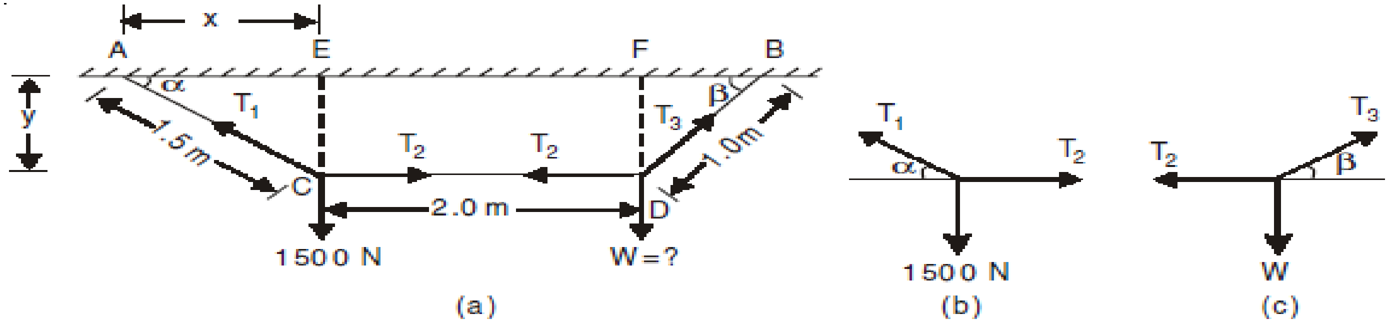


Fig. 2.40

Solution: Drop perpendiculars CE and DF on AB.

Let $CE = y$ and

$$AE = x$$

From Δ AEC, $x^2 + y^2 = 1.5^2 = 2.25$

Now, $AB = 4$ m

and $AC + CD + BD = 4.5$ m

i.e., $CD = 4.5 - 1.5 - 1.0 = 2.0$ m

$\therefore$ $EF = 2.0$ m

$\therefore$ $BF = AB - (AE + EF)$

$$= 4 - (x + 2.0) = 2 - x$$

From Δ BFD, $BF^2 + DF^2 = 1^2$

$$(2 - x)^2 + y^2 = 1$$

From (1) and (3)

$$x^2 - (2 - x)^2 = 1.25$$

i.e., $x^2 - 4 + 4x - x^2 = 1.25$

$$x = 1.3125 \text{ m}$$

$\therefore$

$$\alpha = \cos^{-1}\left(\frac{1.3125}{1.5}\right) = 28.955°$$

$$\beta = \cos^{-1}\left(\frac{2 - 1.3125}{1}\right) = 46.567°$$

$$\frac{T_1}{\sin 90^\circ} = \frac{T_2}{\sin 118.955^\circ} = \frac{1500}{\sin (180 - 28.955)^\circ}$$

$$T_1 = 3098.39 \text{ N}$$

$$T_2 = 2711.09 \text{ N}$$

Reaction and Support

CONDITIONS OF EQUILIBRIUM

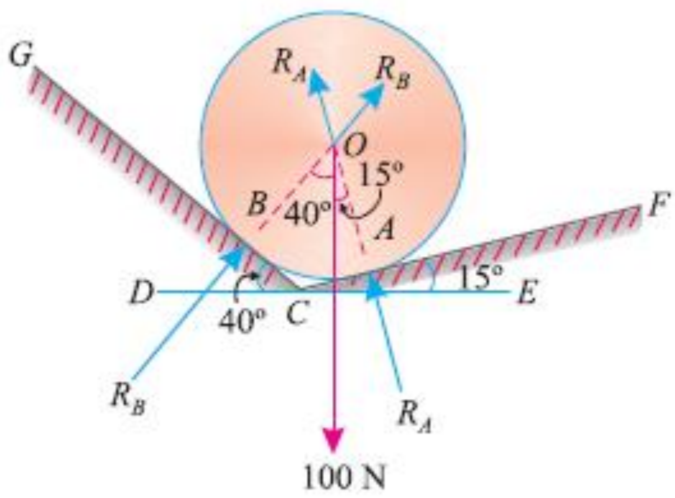
1. If the body moves in any direction, it means that there is a resultant force acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces ($\sum H$) and vertical component of all the forces ($\sum V$) must be zero. Mathematically,

