Non-Regular Language

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Outline

Non-Regular Language

Non-regular languages

$$\{a^n b^n : \ n \geq 0\}$$

$$\{ww^R : \ w \in \{a,b\}^*\}$$

Regular languages

$$a^*b$$

$$b^*c+a$$

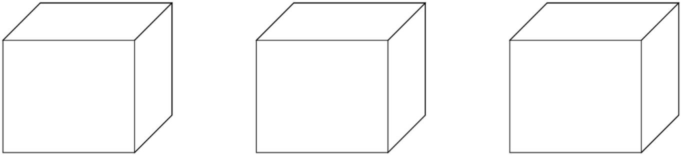
$$b+c(a+b)^*$$

etc...

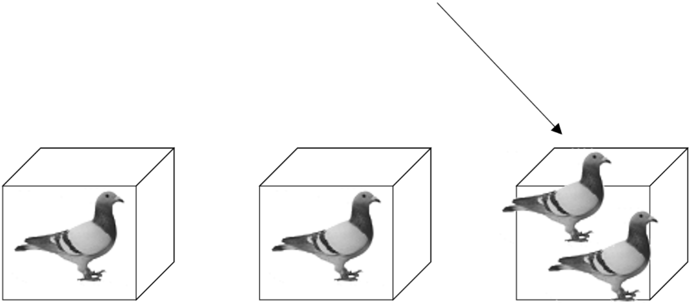
The Pigeonhole Principle

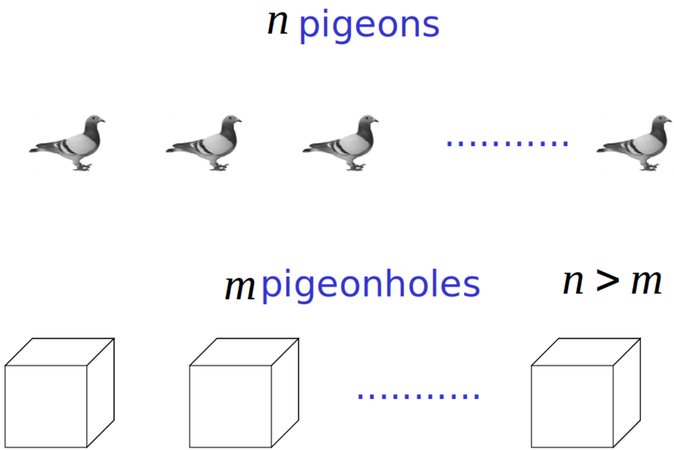
4 pigeons

3 pigeonholes



A pigeonhole must
contain at least two pigeons



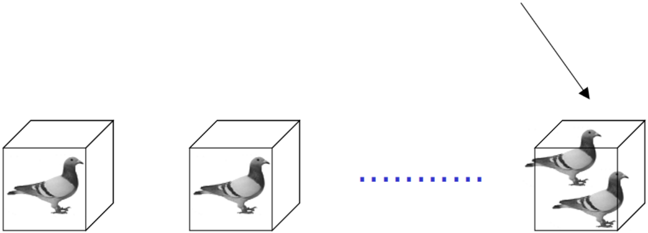

n pigeons
m pigeonholes
$n > m$

n pigeons

m pigeonholes

$$n > m$$

There is a pigeonhole with at least 2 pigeons



Pumping Lemma

Theorem

Let the infinite regular language L, there exist a constant m (i.e. pumping length)
such that for every string $\omega \in L$ of length $|\omega| \geq m$,
a string ω may be split into 3 substring x, y and z;

$$\omega = x\ y\ z$$

such that

1. $y \neq \varepsilon$ or $|y| \geq 1$
2. $|xy| \leq m$
3. The string $xy^iz \in L$; where $i = 0, 1, \ldots \ldots$

then the language is regular.

Pumping lemma is always for negative test.

