3.3.4 Lattice Structure for IIR Systems

Now let us see about how IIR systems can be realized by lattice structures. Let us consider all pole IIR system for simplicity. The system can be given as,

$$H(z) = \frac{1}{1+\sum_{k=1}^{N} a_N(k)\, z^{-k}} = \frac{1}{A_N(z)} \quad \text{... (3.3.39)}$$

We know that $H(z) = \frac{Y(z)}{X(z)}$, hence above equation can be written as,

$$\frac{Y(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{N} a_N(k)\, z^{-k}} \quad \text{... (3.3.40)}$$

$$\therefore \quad Y(z) + \sum_{k=1}^{N} a_N(k) z^{-k} Y(z) = X(z) \qquad \cdots (3.3.41)$$

By taking inverse z-transform of above equation we can write the difference equation as,

$$y(n) = -\sum_{k=1}^{N} a_N(k) y(n-k) + x(n) \qquad \cdots (3.3.42)$$

Now let us interchange $x(n)$ and $y(n)$ in above equation. That is interchanging the input and outputs. Then above equation becomes,

$$x(n) = -\sum_{k=1}^{N} a_N(k)\, x(n-k) + y(n)$$

The above equation can also be written as,

$$y(n) = x(n) + \sum_{k=1}^{N} a_N(k)\, x(n-k) \qquad \cdots (3.3.43)$$

The above equation represent the FIR system. Observe that the above equation is exactly similar to FIR system of equation 3.3.32 we have discussed earlier. The realization of above equation in lattice form is discussed in section 3.2.5.

The realizations of FIR systems for first order and second order of equation 3.3.43 are shown in Fig. 3.2.19 and Fig. 3.2.20 earlier. Here we have seen that all pole IIR system can be obtained by just interchanging the roles of $x(n)$ and $y(n)$. That is by interchanging the inputs and outputs. Fig. 3.3.38 below shows the lattice structure for first order IIR system.

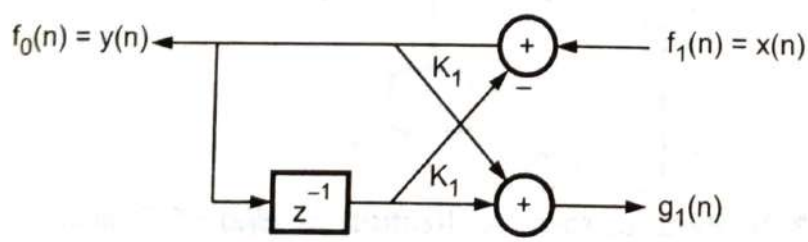

Fig. 3.3.38 A single stage lattice realization of IIR system obtained by interchanging of $x(n)$ and $y(n)$ in Fig. 3.2.19

Compare the single stage lattice realization of IIR system given in above figure with that of FIR system given in Fig. 3.2.19. Observe that the positions of inputs $x(n)$ and outputs $y(n)$ are interchanged. The above structure is redrawn properly in Fig.3.3.39.

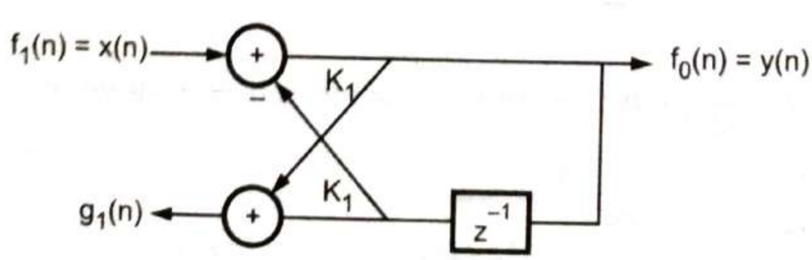

Fig. 3.3.39 Single stage lattice realization of all pole IIR system

Observe that there is a feedback in above realization since it is IIR system. The difference equation for this system can be obtained by putting $N = 1$ in equation 3.3.42 i.e.

$$y(n) = a_1(1)\, y(n-1) + x(n) \qquad \ldots (3.3.44)$$

It is clear from above equation and Fig. 3.3.24 that,

$$K_1 = a_1(1) \qquad \ldots (3.3.45)$$

The two stage lattice filter structure for FIR system is shown in Fig. 3.2.20 earlier. The similar structure can be obtained for all pole IIR system by interchanging $x(n)$ and $y(n)$ $x(n)$ and $y(n)$. Fig. 3.2.20. is redrawn below with $x(n)$ and $y(n)$ interchanged.

