Introduction to TOC

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Outline

Computation

Computation

Any task that can be performed by a calculator or computer.

In theory of compuatation, we are going to mathematically model of a computer or any machine in general & then going to study the theory about it, which means

- What are the capabilities of this machines?
- What are the problem could be solved by this machines?
- what are the limitations of such machines?

Sets

Definition: Set

A set is a collection of elements, without any structure other than membership.
The symbol $\in$ is used to represent membership.

Example:

- $S = \{1, 2, 3\}$
- $S = \{a, b, c, \ldots, z\}$
- $S = \{1, 3, 5, \ldots\}$ OR $S = \{i : i \geq 1, \text{i is odd}\}$
- $S = \{2, 4, 6, \ldots\}$ OR $S = \{i : i \geq 0, \text{i is even}\}$

Basic Terminolgy in set theory

Null Set

A set which contains no elements is called as empty set or null set.
For example, the set of months with 32 days.
It is represented by the symbol $\{\}$ or Φ.

Universal Set

A set which contains all the elements in the domainon which is given set defined, is called as universal set.
For example, the set of natural numbers.
$U = \{1, 2, 3, \dots, \infty\}$

Cardinality of a Set

Cardinality of a given set id the number of element in the set.
For example, if set $A = \{1, 2, 3, 4\}$ then its cardinality is 4.
It is represented as $|A| = 4$.

Basic Terminolgy in set theory

Subset & proper subset

A set A which is said to be the subset of a set B if every element of set A is also an element of set B. This relationship is usually denoted by $A \subset B$, and mathematically this relationship is written as if $x \in A$ implies $x \in B$. The concept of a subset is also written in the from of $A \subseteq B$.

If A is a subset of B ($A \subseteq B$), but A is not equal to B, then we say A is a *proper subset* of B, written as $A \subset B$ *or* $A \subsetneq B$.
Example:
$A = \{1, 3, 5\}, B = \{1, 2, 3, 4, 5\}, C = \{1, 2, 3, 4, 5\}$
A is a subset of B, $A \subseteq B$. because every element in A is also in B.
A is also proper subset of B, $A \subset B$. because every element in A is also in B and $A \neq B$.
C is subset of B, $C \subseteq B$. but is not a proper subset of B because C = B.

Basic Terminolgy in set theory

Equivalent and Equal Sets

Two sets are said to be equivalent if they contain same number of elements. Two sets are said to be equal when they contain exactly the same elemets.

Example:
$A = \{1, 2, 3\}, B = \{4, 5, 6\}, C = \{1, 2, 3\}$
A is equivalent to B, and A is equal to C

Finite and Infinite Set

Finite sets are sets that have finite number of elements, otherwise infinite sets.
For example, If A is the set of positive integers less than 12 then $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ and n(A) = 11 (Number of elements) then A is finite set.
if C is the set of numbers which are also multiples of 3 then $C = \{3, 6, 9, \ldots\}$ and C is an infinite set

Set Operations

Set Operations			
Operation	Symbol	Definition	Example
Union	$\cup$	$A \cup B = \{x : x \in A \text{ or } x \in B\}$, that is, Elements in either A or B	$\{a, b, c\} \cup \{c, d\} = \{a, b, c, d\}$
Intersection	$\cap$	$A \cap B = \{x : x \in A \text{ and } x \in B\}$, that is, Elements in both A and B	$\{a, b, c\} \cup \{c, d\} = \{c\}$
Difference	$-$	$A - B = \{x : x \in A \text{ and } x \notin B\}$, that is, Elements in A and not in B	$\{a, b, c\} - \{c, d\} = \{a, b\}$
Complements	$\complement$	$A^{\complement} = U - A = \{x : x \in U \text{ and } x \notin A\}$ that is, Elements in U and not in A	
Symmetric Difference	$\oplus$	$A \oplus B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$, that is, Elements in either A or B but not in Both	$\{a, b, c\} \oplus \{c, d, e\} = \{a, b, d, e\}$
Cartesian Products	X	$A \ X \ B = \{\{a, b\} : a \in A \text{ and } b \in B\}$, that is, All possible ordered pairs whose first component is member of A and second component is member of B.	$\{a, b\} \ X \ \{0, 1, 2\} = \{(a, 0), (a, 1), (a, 2), (b, 0), (b, 1), (b, 2)\}$

Basic Concepts

The three basic fundamental ideas in study of theory of copuatation is

1. Formal Languages
2. Grammars
3. Automata or Model

Formal Language

For the study of formal languages, needto know the some terminology

Symbol

It is the basic building block, i.e.,

$$a, b, 0, 1, \ \textit{or picture}$$

Alphabet (Σ)

It is a finite collection of symbols, and denoted by Σ
$\Sigma = \{a, b\}$, here we ahve two symbols in alphabet
$\Sigma = \{0, 1, 2\}$

String

String is a finite sequence of symbols over the given alphabet, denoted by ω
$\Sigma = \{a, b\}$, then various string that can be formed { a, b, aa, bb, ab, ba, aaa, aba, abb, . . . }

Length of String

$|\omega|=$ Number of symbols in a string
$\omega = aaabba$ then $|\omega| = 6$

Number of String of length n

We have n symbols in alphabet Σ then
Number of strings of length n is possible over $\Sigma = |\Sigma|^n$

Formal Language

Formal Language

In TOC, the language is a collection of appropriate strings over the given input alphabet.

- if Σ is alphabet, then Σ^* is called universal language.
- if L is any language defined over the alphabet Σ, then $L \subseteq \Sigma^*$

Empty Language and Non-empty Language

Language does not contain any string or empty string(ϵ or null string) called empty language.

$$L = \phi = \{\} \Leftrightarrow |L| = 0$$

Otherwise language contains some string is called non-empty string

Non-empty language is two type

1. Finite Language

Language contains finite number of string where the length of each and every string is finite, called finite language. Empty language is also finite language.

L = { 01, 10 }

2. Infinite Language

Language contains infinite number of strings where the length of each and every string is finite, called infinite language.

$L = \{0^n : n \geq 1\}$

$L = \{0, 00, 000, 0000, 00000, 000000, \ldots\}$

Infinite language are two type

1. Countable
2. Uncountable

Grammar

in TOC,

Grammar

A grammar G is defined as a quadruple

$$G = (V, T, P, S)$$

where, V is finite set of objects called variables,
T is finite set of objects called terminal symbols,
P is a finite set of production rules,
S $\in$ V is start symbol.

Automata or Model

in TOC,

Automata

An Automata is an abstract model of a digital computer.It has a mechanism for reading input, and can produce output and finally automaton has control unit which can have finite number of internal states.

An automata model can have

1. Language Accepter (Accepting the string by a model)
2. Language Generator (generating the string)

Formal Languages

Type	Formal Language	Grammar	Automata
Type 3	Regular Language	Regular Grammar	Finite Automata
Type 2	Context-free Language	Context-free Grammar	Pushdown Automata
Type 1	Context Sensitive Language	Context Sensitive Grammar	Linear Bounded Automata
Type 0	Recursive Enumerable Language	Unrestricted Grammar	Turing Machine