Elasticity and Hooke's Law

- All solid materials deform when they are stressed, and as stress is increased, deformation also increases.
- If a material returns to its original size and shape on removal of load causing deformation, it is said to be **elastic**.
- If the stress is steadily increased, a point is reached when, after the removal of load, not all the induced strain is removed.
- This is called the elastic limit.

Hooke's Law

- States that providing the limit of proportionality of a material is not exceeded, the stress is directly proportional to the strain produced.
- If a graph of stress and strain is plotted as load is gradually applied, the first portion of the graph will be a straight line.
- The slope of this line is the constant of proportionality called modulus of Elasticity, E or Young's Modulus.
- It is a measure of the stiffness of a material.

Hooke's Law

Modulus of Elasticity, E = $\frac{Direct\ stress}{Direct\ strain} = \frac{\sigma}{\varepsilon}$

Also: For Shear stress: Modulus of rigidity or shear modulus, G = $\frac{Shear\ stress}{Shear\ strain} = \frac{\tau}{\gamma}$

Also: Volumetric strain , is proportional to hydrostatic stress, within the elastic range i.e. :

$$\sigma / \varepsilon_v = K$$

'K' called **bulk modulus**.

Stress-Strain Relations of Mild Steel

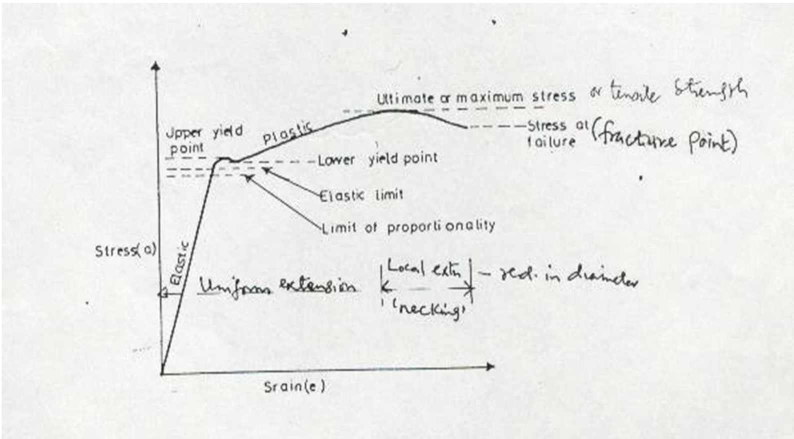


Fig: Behaviour of mild-steel rod under tension.

Equation For Extension

From the above equations:

$$E = \frac{\sigma}{\varepsilon} = \frac{F / A}{dl / L} = \frac{F\,L}{A\,dl}$$

$$dl = \frac{F\,L}{A\,E}$$

This equation for extension is very important

Factor of Safety

- The load which any member of a machine carries is called working load, and stress produced by this load is the working stress.
- Obviously, the working stress must be less than the yield stress, tensile strength or the ultimate stress.
- This working stress is also called the permissible stress or the allowable stress or the design stress.

Factor of Safety Contd.

- Some reasons for factor of safety include the inexactness or inaccuracies in the estimation of stresses and the non-uniformity of some materials.

$$\text{Factor of safety} = \frac{\text{Ultimate or yield stress}}{\text{Design or working stress}}$$

Note: Ultimate stress is used for materials e.g. concrete which do not have a well-defined yield point, or brittle materials which behave in a linear manner up to failure. Yield stress is used for other materials e.g. steel with well defined yield stress.

Results From a Tensile Test

(a) Modulus of Elasticity, $E = \frac{\text{Stress up to limit of proportionality}}{\text{Strain}}$

(b) Yield Stress or Proof Stress (See below)

(c) Percentage elongation = $\frac{\text{Increase in gauge length}}{\text{Original gauge length}} \times 100$

(d) Percentage reduction in area = $\frac{\text{Original area} - \text{area at fracture}}{\text{Original area}} \times 100$

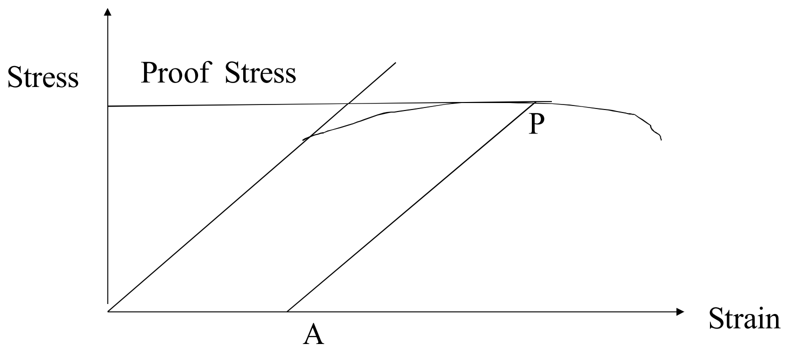
(e) Tensile Strength = $\frac{\text{Maximum load}}{\text{Original cross sectional area}}$

The percentage of elongation and percentage reduction in area give an indication of the ductility of the material i.e. its ability to withstand strain without fracture occurring.

Proof Stress

- High carbon steels, cast iron and most of the non-ferrous alloys do not exhibit a well defined yield as is the case with mild steel.
- For these materials, a limiting stress called proof stress is specified, corresponding to a non-proportional extension.
- The non-proportional extension is a specified percentage of the original length e.g. 0.05, 0.10, 0.20 or 0.50%.

Determination of Proof Stress



The proof stress is obtained by drawing AP parallel to the initial slope of the stress/strain graph, the distance, OA being the strain corresponding to the required non-proportional extension e.g. for 0.05% proof stress, the strain is 0.0005.