

Give General Relations

Date _____

$$dU = Tds - PdV$$

$$dH = Tds + VdP$$

$$dF = -PdV - SdT$$

$$dG = VdP - SdT$$

$$Tds = C_v dT + T \left(\frac{\partial P}{\partial T} \right)_V dV$$

$$Tds = C_p dT - T \left(\frac{\partial V}{\partial T} \right)_P dP$$

$$\left(\frac{\partial T}{\partial V} \right)_S = - \left(\frac{\partial P}{\partial S} \right)_V$$

$$\left(\frac{\partial T}{\partial P} \right)_S = \left(\frac{\partial V}{\partial S} \right)_P$$

$$\left(\frac{\partial P}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T$$

$$\left(\frac{\partial V}{\partial T} \right)_P = - \left(\frac{\partial S}{\partial P} \right)_T$$

MAXWELL
RELATIONS

Properties of Pure Substances: ^{Date} Ideal Gases.

$F(p, v, T) = 0$ Equation of state.

$$pV = m \bar{R} T$$

$m = \text{mass of Gas}$

$\bar{R} = \text{Characteristic Gas const.}$

$p \rightarrow 0$

$T \rightarrow \text{high.}$

\rightarrow molecules are point masses. ($v \rightarrow 0$)

\rightarrow No cohesive force

$$n = \frac{m}{M} \quad (\text{moles}).$$

$$pV = n(M\bar{R})T$$

Avogadro (same volume same mole)

$$m\bar{R} = \bar{R} \quad (\text{universal Gas const})$$

$$pV = n\bar{R}T$$

$$\bar{R} = 8.314 \text{ kJ/Kg mol K.}$$

$p\bar{v} = \bar{R}T$	(specific volume)
$p\bar{V} = \bar{R}T$	(molar volume)

$$\lim_{\substack{p \rightarrow 0 \\ \bar{v} \rightarrow \infty}} (p\bar{v}) = \bar{R}T \quad (\text{for real Gases})$$

h, u are functions of Temperature only.

$$dh = T ds + u dp$$

From second Tds equation.

$$T ds = C_p dT - T \left(\frac{\partial u}{\partial T} \right)_P dP$$

$$dh = C_p dT + \left(u - T \left(\frac{\partial u}{\partial T} \right)_P \right) dP$$

$$h = h(T, P)$$

$$dh = \left(\frac{\partial h}{\partial T} \right)_P dT + \left(\frac{\partial h}{\partial P} \right)_T dP$$

So comparing above

$$\left(\frac{\partial h}{\partial T} \right)_P = C_p$$

$$\left(\frac{\partial h}{\partial P} \right)_T = u - T \left(\frac{\partial u}{\partial T} \right)_P$$

$$Pv = RT$$

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P}$$

So ~~$\frac{\partial v}{\partial P}$~~ $v - T\left(\frac{\partial v}{\partial T}\right)_P = 0$

hence $\left(\frac{\partial h}{\partial P}\right)_T = 0$

$dh = C_p dT$ h is function of T only.

$$\frac{dh}{dT} = C_p$$

$C_p = \left(\frac{\partial h}{\partial T}\right)_P$ but h is not a fn of P

Similarly u is also fn of T only.

$$du = T ds - P dv$$

$$T ds = C_v dT + T \left(\frac{\partial P}{\partial T}\right)_v dv$$

$$du = C_v dT + \left(T \left(\frac{\partial P}{\partial T}\right)_v - P\right) dv$$

$$\downarrow$$
$$\left(\frac{\partial u}{\partial T}\right)_v$$

$$\downarrow$$
$$\left(\frac{\partial u}{\partial v}\right)_T$$

again second quantity becomes zero for ideal gas.

$$u = u(T) \text{ only.}$$

$$du = C_v dT$$

$$\frac{\partial u}{\partial T} = C_v.$$

For ideal gases.

$$du = C_v dT$$

$$dh = C_p dT$$

$$u = \int C_v dT + C_1$$

$$h = \int C_p dT + C_2$$

C_p, C_v are constant for ideal gas.

$$u = C_v T + C_1$$

$$h = C_p T + C_2$$

$$\Delta u = C_v \Delta T$$

$$u_2 - u_1 = C_v (T_2 - T_1)$$

$$h_2 - h_1 = C_p (T_2 - T_1)$$

at $T=0$ $u=0, h=0$

$$\boxed{\begin{array}{l} u = C_v T \\ h = C_p T \end{array}}$$

$$pV = RT$$

$$C_p - C_v = R$$

$$\boxed{\begin{array}{l} h = u + pV \\ h = u + RT \\ C_p T = C_v T + RT \\ C_p - C_v = R \end{array}}$$

Calculate $\left(\frac{\partial U}{\partial T}\right)_P$ $\left(\frac{\partial P}{\partial T}\right)_U$ $\left(\frac{\partial U}{\partial T}\right)_T$ for ideal gas (5)
 Date _____

$$\beta \text{ (Volume expansivity)} = \frac{1}{T} \text{ (for ideal gas)}$$

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_P$$

$$pU = RT$$

$$\left(\frac{\partial U}{\partial T}\right)_P = \frac{R}{p}$$

hence $\beta = 1/T$

Isentropic process:-

$$W = \int p dU$$

$$p = p(U) \text{ (isentropic)}$$

$S = \text{constant}$

$$TdS = dU + p dU$$

$$\rightarrow TdS = C_v dT + p dU \text{ (ideal gas)}$$

$$TdS = dh - v dp$$

$$\rightarrow TdS = C_p dT - v dp \text{ (ideal gas)}$$

$$p dU = -C_v dT$$

$$v dp = C_p dT$$

hence $\frac{dP}{P} + \frac{C_p}{C_v} \frac{dU}{U} = 0$

$$\ln P + \ln U^{\gamma} = \ln C$$

$$PV^{\gamma} = \text{const.}$$

$$\gamma = \frac{C_p}{C_v}$$

(ideal gas)

$$W = \int_1^2 p dV$$

$$pV^n = \text{const}$$

$n = \gamma$ isentropic

$$W = C \int_1^2 \frac{dV}{V^n}$$

$n = 1$ isothermal

$n = 0$ isobaric

$$= \frac{C}{1-n} [V^{1-n}]_1^2$$

$$= \frac{p_2 V_2 - p_1 V_1}{1-n}$$

Entropy Change of an ideal Gas

$$TdS = C_V dT + p dV$$

$$TdS = C_P dT - V dp$$

$$dS = \frac{C_V dT}{T} + \frac{R dV}{V}$$

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

$$S_2 - S_1 = C_P \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$