MEASURING FORECAST ACCURACY

Mean Forecast Error (MFE): Forecast error is a measure of how accurate our forecast was in a given time period. It is calculated as the actual demand minus the forecast, or

$E_t = A_t - F_t$

Forecast error in one time period does not convey much information, so we need to look at the accumulation of errors over time. We can calculate the average value of these forecast errors over time (i.e., a **Mean Forecast Error**, or **MFE**).Unfortunately, the accumulation of the E_t values is not always very revealing, for some of them will be positive errors and some will be negative. These positive and negative errors cancel one another, and looking at them alone (or looking at the MFE over time) might give a false sense of security. To illustrate, consider our original data, and the accompanying pair of hypothetical forecasts made with two different forecasting methods.

Year	Actual Demand A_t	Hypothetical Forecasts Made With Method 1 F_t	Forecast Error With Method 1 $A_t - F_t$	Hypothetical Forecasts Made With Method 2 F_t	Forecast Error With Method 2 $A_t - F_t$
1	310	315	-5	370	-60
2	365	375	-10	455	-90
3	395	390	5	305	90
4	415	405	10	535	-120
5	450	435	15	390	60
6	465	480	-15	345	120
Accumulated Forecast Errors			0		0
Mean Forecast Error, MFE			0/6 = 0		0/6 = 0

Based on the accumulated forecast errors over time, the two methods look equally good. But, most observers would judge that Method 1 is generating better forecasts than Method 2 (i.e., smaller misses).

MEASURING FORECAST ACCURACY

Mean Absolute Deviation (MAD): To eliminate the problem of positive errors canceling negative errors, a simple measure is one that looks at the absolute value of the error (size of the deviation, regardless of sign). When we disregard the sign and only consider the size of the error, we refer to this deviation as the absolute deviation. If we accumulate these absolute deviations over time and find the average value of these absolute deviations, we refer to this measure as the mean absolute deviation (MAD). For our hypothetical two forecasting methods, the absolute deviations can be calculated for each year and an average can be obtained for these yearly absolute deviations, as follows:

		Hypothetical Forecasting Method 1			Hypothetical Forecasting Method 2		
Year	Actual Demand A_t	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\|A_t - F_t\|$	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\|A_t - F_t\|$
1	310	315	-5	5	370	-60	60
2	365	375	-10	10	455	-90	90
3	395	390	5	5	305	90	90
4	415	405	10	10	535	-120	120
5	450	435	15	15	390	60	60
6	465	480	-15	15	345	120	120
	Total Absolute Deviation			60			540
	Mean Absolute Deviation			60/6=10			540/6=90

The smaller misses of Method 1 has been formalized with the calculation of the MAD. Method 1 seems to have provided more accurate forecasts over this six year horizon, as evidenced by its considerably smaller MAD.

MEASURING FORECAST ACCURACY

Mean Squared Error (MSE): Another way to eliminate the problem of positive errors canceling negative errors is to square the forecast error. Regardless of whether the forecast error has a positive or negative sign, the squared error will always have a positive sign. If we accumulate these squared errors over time and find the average value of these squared errors, we refer to this measure as the mean squared error (MSE). For our hypothetical two forecasting methods, the squared errors can be calculated for each year and an average can be obtained for these yearly squared errors, as follows:

		Hypothetical Forecasting Method 1			Hypothetical Forecasting Method 2		
Year	Actual Demand A_t	Forecast F_t	Forecast Error $A_t - F_t$	Squared Error $(A_t - F_t)^2$	Forecast F_t	Forecast Error $A_t - F_t$	Squared Error $(A_t - F_t)^2$
1	310	315	-5	25	370	-60	3600
2	365	375	-10	100	455	-90	8100
3	395	390	5	25	305	90	8100
4	415	405	10	100	535	-120	14400
5	450	435	15	225	390	60	3600
6	465	480	-15	225	345	120	14400
		Total Squared Error		700			52200
		Mean Squared Error		700/6 = 116.67			52200/6 = 8700

Method 1 seems to have provided more accurate forecasts over this six year horizon, as evidenced by its considerably smaller MSE.

The Question often arises as to why one would use the more cumbersome MSE when the MAD calculations are a bit simpler (you don't have to square the deviations). MAD does have the advantage of simpler calculations. However, there is a benefit to the MSE method. Since this method squares the error term, large errors tend to be magnified. Consequently, MSE places a higher penalty on large errors. This can be useful in situations where small forecast errors don't cause much of a problem, but large errors can be devastating.

