Thermal Strain

Most structural materials expand when heated,
in accordance to the law: $\varepsilon = \alpha T$

where ε is linear strain and

α is the coefficient of linear expansion;

T is the rise in temperature.

That is for a rod of Length, L;

if its temperature increased by t, the extension,
dl = α L T.

Thermal Strain Contd.

As in the case of lateral strains, thermal strains
do not induce stresses unless they are constrained.
The total strain in a body experiencing thermal stress
may be divided into two components:

Strain due to stress, ε_σ and

That due to temperature, ε_T.

Thus: $\varepsilon = \varepsilon_\sigma + \varepsilon_T$

$$\varepsilon = \frac{\sigma}{E} + \alpha T$$

Principle of Superposition

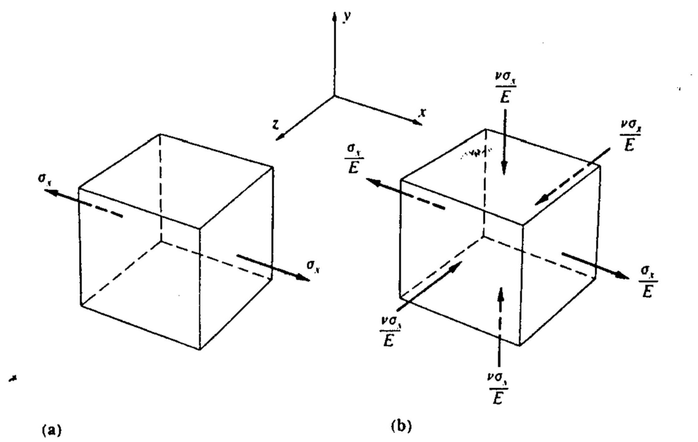
- It states that the effects of several actions taking place simultaneously can be reproduced exactly by adding the effect of each action separately.
- The principle is general and has wide applications and holds true if:

(i) The structure is elastic

(ii) The stress-strain relationship is linear

(iii) The deformations are small.

General Stress-Strain Relationships



Relationship between Elastic Modulus (E) and Bulk Modulus, K

It has been shown that : $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \upsilon(\sigma_y + \sigma_z)\right]$$

For hydrostatic stress, $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$i.e.\quad \varepsilon_x = \frac{1}{E}\left[\sigma - 2\sigma\upsilon\right] = \frac{\sigma}{E}\left[1 - 2\upsilon\right]$$

Similarly, ε_y *and* ε_z *are each* $\frac{\sigma}{E}\left[1 - 2\upsilon\right]$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad = \textit{Volumetric strain}$$

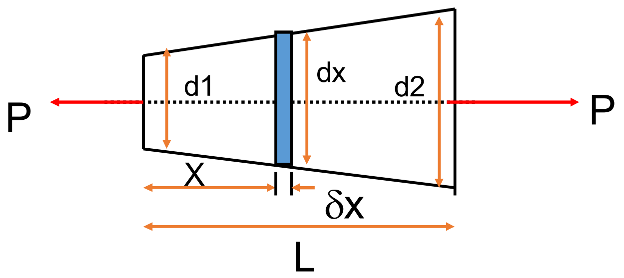
$$\varepsilon_v = \frac{3\sigma}{E}\left[1 - 2\upsilon\right]$$

$$E = \frac{3\sigma}{\varepsilon_v}\left[1 - 2\upsilon\right]$$

$$\textit{Bulk Modulus, } K = \frac{\textit{Volumetric or hydrostatic stress}}{\textit{Volumetric strain}} = \frac{\sigma}{\varepsilon_v}$$

$$i.e.\quad E = 3K\left[1 - 2\upsilon\right] \quad \textit{and} \quad K = \frac{E}{3\left[1 - 2\upsilon\right]}$$

Extension of Bar of Tapering cross Section from diameter d1 to d2:-



Bar of Tapering Section:

dx = d1 + [(d2 - d1) / L] * X

$\delta\Delta = P\delta x / E[\pi /4\{d1 + [(d2 - d1) / L] * X\}^2]$

$$\Delta = 4P\int_0^L dx / [E\,\pi\{d1+kx\}^2]$$

$$= -[4P/\pi E] \times 1/k\ [\{1/(d1+kx)\}]_0^L dx$$

$$=-[4PL/\pi E(d2-d1)]\{1/(d1+d2-d1) - 1/d1\}$$

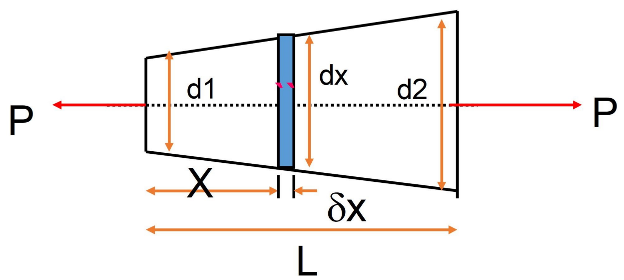
$$\Delta = 4PL/(\pi E\ d1\ d2)$$

Check :-

When d = d1=d2

Δ =PL/ [(π /4)* d^2E] = PL /AE

Q. Find extension of tapering circular bar under axial pull for the following data: d1 = 20mm, d2 = 40mm, L = 600mm, E = 200GPa. P = 40kN

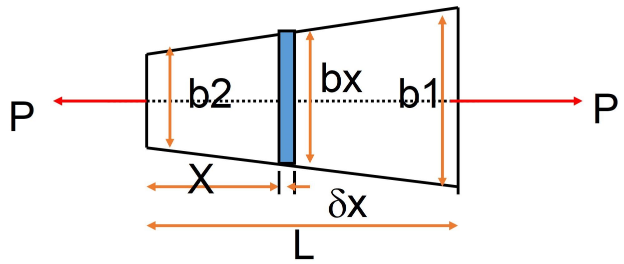


$\Delta L = 4PL/(\pi E\ d1\ d2)$

= 4*40,000*600/(π* 200,000*20*40)

= 0.38mm. Ans.

Extension of Tapering bar of uniform thickness t, width varies from b1 to b2:-



P/Et $\int \delta x / [(b1 + k^*X)]$,

Bar of Tapering Section:

$bx = b1 + [(b2 - b1) / L] * X = b1 + k^*x$,

$\delta\Delta = P\delta x / [Et(b1 + k^*X)]$, $k =$ **(b2 - b1) / L**

$$\Delta L = \int_0^L \Delta L = \int_0^L P\delta x / [Et(b1 - k^*X)],$$

$$= P/Et \int \delta x / [(b1 - k^*X)],$$

$$= - P/E\,t\,k * \log_e [(b1 - k^*X)]_0^L,$$

$$= PL \log_e(b1/b2) / [Et(b1 – b2)]$$