

EXPONENTIAL SMOOTHING METHOD

Exponential smoothing method: The new forecast for next period (period t) will be calculated as follows:

New forecast = Last period's forecast + α (Last period's actual demand – Last period's forecast)

(this box contains all you need to know to apply exponential smoothing)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad (\text{equation 1})$$

$$F_t = \alpha A_{t-1} + (1-\alpha)F_{t-1} \quad (\text{alternate equation 1 – a bit more user friendly})$$

Where α is a smoothing coefficient whose value is between 0 and 1.

The exponential smoothing method only requires that you dig up two pieces of data to apply it (the most recent actual demand and the most recent forecast).

An attractive feature of this method is that forecasts made with this model will include a portion of every piece of historical demand. Furthermore, there will be different weights placed on these historical demand values, with older data receiving lower weights. At first glance this may not be obvious, however, this property is illustrated on the following page.

DEMONSTRATION: EXPONENTIAL SMOOTHING INCLUDES ALL PAST DATA

Note: the mathematical manipulations in this box are not something you would ever have to do when applying exponential smoothing. All you need to use is equation 1 on the previous page. This demonstration is to convince the skeptics that when using equation 1, all historical data will be included in the forecast, and the older the data, the lower the weight applied to that data.

To make a forecast for next period, we would use the user friendly alternate equation 1:

$$F_t = \alpha A_{t-1} + (1-\alpha)F_{t-1} \quad (\text{equation 1})$$

When we made the forecast for the current period (F_{t-1}), it was made in the following fashion:

$$F_{t-1} = \alpha A_{t-2} + (1-\alpha)F_{t-2} \quad (\text{equation 2})$$

If we substitute equation 2 into equation 1 we get the following:

$$F_t = \alpha A_{t-1} + (1-\alpha)[\alpha A_{t-2} + (1-\alpha)F_{t-2}]$$

Which can be cleaned up to the following:

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + (1-\alpha)^2 F_{t-2} \quad (\text{equation 3})$$

We could continue to play that game by recognizing that $F_{t-2} = \alpha A_{t-3} + (1-\alpha)F_{t-3}$ (equation 4)

If we substitute equation 4 into equation 3 we get the following:

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + (1-\alpha)^2[\alpha A_{t-3} + (1-\alpha)F_{t-3}]$$

Which can be cleaned up to the following:

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + (1-\alpha)^3 F_{t-3}$$

If you keep playing that game, you should recognize that

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \alpha(1-\alpha)^3 A_{t-4} + \alpha(1-\alpha)^4 A_{t-5} + \alpha(1-\alpha)^5 A_{t-6} \dots\dots\dots$$

As you raise those decimal weights to higher and higher powers, the values get smaller and smaller.

EXPONENTIAL SMOOTHING ILLUSTRATION

In this illustration we assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, for each subsequent year (beginning with year 2) we made a forecast using the exponential smoothing model. After the forecast was made, we waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

This set of forecasts was made using an α value of .1

Year	Actual Demand (A)	Forecast (F)	Notes
1	310	300	This was a guess, since there was no prior demand data.
2	365	301	From this point forward, these forecasts were made on a year-by-year basis using exponential smoothing with $\alpha=.1$
3	395	307.4	
4	415	316.16	
5	450	326.044	
6	465	338.4396	
7		351.09564	

A SECOND EXPONENTIAL SMOOTHING ILLUSTRATION

In this illustration we assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, for each subsequent year (beginning with year 2) we made a forecast using the exponential smoothing model. After the forecast was made, we waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

This set of forecasts was made using an α value of .2

Year	Actual Demand (A)	Forecast (F)	Notes
1	310	300	This was a guess, since there was no prior demand data.
2	365	302	From this point forward, these forecasts were made on a year-by-year basis using exponential smoothing with $\alpha=.2$
3	395	314.6	
4	415	330.68	
5	450	347.544	
6	465	368.0352	
7		387.42816	

A THIRD EXPONENTIAL SMOOTHING ILLUSTRATION

In this illustration we assume that, in the absence of data at startup, we made a guess for the year 1 forecast (300). Then, for each subsequent year (beginning with year 2) we made a forecast using the exponential smoothing model. After the forecast was made, we waited to see what demand unfolded during the year. We then made a forecast for the subsequent year, and so on right through to the forecast for year 7.

This set of forecasts was made using an α value of .4

Year	Actual Demand (A)	Forecast (F)	Notes
1	310	300	This was a guess, since there was no prior demand data.
2	365	304	From this point forward, these forecasts were made on a year-by-year basis using exponential smoothing with $\alpha=.4$
3	395	328.4	
4	415	355.04	
5	450	379.024	
6	465	407.4144	
7		430.44864	