$$\sum H = 0 \text{ and } \sum V = 0$$

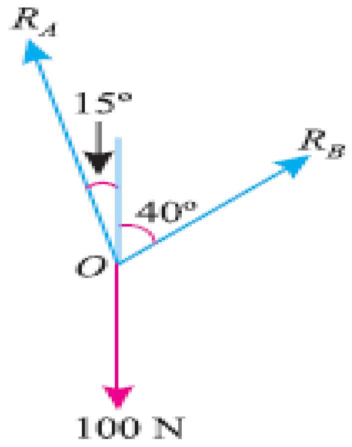
2. If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. A little consideration will show, that if the body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. Or in other words, the resultant moment of all the forces ($\sum M$) must be zero. Mathematically,

$$\sum M = 0$$

Q. A smooth circular cylinder of radius 1.5 meter is lying in a triangular groove, one side of which makes 15° angle and the other 40° angle with the horizontal. Find the reactions at the surfaces of contact, if there is no friction and the cylinder weights 100 N.



Conti..



cont....

Let R_A = Reaction at A, and

R_B = Reaction at B.

The smooth cylinder lying in the groove is shown in Fig. 5.12 (*a*). In order to keep the system in equilibrium, three forces *i.e.* R_A, R_B and weight of cylinder (100 N) must pass through the centre of the cylinder. Moreover, as there is no *friction, the reactions R_A and R_B must be normal to the surfaces as shown in Fig. 5.12 (*a*). The system of forces is shown in Fig. 5.12 (*b*).

Applying Lami's equation, at O,

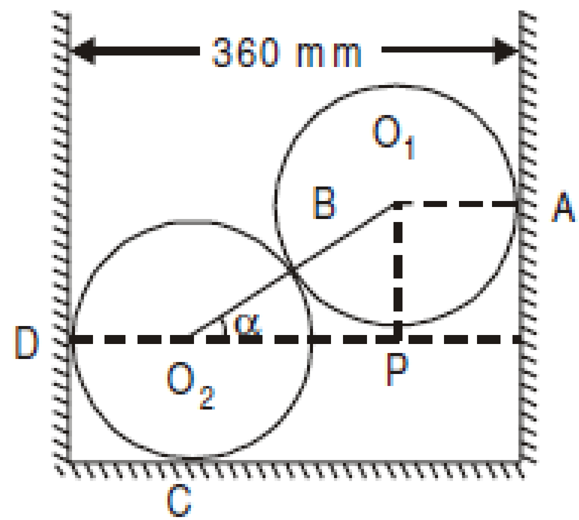
$$\frac{R_A}{\sin(180° - 40°)} = \frac{R_B}{\sin(180° - 15°)} = \frac{100}{\sin(15° + 40°)}$$

or
$$\frac{R_A}{\sin 40°} = \frac{R_B}{\sin 15°} = \frac{100}{\sin 55°}$$

$\therefore$
$$R_A = \frac{100 \times \sin 40°}{\sin 55°} = \frac{100 \times 0.6428}{0.8192} = 78.5\text{ N} \quad \textbf{Ans.}$$

and
$$R_B = \frac{100 \times \sin 15°}{\sin 55°} = \frac{100 \times 0.2588}{0.8192} = 31.6\text{ N} \quad \textbf{Ans.}$$

Two smooth spheres each of radius 100 mm and weight 100 N, rest in a horizontal channel having vertical walls, the distance between which is 360 mm. Find the reactions at the points of contacts A, B, C and D shown in Fig.



