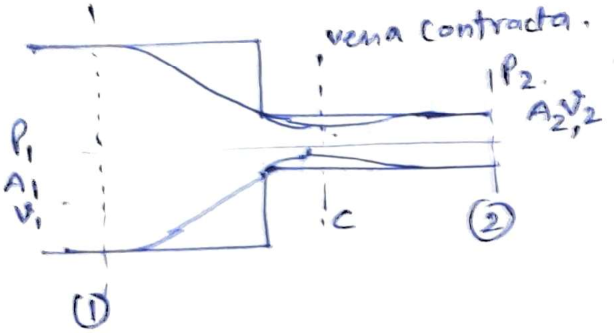
Loss of head due to sudden contraction-



The stream lines converge to form least cross-sectional area of flow immediately after the plane of sudden contraction. This least cross-sectional area is known as vena contracta.

$$h_{LC} = \frac{(V_c - V_2)^2}{2g}$$

[Its similar to head loss due to sudden enlargement after vena contracta]

continuity eqn. $A_c V_c = A_2 V_2 \Rightarrow \frac{V_c}{V_2} = \frac{A_2}{A_c}$

$$h_{LC} = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)^2$$

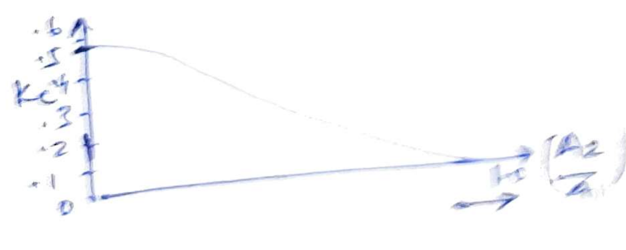
$$h_{LC} = \frac{V_2^2}{2g}\left[\frac{A_2}{A_c} - 1\right]^2$$

co-efficient of contraction
↑
$C_c = \frac{A_c}{A_2}$

$$h_{LC} = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$K_c = \left(\frac{1}{C_c} - 1\right)^2$
↓
Sudden contraction loss factor

$$h_{LC} = \frac{V_2^2}{2g} \cdot K_c$$



Exit loss :- The loss of head at the pipe exit can be obtained from the expression for head loss due to sudden expansion making exit area A_2 as infinite

$A_2 \to \infty$.

$$h_{LE} = \frac{(V_1 - V_2)^2}{2g} \Rightarrow \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2.$$

from continuity eqn $A_1 V_1 = A_2 V_2 \Rightarrow \frac{V_2}{V_1} = \frac{A_1}{A_2}$

$$h_{LE} = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

$A_2 \to \infty \Rightarrow \frac{A_1}{A_2} = 0$

(X)

$$\boxed{h_{L\,exit} = \frac{V_1^2}{2g}.}$$

Entry loss :- $A_1 \to \infty$ (at the inlet section)

So - $\frac{A_2}{A_1} = 0$

$K_c = 0.5$.

(X)

$$\boxed{h_{L,entry} = \frac{0.5\,V^2}{2g}.}$$

Losses in pipe fitting → The head loss due to various pipe fitting. ex valve, coupling etc.

$$h_{L\,fitting} = K\frac{V^2}{2g}.$$

Loss of head due to bend in pipe:-

$$h_{L\,bend} = K\frac{v^2}{2g}$$

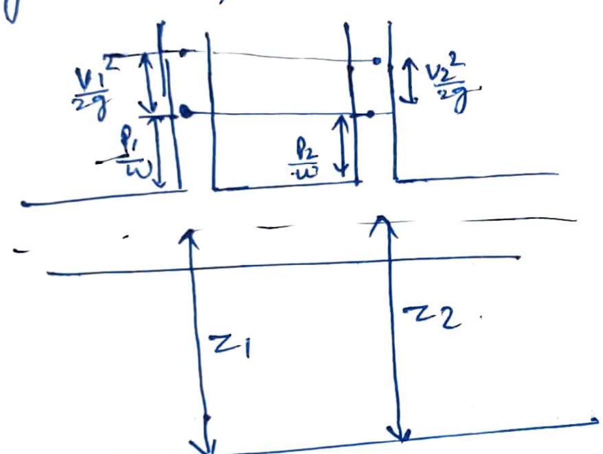
K → co-efficient of bend. (depends upon angle of bend, radius of curvature, & diameter of pipe).

for 90° — K = 1.2

for 45° → K = 0.4.

HGL and TEL:-

(Hydraulic gradient line and Total energy line)

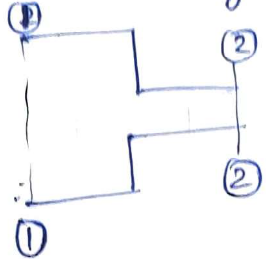


HGL – It's the line which join the piezometric head at various point is known as HGL.

$\frac{P_1}{\rho g} + Z_1$ → Piezometric head

TEL:- It's the line which join the total head $\left(\frac{P}{\rho g} + \frac{V^2}{2g} + Z\right)$ at various point is known as TEL.

Q. In fig., the sudden change in pipe diameter from 40 cm to 20 cm causes the pressure of water to drop from 200 kPa to 150 kPa. Compute the discharge if contraction coefficient is 0.6. (8)



Soln

Applying Bernoulli's eqn between section (1) & (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{LC}.$$

$z_1 = z_2.$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$$Q = A_1 V_1 = A_2 V_2$$

$$= \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + \frac{Q^2}{2g \cdot A_2^2}\left(\frac{1}{C_c} - 1\right)^2$$

$$= \frac{Q^2}{2g}\left[\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + \frac{1}{A_2^2}\left(\frac{1}{C_c} - 1\right)^2\right] \quad \text{——— (1)}$$

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(.4)^2 = 0.12566\, m^2$$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(.2)^2 = 0.03141\, m^2$$

Put the values in eqn (1)

$$\frac{(200 - 150) \times 10^3}{1000} = \frac{Q^2}{2}\left(\left(\frac{1}{(.03141)^2} - \frac{1}{(.12566)^2}\right] + \frac{1}{(.03141)^2} \times \left(\frac{1}{.6} - 1\right)^2\right]$$

$$\boxed{Q = 0.267\, m^3/s}$$