Fig. 3.3.40 is redrawn properly as shown below.

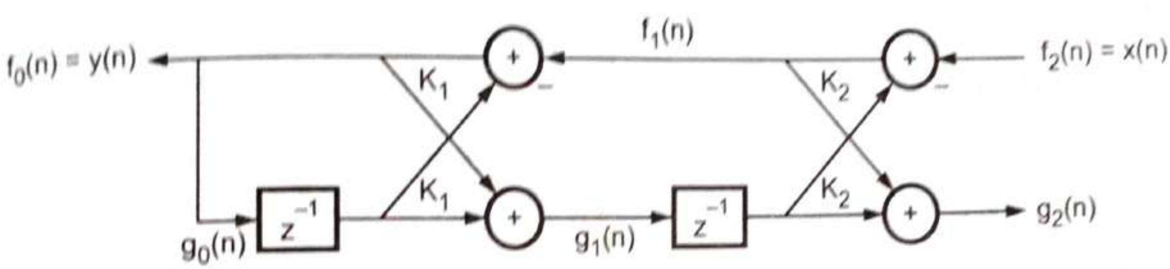

Fig. 3.3.40 Two stage lattice realization of all pole IIR system

Observe that the realization shown in above figure has feedback loop since it is IIR system. Following equations can be written from Fig. 3.3.41.

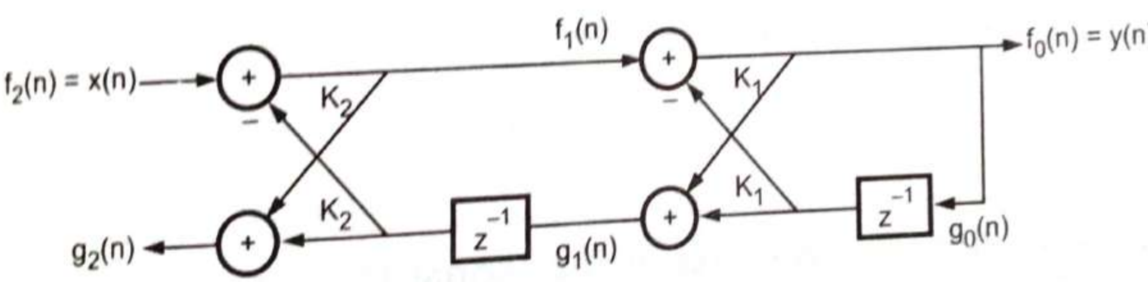

Fig. 3.3.41 Two stage lattice realization of all pole IIR system redrawn properly

$$\left.\begin{aligned} f_2(n) &= x(n) \\ f_1(n) &= f_2(n) - K_2\, g_1(n-1) \\ g_2(n) &= K_2\, f_1(n) + g_1(n-1) \\ f_0(n) &= f_1(n) - K_1\, g_0(n-1) \\ g_1(n) &= K_1\, f_0(n) + g_0(n-1) \\ y(n) &= f_0(n) = g_0(n) \end{aligned}\right\} \qquad \ldots (3.3.46)$$

The equation 3.3.46 can be simplified for $y(n)$ as follows.

$$y(n) = -K_1(1+K_2)\, y(n-1) - K_2\, y(n-2) + x(n) \qquad \ldots (3.3.47)$$

With $N = 2$ in equation 3.3.42 we can obtain difference equation for second order IIR system as,

$$y(n) = -\sum_{k=1}^{2} a_2(k)\, y(n-k) + x(n)$$

$$= -a_2(1)\, y(n-1) - a_2(2)\, y(n-2) + x(n) \quad \dots (3.3.48)$$

Equation 3.3.47 and above equations represent the same system. Hence for these equations to be similar we have,

$$a_2(1) = K_1(1+K_2) \text{ and } a_2(2) = K_2 \quad \dots (3.3.49)$$

The above equation can also be written as,

$$K_1 = \frac{a_2(1)}{1+a_2(2)} \text{ and } K_2 = a_2(2) \quad \dots (3.3.50)$$

For higher values of 'N', the realization for IIR systems can be obtained on the same lines.