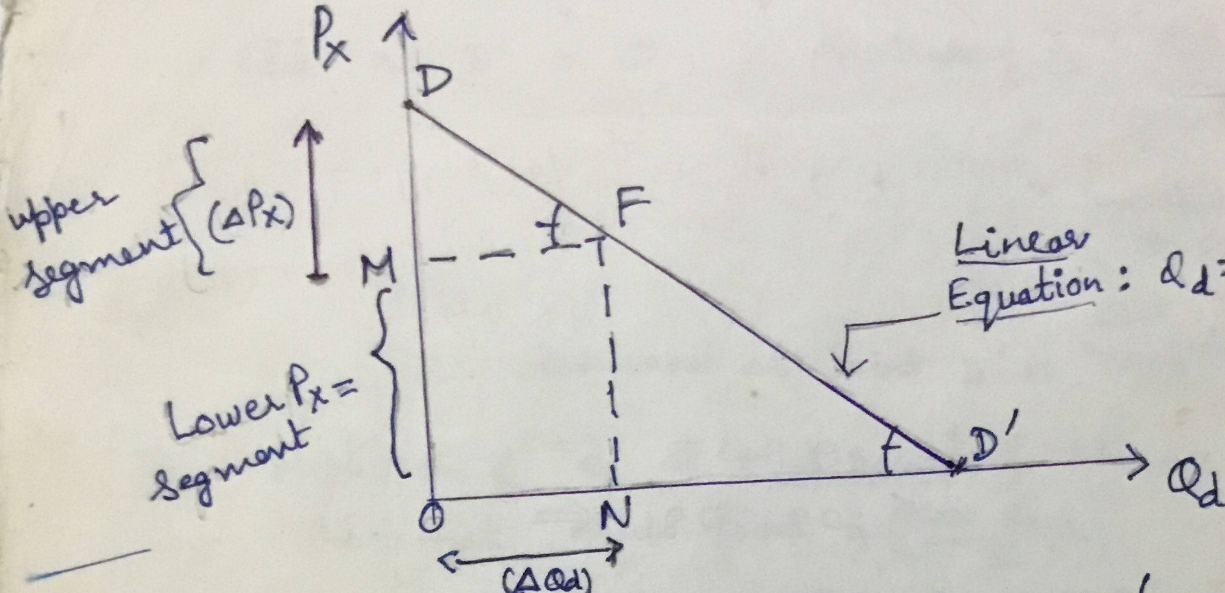
Alfred Marshall Geomatric method.

"Linear" Demand curve Price Elasticity of Demand

①



Linear Equation : $Q_d = a - bP_x$

$a>0$, $b>0$] constant

$-b$ = slope of DD' <0

"Magnitude" of e_D at Point F on DD':

$$e_D \text{ at } F = \frac{\%\Delta Q_d}{\%\Delta P_x}$$

$$= \underbrace{\left(\frac{\Delta Q_d}{\Delta P_x}\right)}_{\text{slope}} \cdot \frac{P_x}{Q_d} < 0$$

∴ Magnitude of e_D at F $= \left(\frac{\cancel{ON}}{DM}\right)\frac{(OM)}{\cancel{(ON)}}$

$$= \frac{OM}{DM}$$

$$= \frac{OM}{(OD - OM)}$$

GEOMETRIC FORMULA

$$= \boxed{\frac{\text{Price at F}}{(\text{Max Price} - \text{Price at F})}} > 0$$

lower segment / upper segment

Different Cases :

① Magnitude of e_D at Point 'D'. Price at D = Max Price

$$\therefore \text{Magnitude of } e_D = \frac{\text{Price at D}}{(\text{Max Price} - \text{Price at D})}$$

$$= \frac{\text{Max Price}}{(\text{Max Price} - \text{Max Price})} = \frac{\text{Max Price}}{0} = \pm\infty$$

$\therefore$ e_D at $D = -\infty$

$\therefore$ Demand at point D is "Infinitely Elastic" (2)

(2) Magnitude of e_D at Point D':

Price at $D' = 0$ $\therefore$ Magnitude of $e_D = \dfrac{\text{Price at } D'}{(\text{Max Price} - \text{Price at } D')}$

$= \dfrac{0}{(\text{Max Price} - 0)} = \dfrac{0}{\text{Max Price}}$

$= 0$

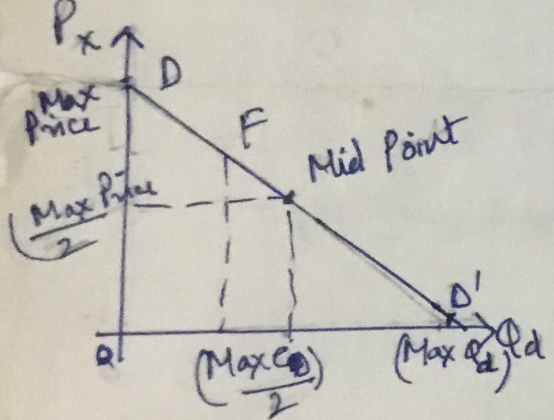
$\therefore$ Demand at Point D' is "Completely Inelastic"

(3) Magnitude of e_D at "Mid point" of DD'

Price at Mid-Point $= \left(\dfrac{\text{Max Price}}{2}\right)$

Magnitude of $e_D = \dfrac{\text{Price at Mid Point}}{(\text{Max Price} - \text{Price at Mid Point})}$

$= \dfrac{\left(\dfrac{\text{Max Price}}{2}\right)}{\left(\text{Max Price} - \left(\dfrac{\text{Max Price}}{2}\right)\right)}$



