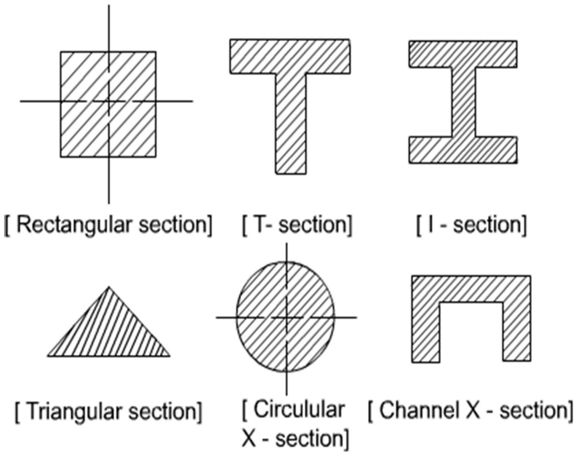
Flexural stresses in beams

- **Members Subjected to Flexural Loads**
- **Introduction:**
- In many engineering structures members are required to resist forces that are applied laterally or transversely to their axes. These type of members are termed as beam.
- There are various ways to define the beams such as
- **Definition I:** A beam is a laterally loaded member, whose cross-sectional dimensions are small as compared to its length.
- **Definition II:** A beam is nothing simply a bar which is subjected to forces or couples that lie in a plane containing the longitudinal axis of the bar. The forces are understood to act perpendicular to the longitudinal axis of the bar.
- **Definition III:** A bar working under bending is generally termed as a beam.
- **Materials for Beam:**
- The beams may be made from several usable engineering materials such commonly among them are as follows:
- Metal
- Wood
- Concrete
- Plastic

Geometric forms of Beams:

- The Area of X-section of the beam may take several forms some of them have been shown below:



Loading restrictions:

Concept of pure bending:

- As we are aware of the fact internal reactions developed on any cross-section of a beam may consists of a resultant normal force, a resultant shear force and a resultant couple. In order to ensure that the bending effects alone are investigated, we shall put a constraint on the loading such that the resultant normal and the resultant shear forces are zero on any cross-section perpendicular to the longitudinal axis of the member,

That means F = 0

since or M = constant.

Thus, the zero shear force means that the bending moment is constant or the bending is same at every cross-section of the beam. Such a situation may be visualized or envisaged when the beam or some portion of the beam, as been loaded only by pure couples at its ends. It must be recalled that the couples are assumed to be loaded in the plane of symmetry.

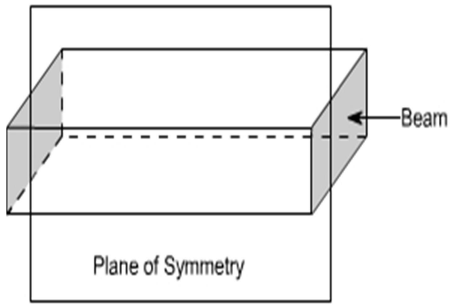


Fig (1)

Fig (2)

Bending Stresses in Beams or Derivation of Elastic Flexural formula :

- In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam **HE** and **GF** , originally parallel as shown in fig 1(a).when the beam is to bend it is assumed that these sections remain parallel i.e. **H'E'** and **G'F'** , the final position of the sections, are still straight lines, they then subtend some angle
- Consider now fibre AB in the material, at a distance y from the N.A, when the beam bends this will stretch to A'B'

Therefore,

$$\text{strain in fibre } AB = \frac{\text{change in length}}{\text{orginal length}}$$

$$= \frac{A'B' - AB}{AB} \qquad \text{But } AB = CD \text{ and } CD = C'D'$$

refer to fig1(a) and fig1(b)

$$\therefore \text{strain} = \frac{A'B' - C'D'}{C'D'}$$

- Consider now fibre AB in the material, at a distance y from the N.A, when the beam bends this will stretch to A'B'
- Since CD and C'D' are on the neutral axis and it is assumed that the Stress on the neutral axis zero. Therefore, there won't be any strain on the neutral axis

$$= \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$$

However $\frac{\text{stress}}{\text{strain}} = E$ where E = Young's Modulus of elasticity

Therefore, equating the two strains as obtained from the two relations i.e,

$$\frac{\sigma}{E} = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \qquad \ldots\ldots\ldots\ldots(1)$$

$$\sigma = \frac{E}{R} y$$

if the shaded strip is of area 'dA' then the force on the strip is

$$F = \sigma\, \delta A = \frac{E}{R} y\, \delta A$$

Moment about the neutral axis would be $= F.y = \frac{E}{R} y^2 \delta A$

The toatl moment for the whole cross-section is therefore equal to

$$M = \sum \frac{E}{R} y^2\, \delta A = \frac{E}{R} \sum y^2 \delta A$$

- Now the term is the property of the material and is called as a second moment of area of the cross-section and is denoted by a symbol I.
- Therefore $M = \frac{E}{R} I \qquad \ldots\ldots\ldots(2)$

  combining equation 1 and 2 we get

$$\boxed{\frac{\sigma}{y} = \frac{M}{T} = \frac{E}{R}}$$

- **This equation is known as the Bending Theory Equation.** The above proof has involved the assumption of pure bending without any shear force being present. Therefore this termed as the pure bending equation. This equation gives distribution of stresses which are normal to cross-section i.e. in x-direction.