MEASURING FORECAST ACCURACY

Mean Absolute Percent Error (MAPE): A problem with both the MAD and MSE is that their values depend on the magnitude of the item being forecast. If the forecast item is measured in thousands or millions, the MAD and MSE values can be very large. To avoid this problem, we can use the MAPE. MAPE is computed as the average of the absolute difference between the forecasted and actual values, expressed as a percentage of the actual values. In essence, we look at how large the miss was relative to the size of the actual value. For our hypothetical two forecasting methods, the absolute percentage error can be calculated for each year and an average can be obtained for these yearly values, yielding the MAPE, as follows:

		Hypothetical Forecasting Method 1			Hypothetical Forecasting Method 2		
Year	Actual Demand A_t	Forecast F_t	Forecast Error $A_t - F_t$	Absolute % Error $100\|A_t - F_t\|/A_t$	Forecast F_t	Forecast Error $A_t - F_t$	Absolute % Error $100\|A_t - F_t\|/A_t$
1	310	315	-5	1.16%	370	-60	19.35%
2	365	375	-10	2.74%	455	-90	24.66%
3	395	390	5	1.27%	305	90	22.78%
4	415	405	10	2.41%	535	-120	28.92%
5	450	435	15	3.33%	390	60	13.33%
6	465	480	-15	3.23%	345	120	17.14%
	Total Absolute % Error			14.59%			134.85%
	Mean Absolute % Error			14.59/6= 2.43%			134.85/6= 22.48%

Method 1seems to have provided more accurate forecasts over this six year horizon, as evidenced by the fact that the percentages by which the forecasts miss the actual demand are smaller with Method 1 (i.e., smaller MAPE).

ILLUSTRATION OF THE FOUR FORECAST ACCURACY MEASURES

Here is a further illustration of the four measures of forecast accuracy, this time using hypothetical forecasts that were generated using some different methods than the previous illustrations (called forecasting methods A and B; actually, these forecasts were made up for purposes of illustration). These calculations illustrate why we cannot rely on just one measure of forecast accuracy.

		Hypothetical Forecasting Method A					Hypothetical Forecasting Method B				
Year	Actual Demand A_t	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\|A_t - F_t\|$	Squared Deviation $(A_t - F_t)^2$	Abs. % Error $\|A_t-F_t\|/A_t$	Forecast F_t	Forecast Error $A_t - F_t$	Absolute Deviation $\|A_t - F_t\|$	Squared Deviation $(A_t - F_t)^2$	Abs. % Error $\|A_t-F_t\|/A_t$
1	310	330	-20	20	400	6.45%	310	0	0	0	0%
2	365	345	20	20	400	5.48%	365	0	0	0	0%
3	395	415	-20	20	400	5.06%	395	0	0	0	0%
4	415	395	20	20	400	4.82%	415	0	0	0	0%
5	450	430	20	20	400	4.44%	390	60	60	3600	13.33%
6	465	485	-20	20	400	4.30%	525	-60	60	3600	12.90%
		Totals	0	120	2400	30.55%	Totals	0	120	7200	26.23%
			MFE = 0/6 = 0	MAD = 120/6 = 20	MSE = 2400/6 = 400	MAPE= 30.55/6 5.09%		MFE = 0/6 = 0	MAD = 120/6 = 20	MSE = 7200/6 = 1200	MAPE= 26.23/6 4.37%

You can observe that for each of these forecasting methods, the same MFE resulted and the same MAD resulted. With these two measures, we would have no basis for claiming that one of these forecasting methods was more accurate than the other. With several measures of accuracy to consider, we can look at all the data in an attempt to determine the better forecasting method to use. Interpretation of these results will be impacted by the biases of the decision maker and the parameters of the decision situation. For example, one observer could look at the forecasts with method A and note that they were pretty consistent in that they were always missing by a modest amount (in this case, missing by 20 units each year). However, forecasting method B was very good in some years, and extremely bad in some years (missing by 60 units in years 5 and 6). That observation might cause this individual to prefer the accuracy and consistency of forecasting method A. This causal observation is formalized in the calculation of the MSE. Forecasting method A has a considerably lower MSE than forecasting method B. The squaring magnified those big misses that were observed with forecasting method B. However, another individual might view these results and have a preference for method B, for the sizes of the misses relative to the sizes of the actual demand are smaller than for method A, as indicated by the MAPE calculations.