Example: The language $L = \{a^n b^n : n \geq 0\}$ is not regular.
Proof: Using Pumping Lemma
Assume for contradiction that $L = \{a^n b^n : n \geq 0\}$ is a regular language
Since L is infinite, so we can apply Pumping Lemma
Let m be the integer in Pumping Lemma, and
a string ω such that $\omega \in L$ so
$|\omega| \geq m$
pick

$$\omega = a^m b^m$$

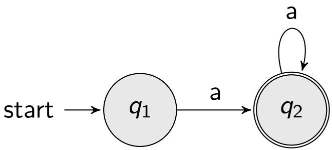
$$a^m b^m = x \ y \ z$$

it must be that : length $|xy| \leq m$ and $|y| \geq 1$

$$a^m b^m = \overbrace{\text{âĂć}}$$

Some Rules for finding a language is regular or not.

1. If a language L is finite then, L is regular language.
   Example 1.: $L_1 = \left\{ a^n b^n \ : \ n \ \leq \ 10^{10^{10^{10}}} \right\}$
   Solution : n is bounded so this is finite language and then the language is regular.
2. If a language L is infinite and we can construct finite automata(either DFA or NFA or ε-NFA) of language then, L is regular language.
   Example 2.: $L_2 = \{a^n \ : \ n \ \geq \ 1\}$
   Solution : $L_2 = \{a, aa, aaa, \dots\}$



Some Rules for finding a language is regular or not.

3. Finite Automata don't have memory to save infinite length string so it don't have capacity to matching and counting the string, so that language is non regular.
   Example 2.: $L_3 = \{a^n b^n \mid n \geq 1\}$
   Solution : n is unbounded so the language is infinite. Infinite counting is not done by finite automata. Finite automata can have finite memory.So that language is non regular.
   Example 3.: $L_4 = \left\{\omega\omega^R : \omega \in \Sigma^*\right\}$
   Solution : This is infinite language. Read entire ω and save it in the memory and compare whenever ω^R and the length of ω is infinite. We know that finite automata doesn't have the capacity to save infinite length string therefore this language is not regular language.
   Example 4.: $L_5 = \{\omega\omega : \omega \in \Sigma^*\}$
   Solution : Using above explanation this language is does not regular language.

Some Rules for finding a language is regular or not.

4. Let $\Sigma = \{a\}$. If the string length is in A.P., so that language is regular language otherwise non-regular language.
 Example 5. $L_1 = \{a^n \mid n \text{ is even number.}\}$
 Solution : So the language

$$\begin{aligned} L_1 &= \left\{a^0, a^2, a^4, \dots\right\} \\ &= \{\textit{length } 0, 2, 4, \dots\} \end{aligned}$$

 String length is in A.P. and common difference represent in loop in finite automta so the language is regular.
 Example 6. $L_2 = \{a^n \mid n \text{ is prime number.}\}$
 Solution : So the language String length is not in A.P. and common difference not represent in loop in finite automta so the language is non-regular.

Some Rules for finding a language is regular or not.

5. Based on dependency.
   Let $\Sigma = \{a, b\}$.
   The reason for non-regularity is not just either infinite counting or infinite comparison or infinite memory.
   Some time even the pattern may not be able to put on a finite automata therefore these language is not regular..
   Example 7. $L_1 = \left\{ a^i b^{j^2} \ : \ i, \ j \ \geq \ 1. \right\}$
   Solution : string of a and b is independent and a^i is in A.P. and b^{j^2} is not in A.P. so the language is non-regular.
   Example 8. $L_2 = \left\{ a^i b^{2^n} \ : \ i, \ n \ \geq \ 1. \right\}$
   Solution : String of a and b is independent and a^i is in A.P. and b^{2^n} is not in A.P. so the language is non-regular.

Some Rules for finding a language is regular or not.

6. We know that finite automata is not capable of doing unbounded counting but it is capable of doing modular counting.
 Example 9. $L_1 = \{\omega\ :\ n_a(\omega)\ =\ n_b(\omega)\ and\ \omega\ \in\ \Sigma\}$
 Solution : The question contain either $=\ or\ \geq\ or\ \leq$ .The language is unbounded counting because of word can be have any length. So the language is non-regular.
 Example 10.
 $L_2 = \{\omega\ :\ n_a(\omega) \mod 3 \geq\ n_b(\omega) \mod 3\ and\ \omega\ \in\ \Sigma\}$
 Solution : The question contain either $=\ or\ \geq\ or\ \leq$ .The language is modular counting. So the language is regular.