Solution: Let O_1 and O_2 be the centres of the first and second spheres. Drop perpendicular O_1P to the horizontal line through O_2. Figures 2.44(*b*) and 2.44(*c*) show free body diagram of the sphere 1 and 2, respectively. Since the surface of contact are smooth, reaction of B is in the radial direction, *i.e.*, in the direction O_1O_2. Let it make angle a with the horizontal. Then,

$$\cos\alpha = \frac{O_2P}{O_1O_2} = \frac{360 - O_1A - O_2D}{O_1B + BO_2} = \frac{360-100-100}{100+100} = 0.8$$

$$\therefore \quad \sin\alpha = 0.6.$$

Consider sphere No. 1.

$$\Sigma V = 0, \text{ gives}$$

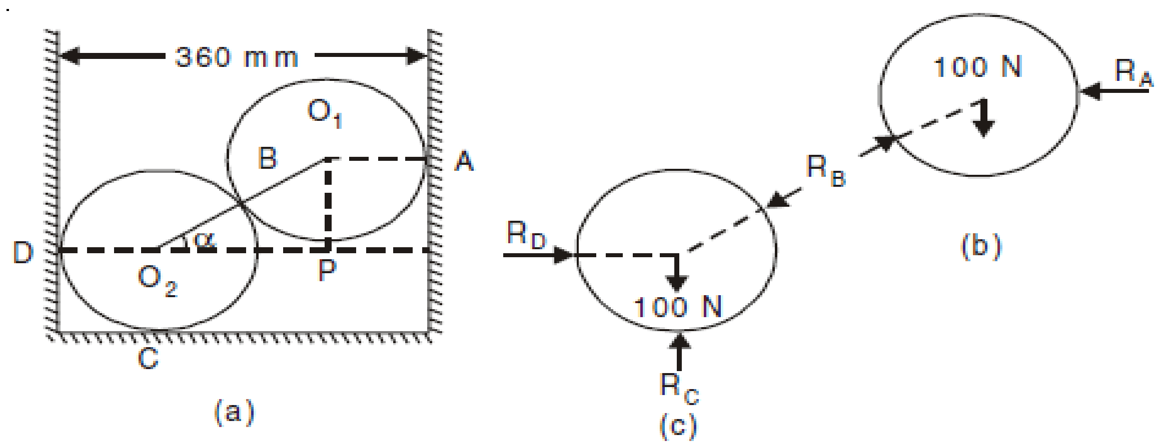
$$R_B \times 0.6 = 100$$

$$\mathbf{R_B = 166.67\ N.}$$

$$\Sigma H = 0, \text{ gives}$$

$$R_A = R_B \times 0.8$$

$$\therefore \quad \mathbf{R_A = 133.33\ N.}$$



Consider sphere No. 2

$$\Sigma V = 0, \text{ gives}$$

$$R_c = 100 + R_B \times 0.6$$

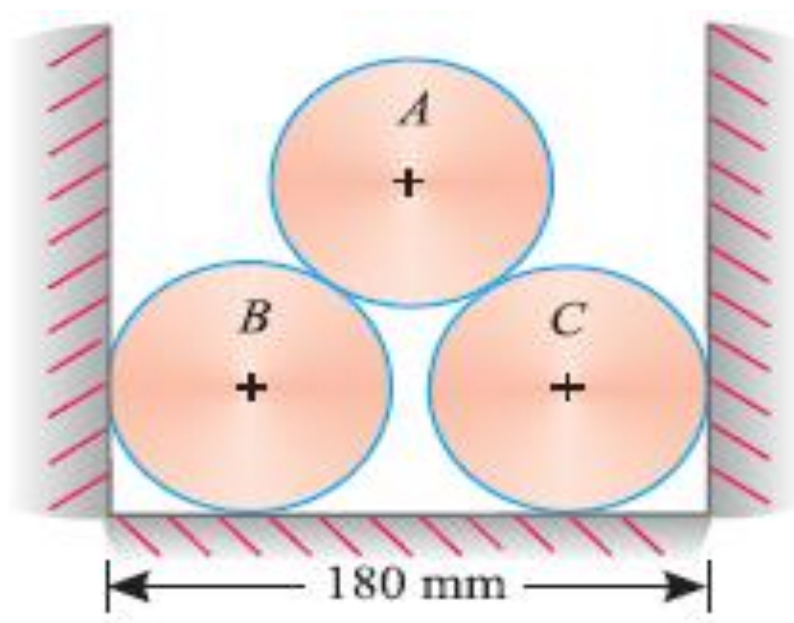
$$\therefore \quad \boldsymbol{R_c = 200 \text{ N.}}$$

