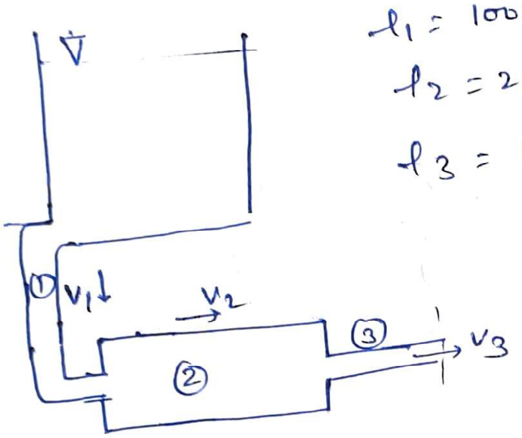
Q: Water flows from a reservoir through a series of pipe joined as shown in fig. Find the percentage error in discharge if minor losses are neglected, take K = 1, for bend & friction factor f = 0.02 for all the pipes. The available head of 20 meter is used to overcome losses.

$l_1 = 100m, \; d_1 = 0.1\,m$

$l_2 = 200m, \; d_2 = 0.2\,m$

$l_3 = 100m, \; d_3 = 0.1\,m.$



Solⁿ

Case I) Discharge when all the losses are taken into account.

$$20 = \underbrace{\frac{.5\,v_1^2}{2g}}_{\text{entry}} + \underbrace{\frac{K v_1^2}{2g}}_{\text{bend}} + \underbrace{\frac{f l_1 v_1^2}{2g d_1}}_{\text{friction}} + \underbrace{\frac{K v_1^2}{2g}}_{\text{bend}} + \underbrace{\frac{(v_1 - v_2)^2}{2g}}_{\text{sudden expansion}} + \underbrace{\frac{f l_2 v_2^2}{2 \times g d_2}}_{\text{frictional}}$$

$$+ \underbrace{\frac{.5\,v_3^2}{2g}}_{\text{sudden contraction}} + \underbrace{\frac{f\,l_3 v_3^2}{2g d_3}}_{\text{frictional}} + \underbrace{\frac{v_3^2}{2g}}_{\text{exit}}$$

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3.$$

$$= \frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} d_2^2 \times v_2 = \frac{\pi}{4} d_3^2 * v_3.$$

$$Q = d_1^2 v_1 = d_2^2 v_2 = d_3^2 v_3.$$

$$= (.1)^2 v_1 = (.2)^2 \times v_2 = (.1)^2 v_3$$

$v_1 = 4v_2 = v_3$.

$$20 = \frac{.5 \times (4v_2)^2}{2g} + \frac{2 \times 1 \times (2v_2)^2}{2g} + \frac{.02 \times 100 \times (4v_2)^2}{2 \times g \times .1}$$

$$+ \frac{(4v_2 - v_2)^2}{2g} + \frac{.02 \times 200 \times v_2^2}{2 \times g \times .2} + \frac{.5\,(4v_2)^2}{2 \times 9.8} + \frac{.02 \times 100 \times (4v_2)^2}{2 \times 9.8 \times .1}$$

$$+ \frac{(4v_2)^2}{2g}.$$

$$20 = \frac{.5 \times 16v_2^2}{2 \times g} + \frac{32v_2^2}{2g} + \frac{32\,v_2^2}{2g} + \frac{9v_2^2}{2g} + \frac{4v_2^2}{.4g} + \frac{8\,v_2^2}{2 \times 9.8}$$

$$+ \frac{320v_2^2}{.2 \times 9.8} + \frac{16v_2^2}{2 \times 9.8}$$

$$20 = \frac{669}{2 \times 9.8} \times v_2^2 \Rightarrow \frac{20 \times 2 \times 9.8}{669} = v_2^2$$

$$v_1 = 4v_2 \Rightarrow v_2 = \frac{1}{4}v_1$$

$$\left(\frac{1}{4}v_1\right)^2 = \frac{20 \times 2 \times 9.8}{669}$$

$$v_1 = 2.92 \text{ m/s}$$

$$Q = A_1 v_1 = \frac{\pi}{4} \times (.1)^2 \times (2.92)^2$$

$$\boxed{Q_{act} = .0229 \text{ m}^3/\text{s}}$$

(2) when minor losses are neglected

$$20 = \frac{f\, l_1 v_1^2}{2\times g d_1} + \frac{f\, l_2 v_2^2}{2\times g d_2} + \frac{f\, l_3 v_3^2}{2\times g d_3}.$$

$V_1 = 4 v_2 = V_3.$

after solving.

