Assumption:

The materiel is homogenous i.e of uniform elastic properties exists throughout the material.

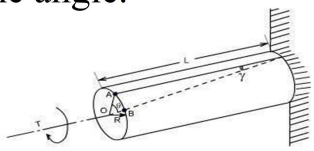
The material is elastic, follows Hook's law, with shear stress proportional to shear strain.

The stress does not exceed the elastic limit.

The circular section remains circular

Cross section remain plane.

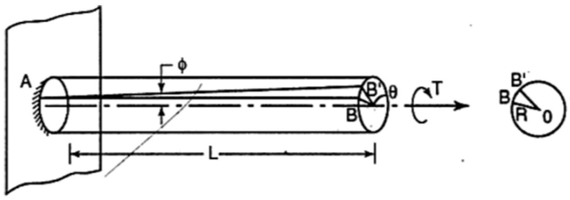
Cross section rotate as if rigid i.e. every diameter rotates through the same angle.



DERIVATION OF TORSIONAL EQUATIONS

Consider a shaft of length L, radius *R* fixed at one end and subjected to a torque T at the other end as shown in Fig.

Let O be the centre of circular section and B a point on surface. AB be the line on the shaft parallel to the axis of shaft. Due to torque T applied, let *B* move to *B'*. If γ is shear strain *(angle BOB')* and θ is the angle of twist in length *L,* then



$$R\theta = BB' = L\gamma$$

If τ_s is the shear stress and G is modulus of rigidity then,

$$\gamma = \frac{\tau}{G}$$

$$R\theta = L\frac{\tau_s}{G}$$

$$\frac{\tau_s}{R} = \frac{G\theta}{L}$$

Similarly if the point B considered is at any distance r from centre instead of on the surface, it can be shown that

$$\frac{\tau}{r} = \frac{G\theta}{L} \qquad \ldots \text{(i)}$$

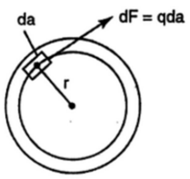
$$\frac{\tau_s}{R} = \frac{\tau}{r}$$

Thus shear stress increases linearly from zero at axis to the maximum value τ_s at surface.

Now consider the torsional resistance developed by an elemental area 'δa' at distance r from centre.

If τ is the shear stress developed in the element the resisting force is

$$dF = \tau da$$



Resisting torsional moment, dT= dF x r
= τrda

$$\tau = \tau_s \frac{r}{R}$$

Therefore,

$$dT = \tau_s \frac{r^2}{R} da$$

Total resisting torsional moment,

$$T = \sum \tau_s \frac{r^2}{R} da$$

$$T = \frac{\tau_s}{R} \sum r^2 da$$

But $\sum r^2 da$ is nothing but polar moment of inertia of the section. Representing it by notation J we get,

$$T = \frac{\tau_s}{R} J$$

$$\frac{T}{J} = \frac{\tau_s}{R}$$

$$\frac{\tau_s}{R} = \frac{\tau}{r}$$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

Where,

T - torsional moment , N-mm

J - polar moment of inertia, mm^4

τ - shear stress in the element, N/mm^2

r- distance of element from centre of shaft, mm

G - modulus of rigidity, N/mm^2

θ- angle of twist, rad

L- length of shaft, mm

Power Transmitted by a shaft : If T is the applied to the shaft, then the power transmitted by the shaft is

Distribution of shear stresses in circular Shafts subjected to torsion :

This states that the shearing stress varies directly as the distance 1r' from the axis of the shaft and the following is the stress distribution in the plane of cross section and also the complementary shearing stresses in an axial plane.

Torsional stiffness: The torsional stiffness k is defined as the torque per radian twist .

For a ductile material, the plastic flow begins first in the outer surface. For a material which is weaker in shear longitudinally than transverse ly for instance a wooden shaft, with the fibers parallel t o axis the first cracks will be produced by the shearing stresses acting in the axial section and they will upper on the surface of the shaft in the longitudinal direction.