Derivation of Maximum Signal to Quantization Noise Ratio for Linear Quantization:

The signal to quantization noise ratio of the quantizer is defined as,

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{Noise power (normalized)}}$$

$$= \frac{\text{Normalized signal power}}{(\delta^2 / 12)} \quad \cdots (17)$$

The number of bits 'v' and quantization levels 'q' are related as,

$$q = 2^v \quad \cdots (18)$$

Putting this value in equation (3) we have,

$$\delta = \frac{2\,x_{max}}{2^v} \quad \cdots (19)$$

Let normalized signal power be denoted as 'P'.

Then

$$\frac{S}{N} = \frac{P}{\frac{4\,x_{max}^2}{2^{2v}} \times \frac{1}{12}} = \frac{3P}{x_{max}^2} \cdot 2^{2v}$$

This is the required relation for maximum signal to quantization noise ratio. Thus,

Maximum signal to quantization noise ratio : $\frac{S}{N} = \frac{3P}{x_{max}^2} \cdot 2^{2v}$ $\cdots (20)$

If we assume that input $x(t)$ is normalized, i.e.,

$$x_{max} = 1 \quad \cdots (21)$$

Then signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P \quad \cdots (22)$$

If the destination signal power 'P' is normalized, i.e.,

$$P \leq 1 \quad \cdots (23)$$

Then the signal to noise ratio is given as,

$$\frac{S}{N} \leq 3 \times 2^{2v} \quad \cdots (24)$$

Expressing the signal to noise ratio in decibels,

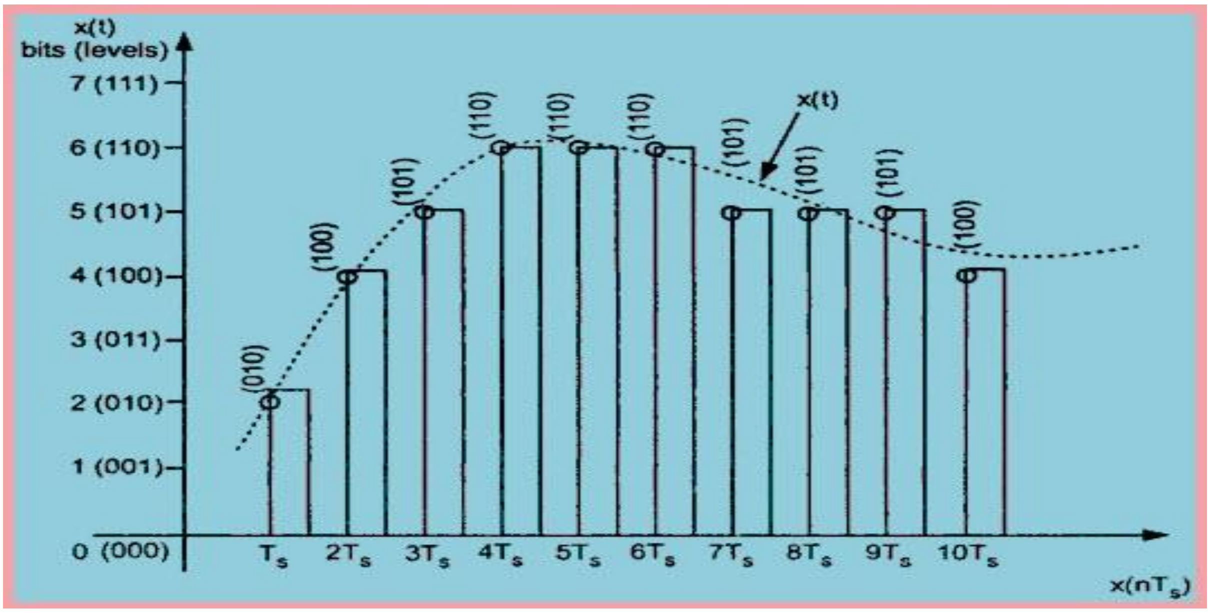
$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB \quad \text{since power ratio.}$$

$$\leq 10 \log_{10} [3 \times 2^{2v}]$$

$$\leq (4.8 + 6\,v)\, dB$$

Differential pulse code modulation:

Differential pulse code modulation is a technique of analog to digital signal conversion. This technique samples the analog signal and then quantizes the difference between the sampled value and its predicted value, then encodes the signal to form a digital value. Before going to discuss differential pulse code modulation, we have to know the demerits of PCM (Pulse Code Modulation). The samples of a signal are highly correlated with each other. The signal's value from the present sample to the next sample does not differ by a large amount. The adjacent samples of the signal carry the same information with a small difference. When these samples are encoded by the standard PCM system, the resulting encoded signal contains some redundant information bits. The below figure illustrates this.

