Lecture 7

Engineering Mechanics

Reaction and Support

CONDITIONS OF EQUILIBRIUM

1. If the body moves in any direction, it means that there is a resultant force acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces ($\sum H$) and vertical component of all the forces ($\sum V$) must be zero. Mathematically,

$$\sum H = 0 \text{ and } \sum V = 0$$

2. If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. A little consideration will show, that if the body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. Or in other words, the resultant moment of all the forces ($\sum M$) must be zero. Mathematically,

$$\sum M = 0$$

Conti…

3. If the body moves in any direction and at the same time it rotates about itself, if means that there is a resultant force and also a resultant couple acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movements and the resultant moment of the couple causing rotation must be zero. Or in other words, horizontal component of all the forces ($\sum H$), vertical component of all the forces ($\sum V$) and resultant moment of all the forces ($\sum$ M) must be zero. Mathematically,

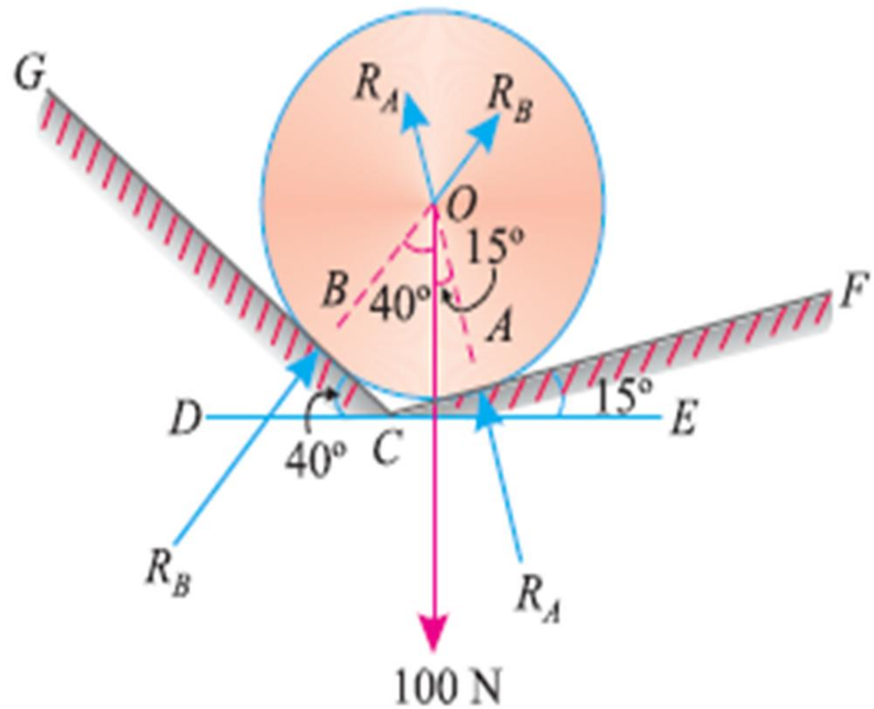
$$\sum H = 0 , \quad \sum V = 0, \quad \text{and } \sum M = 0$$

4. If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. A little consideration will show, that in this case the following conditions are already satisfied :

$$\sum H = 0, \sum V = 0, \quad \text{and } \sum M = 0$$

The above mentioned three equations are known as the conditions of equilibrium.

Q. A smooth circular cylinder of radius 1.5 meter is lying in a triangular groove, one side of which makes 15° angle and the other 40° angle with the horizontal. Find the reactions at the surfaces of contact, if there is no friction and the cylinder weights 100 N.



