Lecture 5

Engineering Mechanics

Q. Four forces equal to P, 2P, 3P and 4P are respectively acting along the four sides of square ABCD taken in order. Find the magnitude, direction and position of the resultant force.

Solution. The system of given forces is shown in Fig. 3.12.

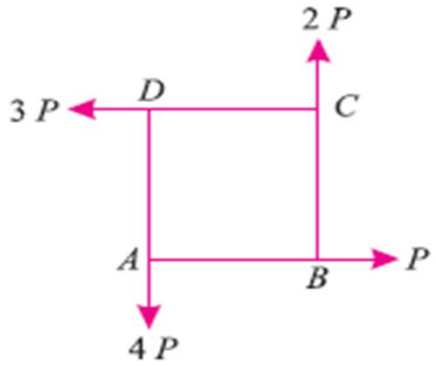


Fig. 3.12.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = P - 3P = -2P \quad \ldots(i)$$

and now resolving all forces vertically,

$$\Sigma V = 2P - 4P = -2P \quad \ldots(ii)$$

We know that magnitude of the resultant forces,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-2P)^2 + (-2P)^2}$$

$$= 2\sqrt{2}P \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant makes with the horizontal.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-2P}{-2P} = 1 \quad \text{or} \quad \theta = 45°$$

Since ΣH as well as ΣV are –ve, therefore resultant lies between 180°and 270°. Thus actual angle of the resultant force = 180° + 45° = 225° **Ans.**

Position of the resultant force

Let x = Perpendicular distance between A and the line of action of the resultant force.

Now taking moments of the resultant force about A and equating the same,

$$2\sqrt{2}P \times x = (2P \times a) + (3P \times a) = 5P \times a$$

$$\therefore \quad x = \frac{5a}{2\sqrt{2}} \quad \textbf{Ans.}$$

Q. Three forces of 2P, 3P and 4P act along the three sides of an equilateral triangle of side 100 mm taken in order. Find the magnitude and position of the resultant force.

Solution:

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 2P + 3P \cos 120° + 4P \cos 240°$$
$$= 2P + 3P\,(-0.5) + 4P\,(-0.5)$$
$$= -1.5\,P \qquad ...(i)$$

and now resolving all the forces vertically.

$$\Sigma V = 3P \sin 60° - 4P \sin 60°$$
$$= (3P \times 0.866) - (4P \times 0.866)$$
$$= -0.866\,P \qquad ...(ii)$$

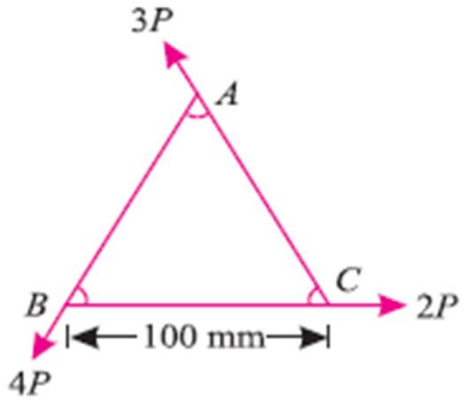


Fig. 3.11.

We know that magnitude of the resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-1.5P)^2 + (-0.866\,P)^2} = 1.732\,P \text{ Ans.}$$

Position of the resultant force

Let x = Perpendicular distance between B and the line of action of the resultant force.

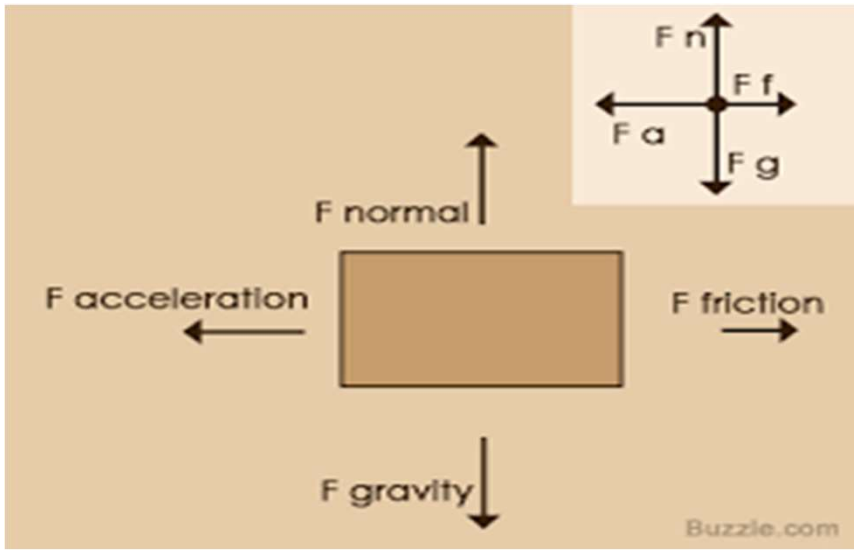
Now taking moments of the resultant force about B and equating the same,

$$1.732\,P \times x = 3P \times 100 \sin 60° = 3P \times (100 \times 0.866) = 259.8\,P$$

$$\therefore \quad x = \frac{259.8}{1.732} = 150 \text{ mm} \quad \text{Ans.}$$

Free body diagram

Free-body diagrams are **diagrams** used to show the relative magnitude and direction of all forces acting upon an object in a given situation.



LAMI'S THEOREM

It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."

Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$