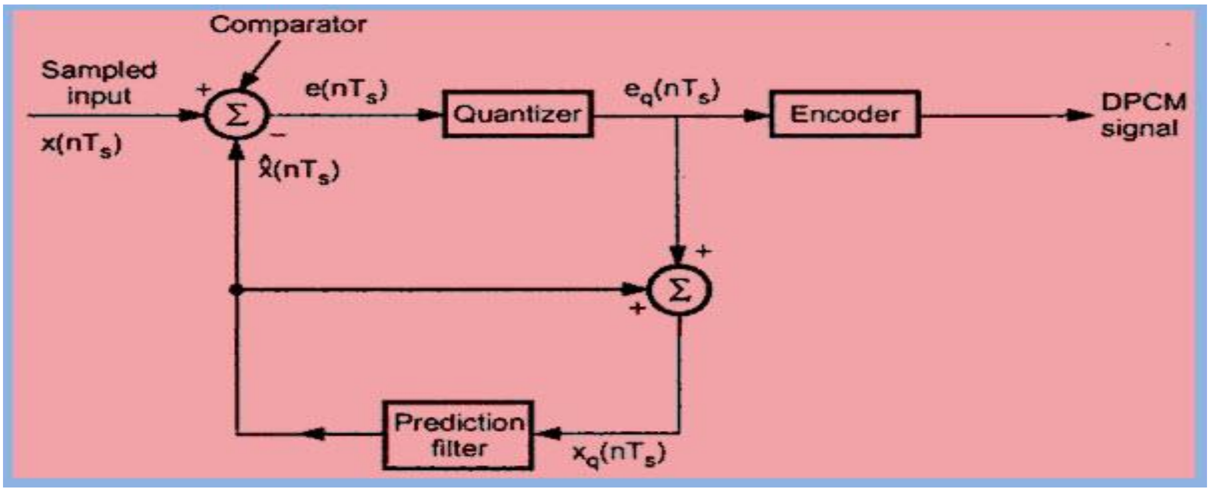


The above figure shows a continuing time signal x(t) denoted by a dotted line. This signal is sampled by flat-top sampling at intervals T_s, $2T_s$, $3T_s \ldots nT_s$. The sampling frequency is selected to be higher than the Nyquist rate. These samples are encoded by using 3-bit (7 levels) PCM. The samples are quantized to the nearest digital level as shown by small circles in the above figure. The encoded binary value of each sample is written on the top of the samples. Just observe the above figure at samples taken at $4T_s$, $5T_s$, and $6T_s$ are encoded to the same value of (110). This information can be carried only by one sample value. But three samples are carrying the same information means redundant.

Principle of Differential Pulse Code Modulation:

If the redundancy is reduced, then the overall bit rate will decrease and the number of bits required to transmit one sample will also reduce. This type of digital pulse modulation technique is called differential pulse code modulation. The DPCM works on the principle of prediction. The value of the present sample is predicted from the previous samples. The prediction may not be exact, but it is very close to the actual sample value.

Differential Pulse Code Modulation **Transmitter:**



The sampled signal is denoted by $x(nT_s)$ and the predicted signal is indicated by $x\^(nT_s)$. The comparator finds out the difference between the actual sample value $x\ (nT_s)$ and the predicted value $x\^(nT_s)$. This is called signal error and it is denoted as $e(nT_s)$

$e(nT_s)= x(nT_s)- x\^(\ nT_s)$ …….(1)

Here the predicted value $x\^(nT_s)$ is produced by using a prediction filter(signal processing filter). The quantizer output signal $e_q(nT_s)$ and the previous prediction is added and given as input to the prediction filter, this signal is denoted by $x_q(nT_s)$. This makes the prediction closer to the actually sampled signal. The quantized error signal $e_q(nT_s)$ is very small and can be encoded by using a small number of bits. Thus the number of bits per sample is reduced in DPCM.

The quantizer output would be written as,

$e_q(nT_s)= e(nT_s)+ q(nT_s)$ ……(2)

Here $q(nT_s)$ is quantization error. From the above block diagram the prediction filter input $x_q(nT_s)$ is obtained by sum of x^(nT_s) and the quantizer output $e_q(nT_s)$.

i.e, $x_q(nT_s)$ = x^(nT_s)+ $e_q(nT_s)$………. (3)

by substituting the value of e_q(nTs) from the equation (2) in equation (3) we get,
$x_q(nT_s)$ = x^(nT_s)+ $e(nT_s)$+ $q(nT_s)$……. (4)

Equation (1) can written as,

e (nT_s)+ x^(nT_s) = $x(nT_s)$……. (5)

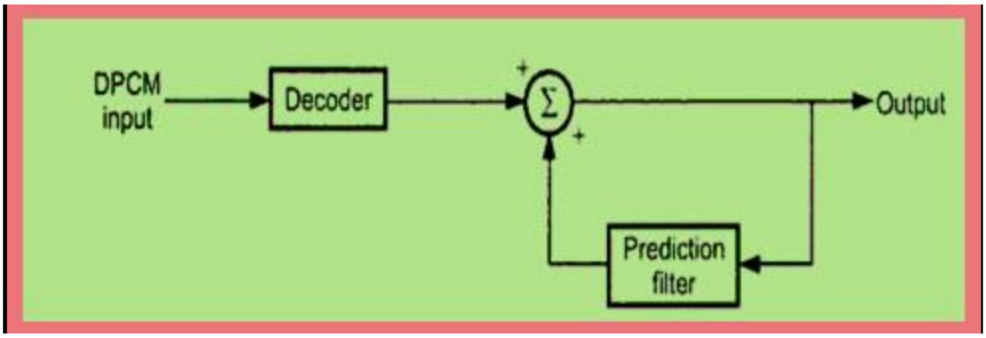
from the above equations 4 and 5 we get,

$x_q(nT_s) = x(nT_s)+ q (nT_s)$

Therefore, the quantized version of signal $x_q(nT_s)$ is the sum of original sample value and quantized error $q(nT_s)$. The quantized error can be positive or negative. So the output of the prediction filter does not depend on its characteristics.

Differential Pulse Code Modulation **Receiver:**

In order to reconstruct the received digital signal, the DPCM receiver (shown in the below figure) consists of a decoder and prediction filter. In the absenteeism of noise, the encoded receiver input will be the same as the encoded transmitter output.


DPCM input
Decoder
Σ
Output
Prediction filter