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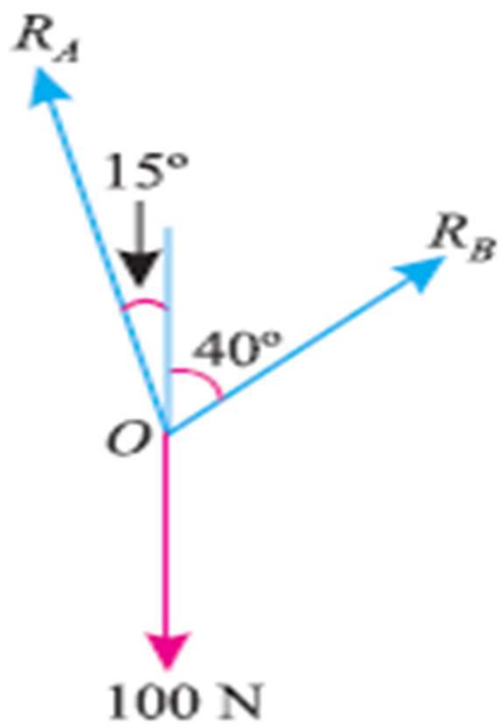

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Let R_A = Reaction at A, and

R_B = Reaction at B.

The smooth cylinder lying in the groove is shown in Fig. 5.12 (*a*). In order to keep the system in equilibrium, three forces *i.e.* R_A, R_B and weight of cylinder (100 N) must pass through the centre of the cylinder. Moreover, as there is no *friction, the reactions R_A and R_B must be normal to the surfaces as shown in Fig. 5.12 (*a*). The system of forces is shown in Fig. 5.12 (*b*).

Applying Lami's equation, at O,

$$\frac{R_A}{\sin(180° - 40°)} = \frac{R_B}{\sin(180° - 15°)} = \frac{100}{\sin(15° + 40°)}$$

or

$$\frac{R_A}{\sin 40°} = \frac{R_B}{\sin 15°} = \frac{100}{\sin 55°}$$

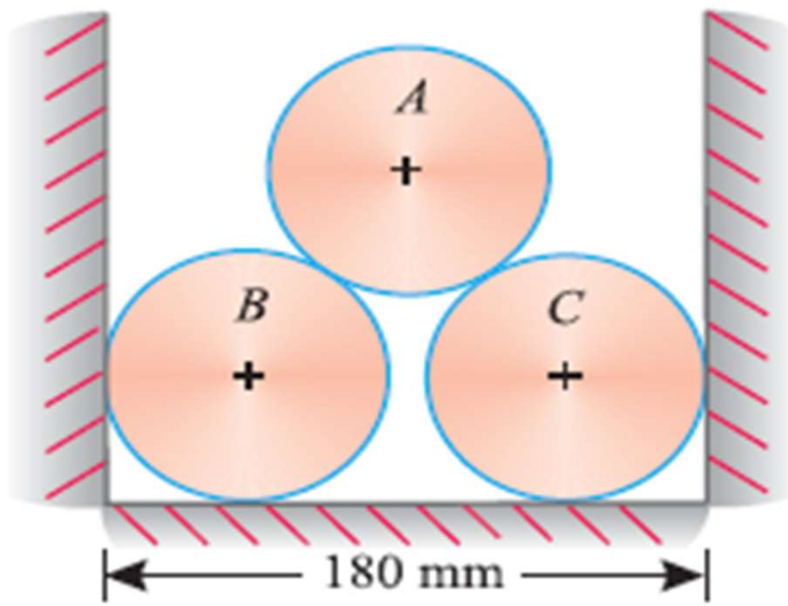
$\therefore$

$$R_A = \frac{100 \times \sin 40°}{\sin 55°} = \frac{100 \times 0.6428}{0.8192} = 78.5 \text{ N} \quad \textbf{Ans.}$$

and

$$R_B = \frac{100 \times \sin 15°}{\sin 55°} = \frac{100 \times 0.2588}{0.8192} = 31.6 \text{ N} \quad \textbf{Ans.}$$

Q. Three cylinders weighting 100 N each and of 80 mm diameter are placed in a channel of 180 mm width as shown in Fig. Determine the pressure exerted by (i) the cylinder A on B at the point of contact (ii) the cylinder B on the base and (iii) the cylinder B on the wall.



Solution:

(*i*) *Pressure exerted by the cylinder A on the cylinder B*

Let $R1$ = Pressure exerted by the cylinder *A* on *B*. It is also equal to pressure exerted by the cylinder *A* on *B*.

