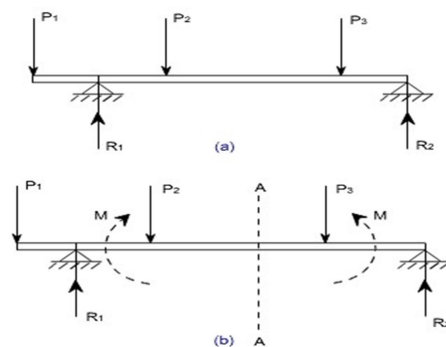
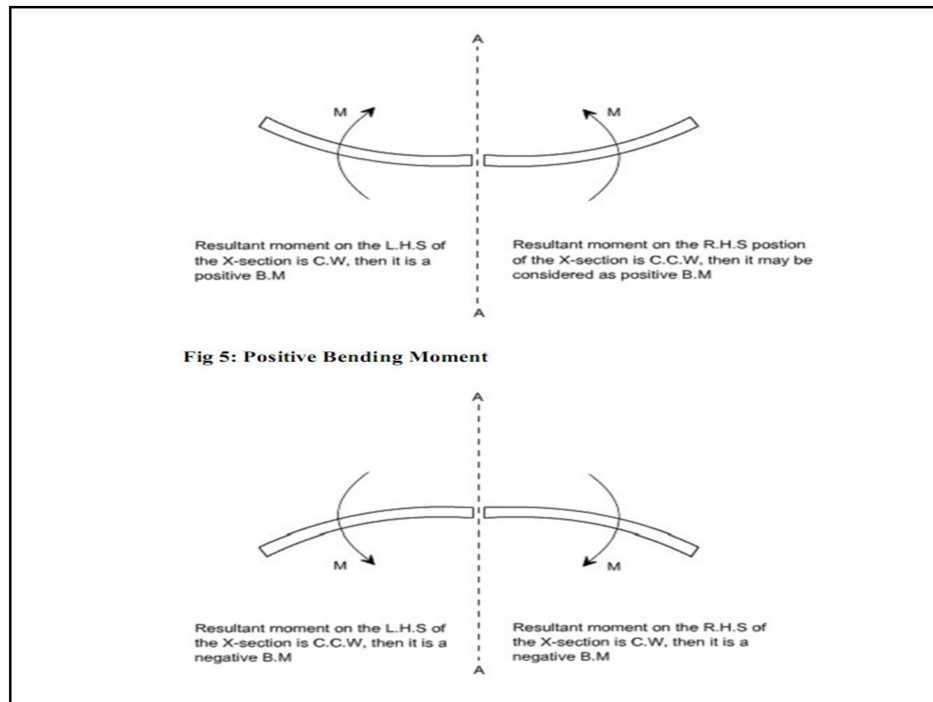


BENDING MOMENT

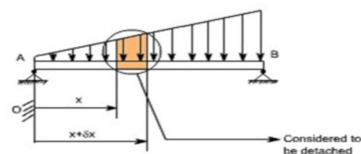


Let us again consider the beam which is simply supported at the two points, carrying loads P_1 , P_2 and P_3 and having the reactions R_1 and R_2 at the supports Fig 4. Now, let us imagine that the beam is cut into two portions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is M in C.W direction, then moment of forces to the right of x-section AA must be ' M ' in C.C.W. Then ' M ' is called as the Bending moment and is abbreviated as B.M. Now one can define the bending moment to be simply as the algebraic sum of the moments about an x-section of all the forces acting on either side of the section

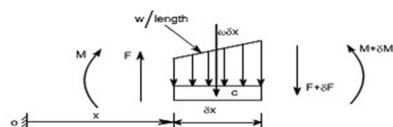


Basic Relationship Between The Rate of Loading, Shear Force and Bending Moment:

The construction of the shear force diagram and bending moment diagrams is greatly simplified if the relationship among load, shear force and bending moment is established. Let us consider a simply supported beam AB carrying a uniformly distributed load w/length . Let us imagine to cut a short slice of length dx cut out from this loaded beam at distance ' x ' from the origin 'O'.



Let us detach this portion of the beam and draw its free body diagram.



The forces acting on the free body diagram of the detached portion of this loaded beam are the following

- The shearing force F and $F + dF$ at the section x and $x + dx$ respectively.
- The bending moment at the sections x and $x + dx$ be M and $M + dM$ respectively.
- Force due to external loading, if ' w ' is the mean rate of loading per unit length then the total loading on this slice of length dx is $w \cdot dx$, which is approximately acting through the centre ' c '. If the loading is assumed to be uniformly distributed then it would pass exactly through the centre ' c '. This small element must be in equilibrium under the action of these forces and couples.