$$\Sigma H = 0$$

$$R_D = R_B \times 0.8$$

$$\therefore \quad \boldsymbol{R_D = 133.33 \text{ N.}}$$

Q. Three cylinders weighting 100 N each and of 80 mm diameter are placed in a channel of 180 mm width as shown in Fig. Determine the pressure exerted by (i) the cylinder A on B at the point of contact (ii) the cylinder B on the base and (iii) the cylinder B on the wall.



Solution:

(i) *Pressure exerted by the cylinder A on the cylinder B*

Let $R1$ = Pressure exerted by the cylinder A on B. It is also equal to pressure exerted by the cylinder A on B.

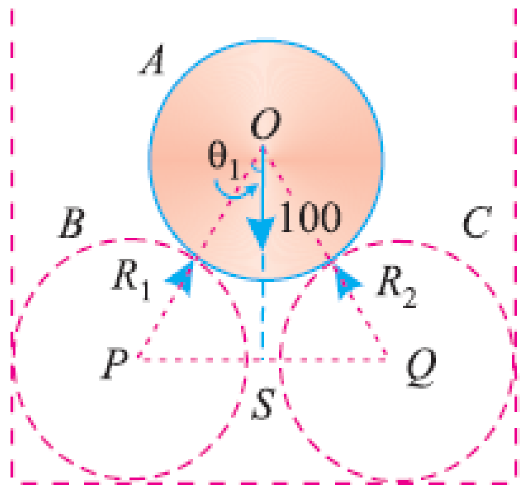
First of all, consider the equilibrium of the cylinder A. It is in equilibrium under the action of

the following forces, which must pass through the centre of the cylinder as shown in Fig. 5.17 (a).

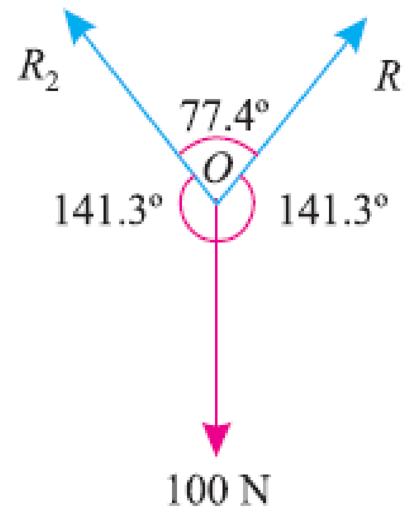
1. Weight of the cylinder 100 N acting downwards.
2. Reaction $R1$ of the cylinder B on the cylinder A.
3. Reaction $R2$ of the cylinder C on the cylinder A.

Now join the centres O, P and Q of the three cylinders. Bisect PQ at S and join OS as shown in fig.

Conti..



(*a*) Free body diagram



(*b*) Force diagram

Fig. 5.17.

From the geometry of the triangle *OPS*, we find that

$$OP = 40 + 40 = 80 \text{ mm}$$

and $$PS = 90 - 40 = 50 \text{ mm}$$

$$\therefore \qquad \sin \angle POS = \frac{PS}{OP} = \frac{50}{80} = 0.625$$

or $$\angle POS = 38.7^\circ$$

Conti..