First of all, consider the equilibrium of the cylinder *A*. It is in equilibrium under the action of

the following forces, which must pass through the centre of the cylinder as shown in Fig. 5.17 (*a*).

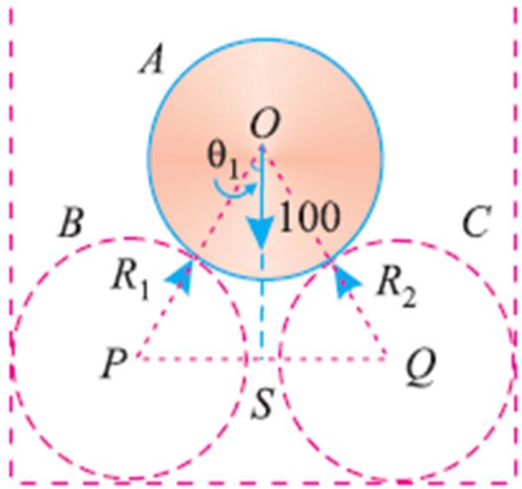
1. Weight of the cylinder 100 N acting downwards.

2. Reaction $R1$ of the cylinder *B* on the cylinder *A*.

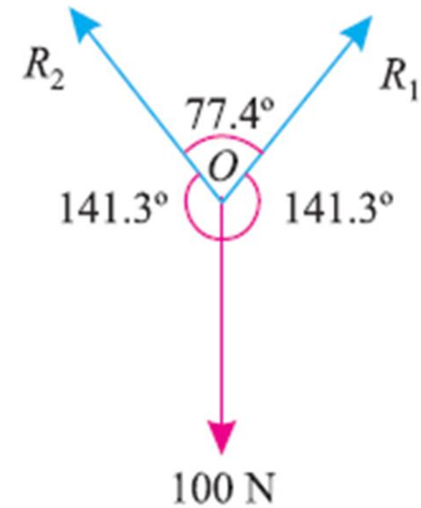
3. Reaction $R2$ of the cylinder *C* on the cylinder *A*.

Now join the centres *O*, *P* and *Q* of the three cylinders. Bisect *PQ* at *S* and join *OS* as shown in fig.

Conti..



(*a*) Free body diagram



(*b*) Force diagram

Fig. 5.17.

From the geometry of the triangle *OPS*, we find that

$$OP = 40 + 40 = 80 \text{ mm}$$

and $$PS = 90 - 40 = 50 \text{ mm}$$

$$\therefore \quad \sin \angle POS = \frac{PS}{OP} = \frac{50}{80} = 0.625$$

or $$\angle POS = 38.7°$$

Conti..

Since the triangle OSQ is similar to the triangle OPS, therefore $\angle SOQ$ is also equal to 38.7°. Thus the angle between R_1 and R_2 is $2 \times 38.7° = 77.4°$.

And angle between R_1 and OS (also between R_2 and OS)

$$= 180° - 38.7° = 141.3°$$

The system of forces at O is shown in Fig. 5.17 (b). Applying Lami's equation at O,

$$\frac{R_1}{\sin 141.3°} = \frac{R_2}{\sin 141.3°} = \frac{100}{\sin 77.4°}$$

$$\frac{R_1}{\sin 38.7°} = \frac{R_2}{\sin 38.7°} = \frac{100}{\sin 77.4°} \qquad ...[\because \sin(180° - \theta) = \sin\theta]$$

$$\therefore \qquad R_1 = \frac{100 \times \sin 38.7°}{\sin 77.4°} = \frac{100 \times 0.6252}{0.9759} = 64.0 \text{ N} \quad \textbf{Ans.}$$

