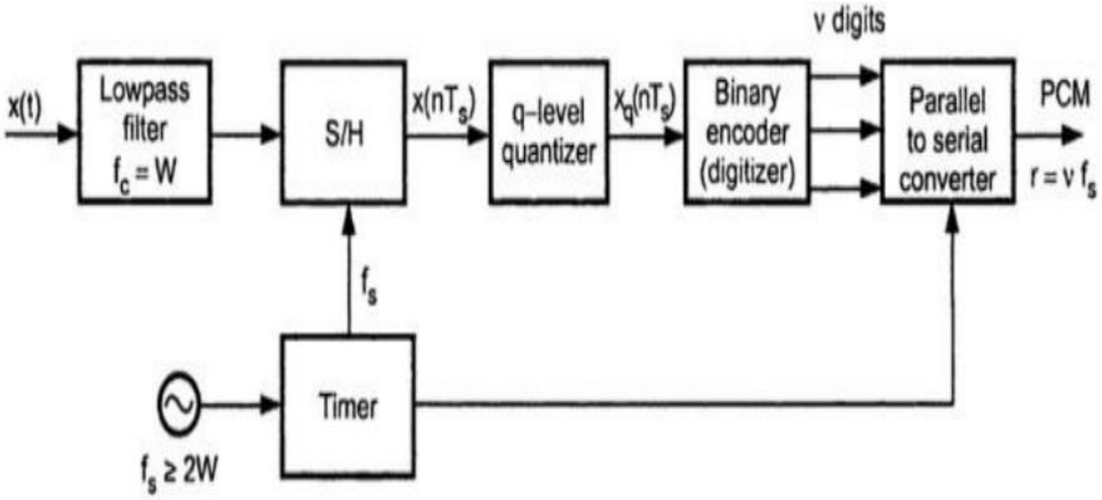
PCM Generator:



PCM Transmission Path:

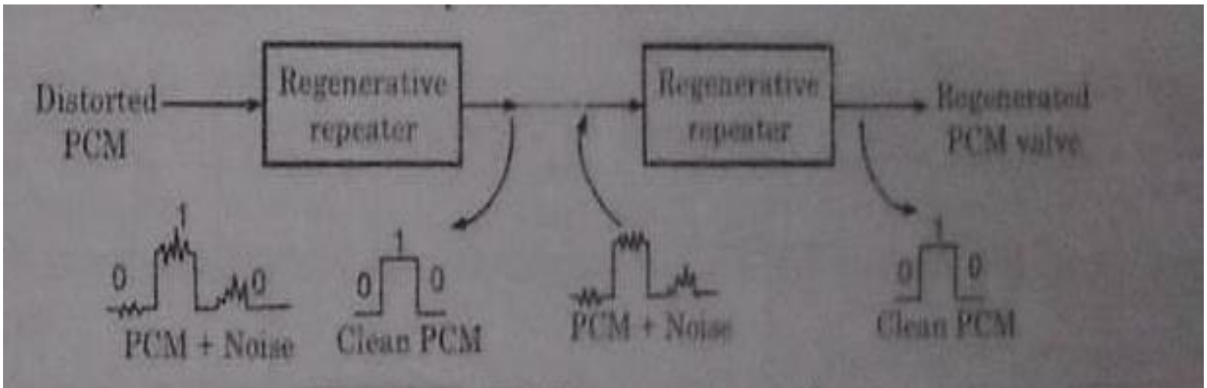
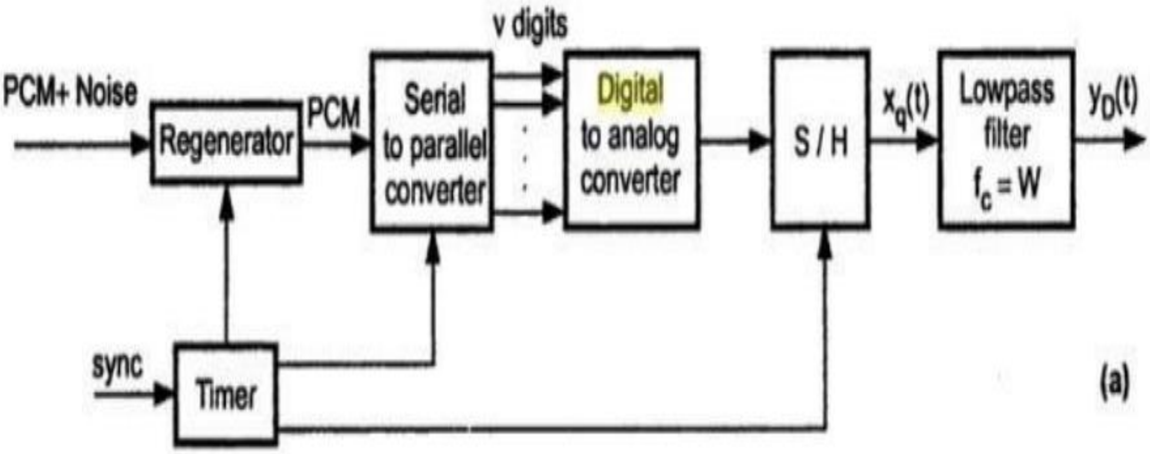