Since the triangle OSQ is similar to the triangle OPS, therefore $\angle SOQ$ is also equal to 38.7°. Thus the angle between R_1 and R_2 is $2 \times 38.7° = 77.4°$.

And angle between R_1 and OS (also between R_2 and OS)

$$= 180° - 38.7° = 141.3°$$

The system of forces at O is shown in Fig. 5.17 (b). Applying Lami's equation at O,

$$\frac{R_1}{\sin 141.3°} = \frac{R_2}{\sin 141.3°} = \frac{100}{\sin 77.4°}$$

$$\frac{R_1}{\sin 38.7°} = \frac{R_2}{\sin 38.7°} = \frac{100}{\sin 77.4°} \qquad \text{...}[\because \sin(180° - \theta) = \sin\theta]$$

$$\therefore \qquad R_1 = \frac{100 \times \sin 38.7°}{\sin 77.4°} = \frac{100 \times 0.6252}{0.9759} = 64.0 \text{ N} \quad \textbf{Ans.}$$

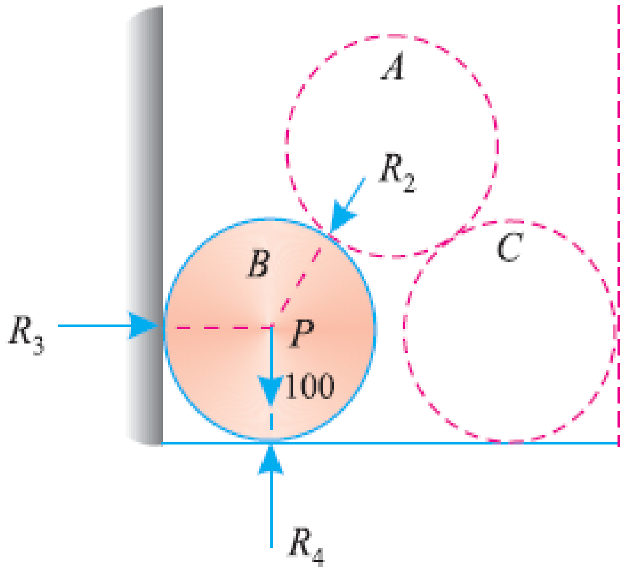
Similarly $\qquad R_2 = R_1 = 64.0 \text{ N} \quad$ **Ans.**

Conti...

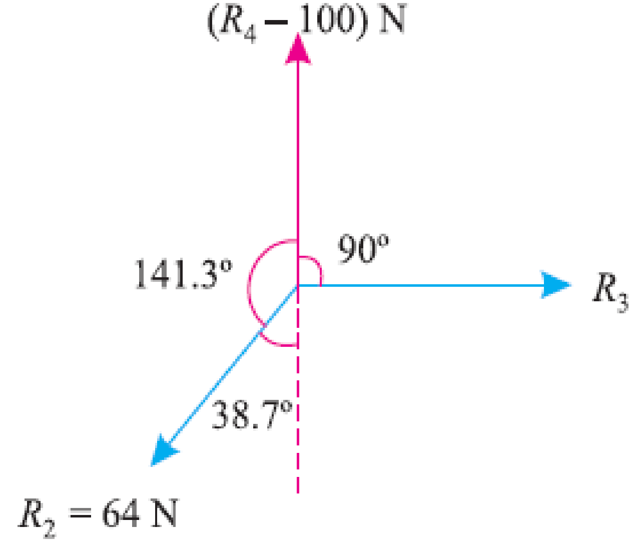
(ii) *Pressure exerted by the cylinder B on the base*

Let R_3 = Pressure exerted by the cylinder *B* on the wall, and

R_4 = Pressure exerted by the cylinder *B* on the base.



(*a*) Free body diagram



(*b*) Force diagram