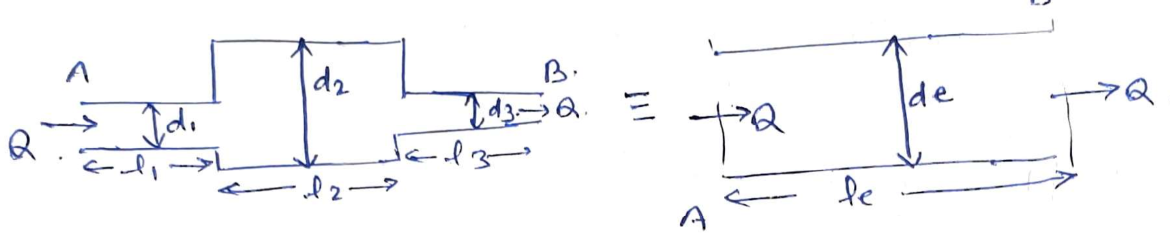
$v_1 = 3.06\ m/s.$

$Q_{th} = A_1 v_1 = \frac{\pi}{4}(.1)^2 \times 3.06 = 0.0242\ m^3/s$

$$\%\ error = \frac{Q_{th} - Q_{act}}{Q_{th}} = \left(\frac{.0242 - .0229}{.0242}\right)\times 100$$

$\%\ error = 5.3\ \%$

Equivalent pipe!- An equivalent pipe is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different length and diameters. The uniform diameter of the equivalent pipe is known as the equivalent diameter of the series or compound pipe.



$$Q = Q_1 = Q_2 = Q_3 - - -$$

$$h_L = h_{l1} + h_{l2} + h_{l3} + - -$$

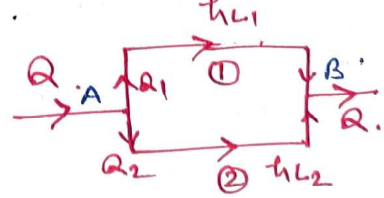
$$h_L = \frac{f l_1 Q^2}{12 d_1^5} + \frac{f l_2 Q^2}{12 d_2^5} + \frac{f l_3 Q^2}{12 d_3^5} + - -$$

$$h_L = \frac{f l_e Q^2}{12 d_e^5} = \frac{f l_1 Q^2}{12 d_1^5} + \frac{f l_2 Q^2}{12 d_2^5} + \frac{f l_3 Q^2}{12 d_3^5} + - -$$

$$\boxed{\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} + - -}$$ Dupit's equation.

Minor losses are neglected.

Pipes in parallel!- The pipes are said to be in parallel when a main line divides into two or more parallel pipes which again join together downstream and continues as a main line.



$$Q = Q_1 + Q_2$$

$$\frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B + h_{L1} \quad \text{(I)}$$

$$\frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B + h_{L2} \quad \text{(II)}$$

$$\boxed{h_{L1} = h_{L2}.}$$

for parallel pipe.

$$Q = Q_1 + Q_2 \quad \& \quad h_{L1} = h_{L2}$$

for series pipe :-

$$Q = Q_1 = Q_2 = Q_3$$

$$\boxed{h_L = h_{L1} + h_{L2} + h_{L3}}$$

Hydraulic siphon :- The hydraulic siphon is a pipe that used for drawing the fluid out of a tank. The siphonic action takes place in accordance to the Bernoulli's principle by keeping one end of the pipe inside the tank while other end is left in a the atmosphere.