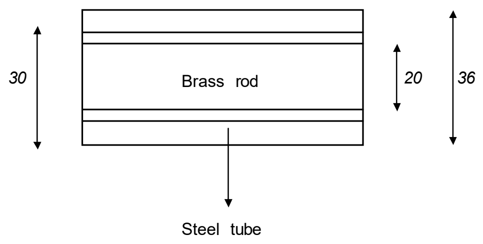
Example

- A steel tube having an external diameter of 36 mm and an internal diameter of 30 mm has a brass rod of 20 mm diameter inside it, the two materials being joined rigidly at their ends when the ambient temperature is 18 ^{0}C. Determine the stresses in the two materials: (a) when the temperature is raised to 68 ^{0}C (b) when a compressive load of 20 kN is applied at the increased temperature.

For brass: Modulus of elasticity = 80 GN/m^2; Coefficient of expansion = 17 x 10 -6 /^{0}C

For steel: Modulus of elasticity = 210 GN/m^2; Coefficient of expansion = 11 x 10 -6 /^{0}C

Solution



Area of brass rod (A_b) = $\dfrac{\pi \, x \, 20^2}{4} = 314.16 \, mm^2$

Area of steel tube (A_s) = $\dfrac{\pi \, x \, (36^2 - 30^2)}{4} = 311.02 \, mm^2$

$$A_s E_s = 311.02 \, x \, 10^{-6} m^2 \, x \, 210 \, x \, 10^9 \, N/m^2 = 0.653142 \, x \, 10^8 \, N$$

$$\frac{1}{A_s E_s} = 1.53106 \, x \, 10^{-8}$$

Solution Contd.

$$A_b E_b = 314.16\,x\,10^{-6} m^2\ x\,80\,x\,10^9\,N/m^2 = 0.251327\,x\,10^8\,N$$

$$\frac{1}{A_b E_b} = 3.9788736\,x\,10^{-8}$$

$$T(\alpha_b - \alpha_s) = 50(17-11)\,x\,10^{-6} = 3\,x\,10^{-4}$$

With increase in temperature, brass will be in compression while steel will be in tension. This is because expands more than steel.

$$i.e.\quad F[\frac{1}{A_s E_s} + \frac{1}{A_b E_b}] = T(\alpha_b - \alpha_s)$$

i.e. F[1.53106 + 3.9788736] x 10 $^{-8}$ = 3 x 10 $^{-4}$

F = 5444.71 N

Solution Concluded

Stress in steel tube = $\frac{5444.71N}{311.02\,mm^2} = 17.51N/mm^2 = 17.51MN/m^2(Tension)$

Stress in brass rod = $\frac{5444.71N}{314.16\,mm^2} = 17.33N/mm^2 = 17.33MN/m^2(Compression)$

(b) Stresses due to compression force, F' of 20 kN

$$\sigma_s = \frac{F'E_s}{E_s A_s + E_b A_b} = \frac{20\,x\,10^3\,N\ x\ 210\,x\,10^9\,N/m^2}{0.653142 + 0.251327\,x\,10^8} = 46.44\,MN/m^2\,(Compression)$$

$$\sigma_b = \frac{F'E_b}{E_s A_s + E_b A_b} = \frac{20\,x\,10^3\,N\ x\ 80\,x\,10^9\,N/m^2}{0.653142 + 0.251327\,x\,10^8} = 17.69\,MN/m^2\,(Compression)$$

Resultant stress in steel tube = - 46.44 + 17.51 = 28.93 MN/m ^{2} (Compression)

Resultant stress in brass rod = -17.69 - 17.33 = 35.02 MN/m ^{2} (Compression)

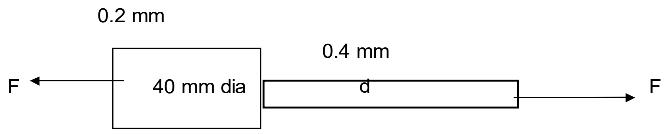
Example

A composite bar, 0.6 m long comprises a steel bar 0.2 m long and 40 mm diameter which is fixed at one end to a copper bar having a length of 0.4 m.

i. Determine the necessary diameter of the copper bar in order that the extension of each material shall be the same when the composite bar is subjected to an axial load.

ii. What will be the stresses in the steel and copper when the bar is subjected to an axial tensile loading of 30 kN? (For steel, E = 210 GN/m^2; for copper, E = 110 GN/m^2)

Solution



Let the diameter of the copper bar be d mm

Specified condition: Extensions in the two bars are equal

$$dl_c = dl_s$$

$$dl = \varepsilon\, L = \frac{\sigma}{E}\, L = \frac{FL}{AE}$$

Thus: $$\frac{F_c L_c}{A_c E_c} = \frac{F_s L_s}{A_s E_s}$$

Solution

Also: Total force, F is transmitted by both copper and steel

i.e. $F_c = Fs = F$

$$i.e. \quad \frac{L_c}{A_c E_c} = \frac{L_s}{A_s E_s}$$

Substitute values given in problem:

$$\frac{0.4\, m}{\pi\, d^2 / 4\, m^2\ 110\, x\, 10^9\, N / m^2} = \frac{0.2\, m}{\pi / 4\, x\, 0.040^2\ x\, 210\, x\, 10^9\ N / m^2}$$

$$d^2 = \frac{2\, x\, 210\, x\, 0.040^2}{110} m^2;\quad d = 0.07816\, m = 78.16\, mm.$$

Thus for a loading of 30 kN

Stress in steel, $\sigma_s = \dfrac{30\, x\, 10^3\, N}{\pi / 4\, x\, 0.040^2\ x\, 10^{-6}} = 23.87\ MN / m^2$

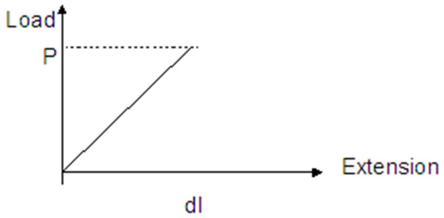
Stress in copper, $\sigma_c = \dfrac{30\, x\, 10^3\, N}{\pi / 4\, x\, 0.07816^2\ x\, 10^{-6}} = 9\ MN / m^2$

Elastic Strain Energy

- If a material is strained by a gradually applied load, then work is done on the material by the applied load.
- The work is stored in the material in the form of strain energy.
- If the strain is within the elastic range of the material, this energy is not retained by the material upon the removal of load.

Elastic Strain Energy Contd.

Figure below shows the load-extension graph of a uniform bar. The extension dl is associated with a gradually applied load, P which is within the elastic range. The shaded area represents the work done in increasing the load from zero to its value



Work done = strain energy of bar = shaded area

Elastic Strain Energy Concluded

W = U = 1/2 P dl (1)

Stress, σ = P/A i.e P = σ A

Strain = Stress/E

i.e dl/L = σ/E , dl = (σL)/E L= original length

Substituting for P and dl in Eqn (1) gives:

W = U = 1/2 σ A . (σ L)/E = σ^2/2E x A L

A L is the volume of the bar.

i.e U = σ^2/2E x Volume

The units of strain energy are same as those of work i.e. Joules. Strain energy per unit volume, σ^2/2E is known as resilience. The greatest amount of energy that can stored in a material without permanent set occurring will be when σ is equal to the elastic limit stress.