$= \dfrac{\left(\dfrac{\text{Max Price}}{2}\right)}{\left(\dfrac{\text{Max Price}}{2}\right)} = 1$ (Unitary)

$\therefore$ e_D at Mid-Point $= -1 < 0$

$\therefore$ Demand at "Mid-Point is "Unitary Elastic"

(4) Above Mid Point, magnitude of e_D

Above Mid Point, Price at F $> \left(\dfrac{\text{Max Price}}{2}\right)$

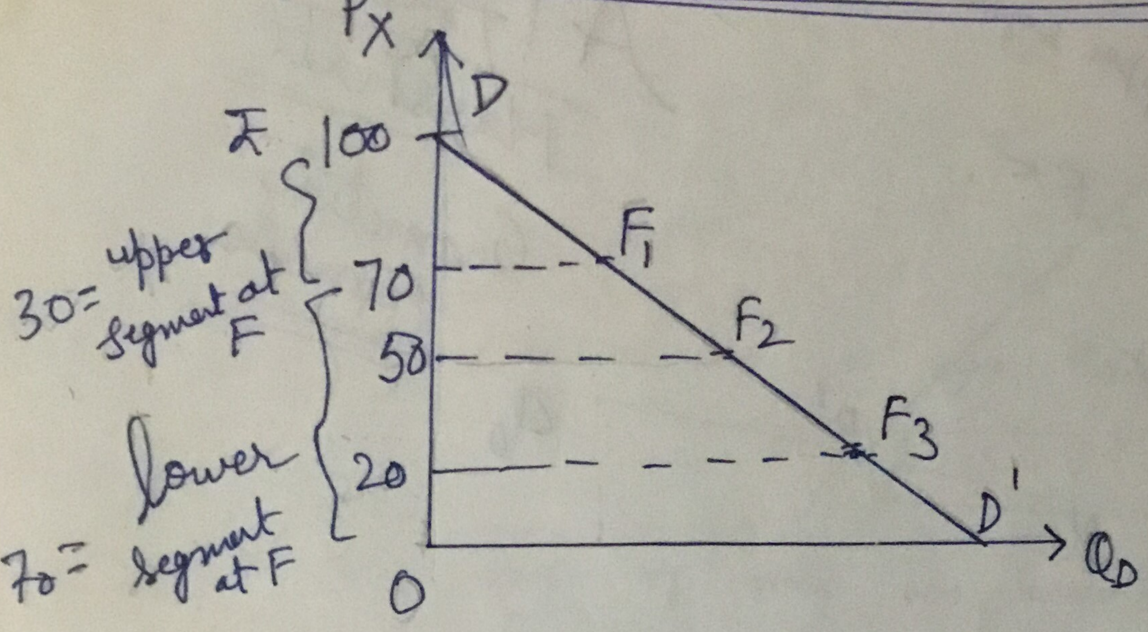
and $(\text{Max Price} - \text{Price at F}) < \left(\dfrac{\text{Max Price}}{2}\right)$

$\therefore$ Magnitude of e_D at F $= \dfrac{\text{Price at F}}{(\text{Max Price} - \text{Price at F})} > \dfrac{\left(\dfrac{\text{Max Price}}{2}\right)}{\left(\dfrac{\text{Max Price}}{2}\right)}$

> 1

Demand at F is elastic $\Leftarrow$ $\therefore$ e_D at F < -1 (eg. $-2, -3, -4 \cdots$)

eg. DD' is str. line demand curve



$$\text{magnitude of } e_D = \frac{7\cancel{0}}{3\cancel{0}} = \frac{7}{3} = 2.33$$

$$\therefore \quad e_D = -\frac{7}{3} < 0$$

Shows magnitude of $e_D = 2.33 > 1$

$\therefore$ DD' is elastic at F_1

F_1 $\quad 2.33 > 1$: elastic D

$e_D = -2.33 < -1$

F_2 $\quad \frac{50}{50} = 1$ (unit elastic)

$\therefore$ $e_D = -1$

F_3 $\quad \frac{2\cancel{0}}{8\cancel{0}} = \frac{1}{4} = 0.25 < 1$ $\therefore$ inelastic D

$e_D = -0.25 > -1$

Using Calculus: e_D at a point $= \frac{\%\Delta Q_d}{\%\Delta P_x}$

$$= \left(\frac{\partial Q_d}{\partial P_x}\right)\cdot\frac{P_x}{Q_d}$$

Now, $Q_d = \frac{100}{P_x}$

$$\therefore \frac{\partial Q_d}{\partial P_x} = -\frac{100}{(P_x)^2} < 0$$

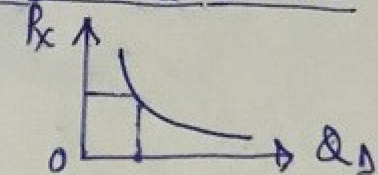
$$\left[\text{Formula: } y = \frac{100}{x} \quad \therefore \frac{\partial y}{\partial x} = -\frac{100}{(x)^2}\right]$$

$$\therefore e_D = -\frac{100}{(P_x)^{\cancel{2}}}\cdot\frac{\cancel{P_x}}{Q_d} = \frac{-100}{(P_x\cdot Q_d)}$$

$$= -\frac{\cancel{100}}{\cancel{100}} = -1 < 0$$

∴ Demand is "unitary elastic" at any point.

Ractangular Hyperbola Demand curve



$$\left(Q_d = \frac{100}{P_x}\right)$$