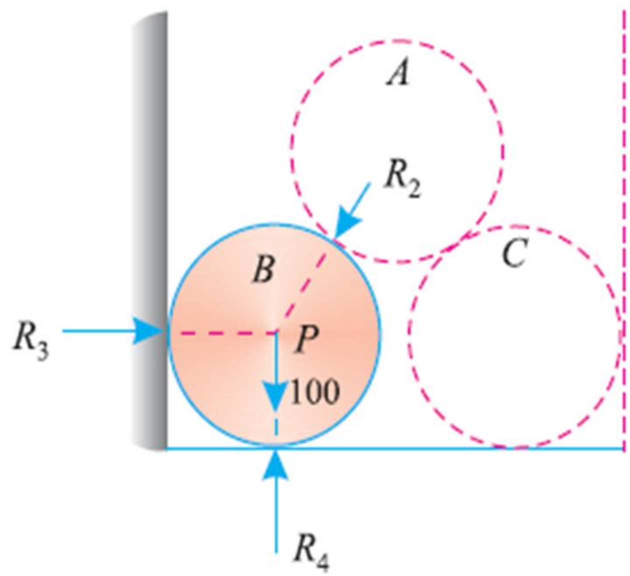
Similarly $\qquad R_2 = R_1 = 64.0$ N **Ans.**

Conti…

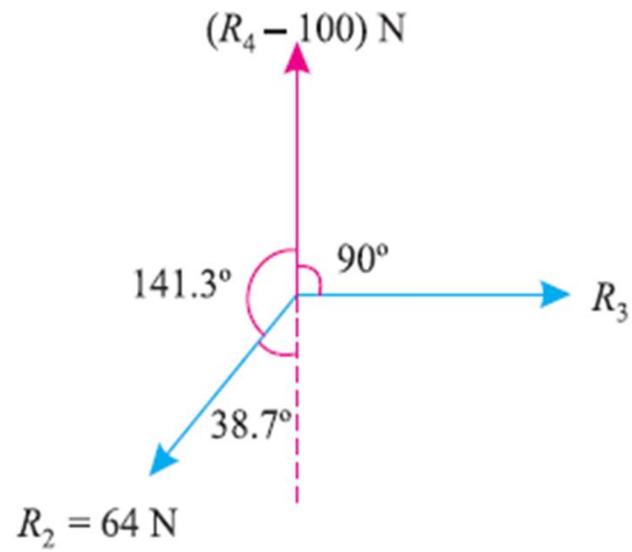
(ii) *Pressure exerted by the cylinder B on the base*

Let R_3 = Pressure exerted by the cylinder B on the wall, and

R_4 = Pressure exerted by the cylinder B on the base.



(a) Free body diagram



(b) Force diagram

Conti..

Now consider the equilibrium of the cylinder *B*. It is in equilibrium under the action of the

following forces, which must pass through the centre of the cylinder as shown in Fig. (*a*).

1. Weight of the cylinder 100 N acting downwards.
2. Reaction $R2$ equal to 64.0 N of the cylinder *A* on the cylinder *B*.
3. Reaction $R3$ of the cylinder *B* on the vertical side of the channel.
4. Reaction $R4$ of the cylinder *B* on the base of the channel.

A little consideration will show that weight of the cylinder *B* is acting downwards and the reaction $R4$ is acting upwards. Moreover, their lines of action also coincide with each other.

Therefore net downward force will be equal to ($R4$ – 100) N.

The system of forces is shown in Fig. (*b*). Applying Lami's equation at *P*,

Conti..

$$\frac{64}{\sin 90°} = \frac{R_3}{\sin(180° - 38.7°)} = \frac{(R_4 - 100)}{\sin(90° + 38.7°)}$$

$$\frac{64}{1} = \frac{R_3}{\sin 38.7°} = \frac{R_4 - 100}{\cos 38.7°}$$

$$R_4 - 100 = 64 \cos 38.7° = 64 \times 0.7804 = 50 \text{ N}$$

or $\quad R_4 = 50 + 100 = 150 \text{ N}$ **Ans.**

(*iii*) *Pressure exerted by the cylinder B on the wall*

From the above Lami's equation, we also find that

$$R_3 = 64 \sin 38.7° = 64 \times 0.6252 = 40 \text{ N} \quad \textbf{Ans.}$$