Now let us take the moments at the point ' c '. Such that

$$\begin{aligned}
 M + F \cdot \frac{\delta x}{2} + (F + \delta F) \cdot \frac{\delta x}{2} &= M + \delta M \\
 \Rightarrow F \cdot \frac{\delta x}{2} + (F + \delta F) \cdot \frac{\delta x}{2} &= \delta M \\
 \Rightarrow F \cdot \frac{\delta x}{2} + F \cdot \frac{\delta x}{2} + \delta F \cdot \frac{\delta x}{2} &= \delta M \quad [\text{Neglecting the product of } \delta F \text{ and } \delta x \text{ being small quantities}] \\
 \Rightarrow F \cdot \delta x &= \delta M \\
 \Rightarrow F &= \frac{\delta M}{\delta x}
 \end{aligned}$$

Under the limits $\delta x \rightarrow 0$

$$F = \frac{dM}{dx} \quad \dots\dots\dots (1)$$

Resolving the forces vertically we get

$$w \cdot \delta x + (F + \delta F) = F$$

$$\Rightarrow w = -\frac{\delta F}{\delta x}$$

Under the limits $\delta x \rightarrow 0$

$$\begin{aligned}
 \Rightarrow w &= -\frac{dF}{dx} \text{ or } -\frac{d}{dx} \left(\frac{dM}{dx} \right) \\
 w &= -\frac{dF}{dx} = -\frac{d^2M}{dx^2} \quad \dots\dots\dots (2)
 \end{aligned}$$

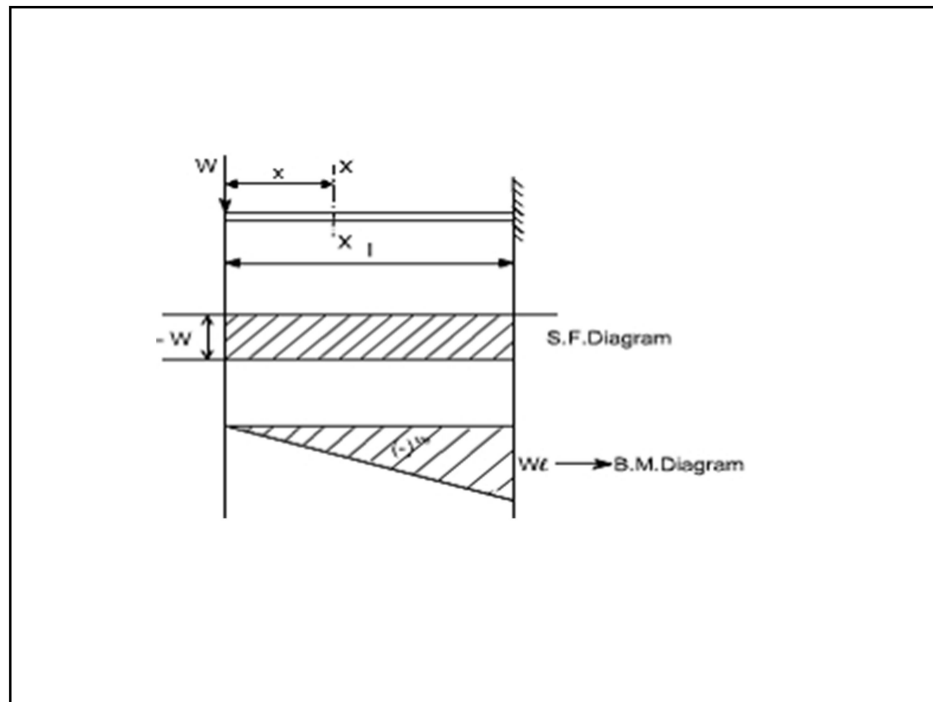
A cantilever of length carries a concentrated load 'W' at its free end. Draw shear force and bending moment.

Solution:

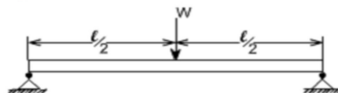
At a section a distance x from free end consider the forces to the left, then $F = -W$ (for all values of x) -ve sign means the shear force to the left of the x -section are in downward direction and therefore negative.

Taking moments about the section gives (obviously to the left of the section) $M = -Wx$ (-ve sign means that the moment on the left hand side of the portion is in the anticlockwise direction and is therefore taken as -ve according to the sign convention) so that the maximum bending moment occurs at the fixed end i.e.

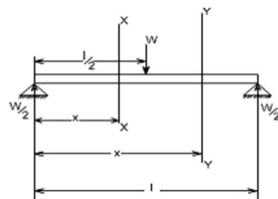
$M = -Wl$ From equilibrium consideration, the fixing moment applied at the fixed end is Wl and the reaction is W . the shear force and bending moment are shown as,



**Simply supported beam subjected to a central load (i.e. load acting at the mid-
w**



By symmetry the reactions at the two supports would be $W/2$ and $W/2$. now consider any section X-X from the left end then, the beam is under the action of following forces.



.So the shear force at any X-section would be $= W/2$ [Which is constant upto $x < l/2$]

If we consider another section Y-Y which is beyond $l/2$ then

$$S.F_{Y-Y} = \frac{W}{2} - W = -\frac{W}{2} \text{ for all values greater } = l/2$$

Hence S.F diagram can be plotted as,