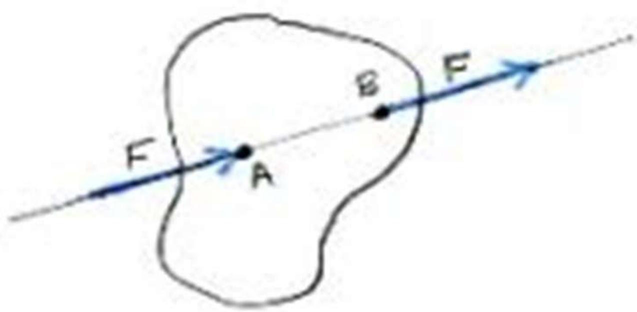
Lecture -3

Engineering Mechanics

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

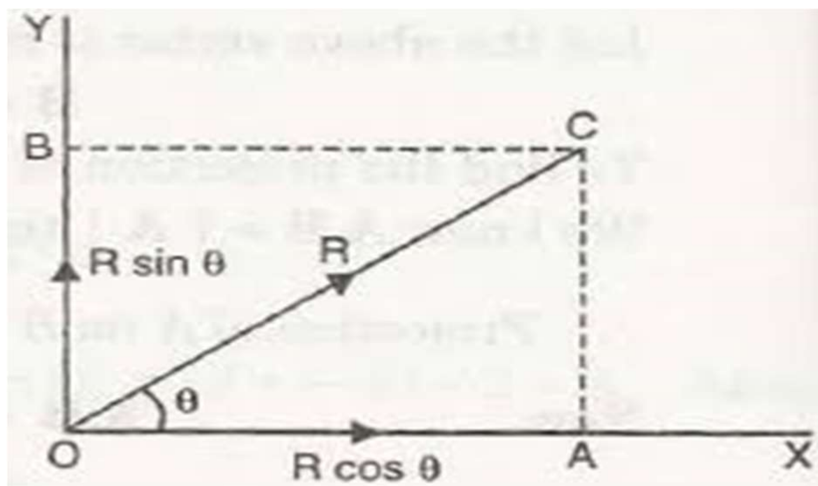
It states, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body."



RESOLUTION OF A FORCE

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions.

"The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction."



METHOD OF RESOLUTION FOR THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*,$\sum H$).

2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.*, $\sum V$).

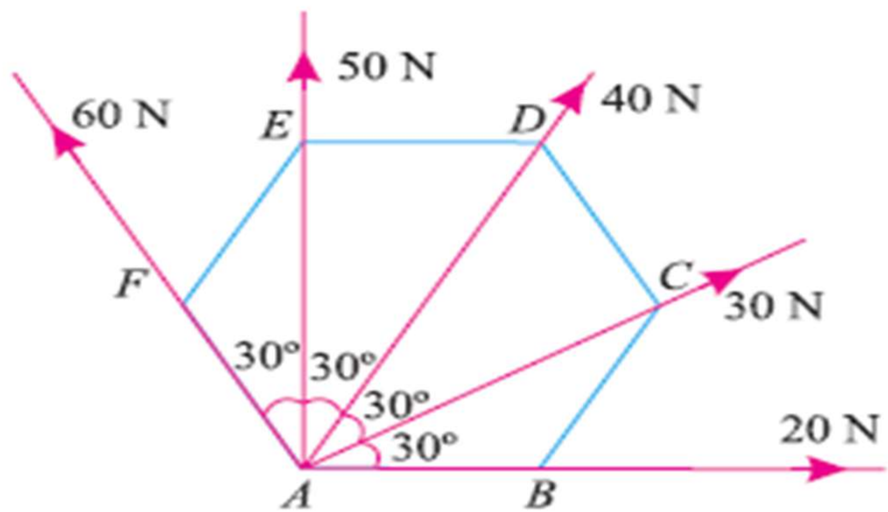
3. The resultant *R* of the given forces will be given by the equation:

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\sum V}{\sum H}$$

Q. The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.



Conti…

Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$

$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60\,(-0.5) \text{ N}$$

$$= 36.0 \text{ N} \quad \ldots(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to *AB*),

$$\Sigma V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$

$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$

$$= 151.6 \text{ N} \quad \ldots(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

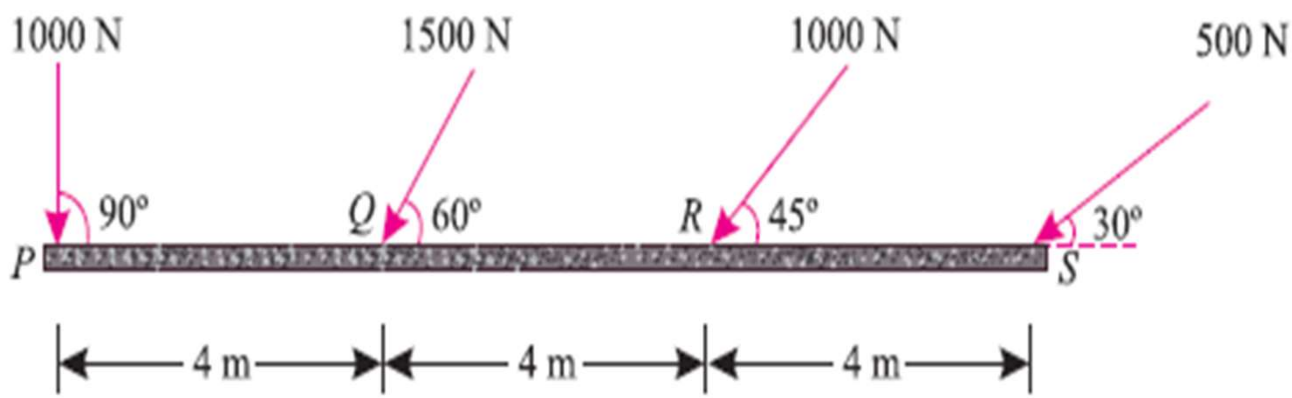
Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, *AB*).

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Q. A horizontal line PQRS is 12 m long, where PQ = QR = RS = 4 m. Forces of 1000 N, 1500 N, 1000 N and 500 N act at P, Q, R and S respectively with downward direction. The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant force.



Solution:

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30° \text{ N}$$

$$= (1000 \times 0) + (1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866) \text{ N}$$

$$= 1890 \text{ N} \qquad ...(i)$$

and now resolving all the forces vertically,

$$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30° \text{ N}$$

$$= (1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5) \text{ N}$$

$$= 3256 \text{ N} \qquad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N} \quad \textbf{Ans.}$$

Conti…

Direction of the resultant force

Let θ = Angle, which the resultant force makes with PS.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8^\circ \quad \textbf{Ans.}$$

Note. Since both the values of ΣH and ΣV are +ve. therefore resultant lies between 0° and 90°.

Position of the resultant force

Let x = Distance between P and the line of action of the resultant force.

Now taking moments* of the vertical components of the forces and the resultant force about P, and equating the same,

$$3256\,x = (1000 \times 0) + (1500 \times 0.866)\,4 + (1000 \times 0.707)8 + (500 \times 0.5)12$$

$$= 13\,852$$

$$\therefore \quad x = \frac{13\,852}{3256} = 4.25 \text{ m} \quad \textbf{Ans.}$$