Fig. 10.4. *PCM transmission path.*

PCM Receiver:



Transmission BW in PCM:

Let the quantizer use 'v' number of binary digits to represent each level. Then the number of levels that can be represented by 'v' digits will be,

$$q = 2^v \quad \text{... } 1$$

Here 'q' represents total number of digital levels of q-level quantizer.

For example if v = 3 bits, then total number of levels will be,

$$q = 2^3 = 8 \text{ levels}$$

Each sample is converted to 'v' binary bits. i.e. Number of bits per sample = v

We know that, Number of samples per second = f_s

∴ Number of bits per second is given by,

$$\text{(Number of bits per second)} = \text{(Number of bits per samples)} \times \text{(Number of samples per second)}$$

$$= v \text{ bits per sample} \times f_s \text{ samples per second} \quad \text{... } 2$$

The number of bits per second is also called signaling rate of PCM and is denoted by 'r' i.e.,

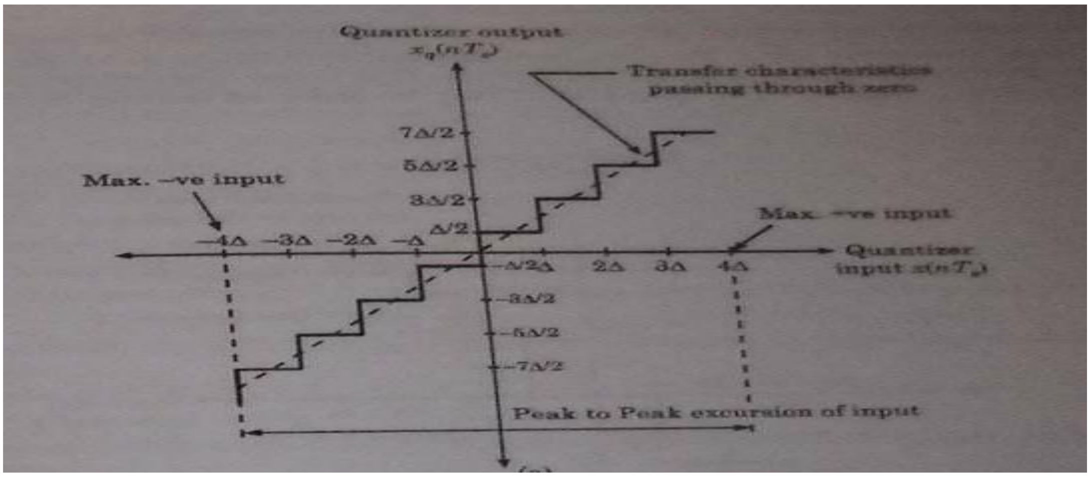
$$\boxed{\text{Signaling rate in PCM : } r = v f_s} \quad \text{... } 3$$

Here $f_s \geq 2W$.

Bandwidth needed for PCM transmission will be given by half of the signaling rate i.e.,

$$\text{Transmission Bandwidth of PCM : } \begin{cases} B_T \geq \frac{1}{2} r & \text{... } 4 \\ B_T \geq \frac{1}{2} v f_s \quad \text{Since } f_s \geq 2W & \text{.... } 5 \\ B_T \geq v W & \text{... } 6 \end{cases}$$

Quantization Noise:



Derivation of Quantization Error/Noise or Noise Power for Uniform (Linear) Quantization

Step 1 : Quantization Error

Because of quantization, inherent errors are introduced in the signal. This error is called *quantization error*. We have defined quantization error as,

$$\varepsilon = x_q(nT_s) - x(nT_s) \qquad \text{..........} (1)$$

Step 2 : Step size

Let an input $x(nT_s)$ be of continuous amplitude in the range $-x_{max}$ to $+x_{max}$.

Therefore the total amplitude range becomes,

$$\text{Total amplitude range} = x_{max} - (-x_{max}) = 2x_{max} \qquad \text{..........} (2)$$

If this amplitude range is divided into 'q' levels of quantizer, then the step size 'δ' is given as,

$$\delta = \frac{x_{max} - (-x_{max})}{q} = \frac{2x_{max}}{q} \qquad \text{..........} (3)$$

If signal $x(t)$ is normalized to minimum and maximum values equal to 1, then

$$x_{max} = 1, \quad -x_{max} = -1 \qquad \text{..........} (4)$$

Therefore step size will be,

$$\delta = \frac{2}{q} \quad \text{(for normalized signal)} \qquad \text{..........} (5)$$

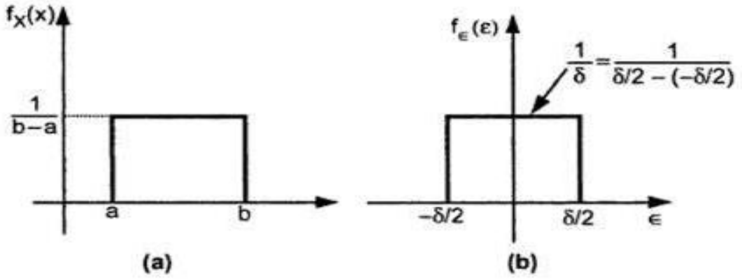
Step 3 : Pdf of Quantization error

If step size 'δ' is sufficiently small, then it is reasonable to assume that the quantization error 'ε' will be uniformly distributed random variable. The maximum quantization error is given by

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right| \qquad \text{..........} (6)$$

i.e. $$-\frac{\delta}{2} \geq \varepsilon_{max} \geq \frac{\delta}{2} \qquad \text{..........} (7)$$

Thus over the interval $\left(-\frac{\delta}{2}, \frac{\delta}{2}\right)$ quantization error is uniformly distributed random variable.



In above figure, a random variable is said to be uniformly distributed over an interval (a, b). Then PDF of 'X' is given by, (from equation of Uniform PDF).

$$f_X(x) = \begin{cases} 0 & \text{for} \quad x \le a \\ \dfrac{1}{b-a} & \text{for} \quad a < x \le b \\ 0 & \text{for} \quad x > b \end{cases} \qquad \text{.......................(8)}$$

Thus with the help of above equation we can define the probability density function for quantization error 'ε' as,

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for} \quad \varepsilon \le \dfrac{\delta}{2} \\ \dfrac{1}{\delta} & \text{for} \quad -\dfrac{\delta}{2} < \varepsilon \le \dfrac{\delta}{2} \\ 0 & \text{for} \quad \varepsilon > \dfrac{\delta}{2} \end{cases} \qquad \text{..................................(9)}$$

Noise Power

If type of signal at input i.e., $x(t)$ is known, then it is possible to calculate signal power.

The signal to quantization noise ratio of the quantizer is defined as,

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{Noise power (normalized)}} \qquad \cdots 10$$

$$\text{Noise power} = \frac{V_{noise}^2}{R} \qquad \cdots (11)$$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_\varepsilon(\varepsilon)$, its mean square value is given as,

$$\text{mean square value} = E[\varepsilon^2] = \bar{\varepsilon}^2 \qquad \dots (12)$$

The mean square value of a random variable 'X' is given as,

$$\bar{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx \quad \text{By definition} \qquad \dots (13)$$

Here

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\,d\varepsilon \qquad \dots (14)$$

From equation 9 we can write above equation as,

$$E[\varepsilon^2] = \int_{-\delta/2}^{\delta/2} \varepsilon^2 \times \frac{1}{\delta}\,d\varepsilon$$

$$= \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta/2}^{\delta/2} = \frac{1}{\delta}\left[\frac{(\delta/2)^3}{3} + \frac{(\delta/2)^3}{3}\right]$$

$$= \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right] = \frac{\delta^2}{12} \qquad \dots (15)$$

the mean square value of noise voltage is,

$$V_{noise}^2 = \text{mean square value} = \frac{\delta^2}{12}$$

Thus

Normalized noise power

or Quantization noise power $= \frac{\delta^2}{12}$ **; For linear quantization.**

or Quantization error (in terms of power) ... (16)