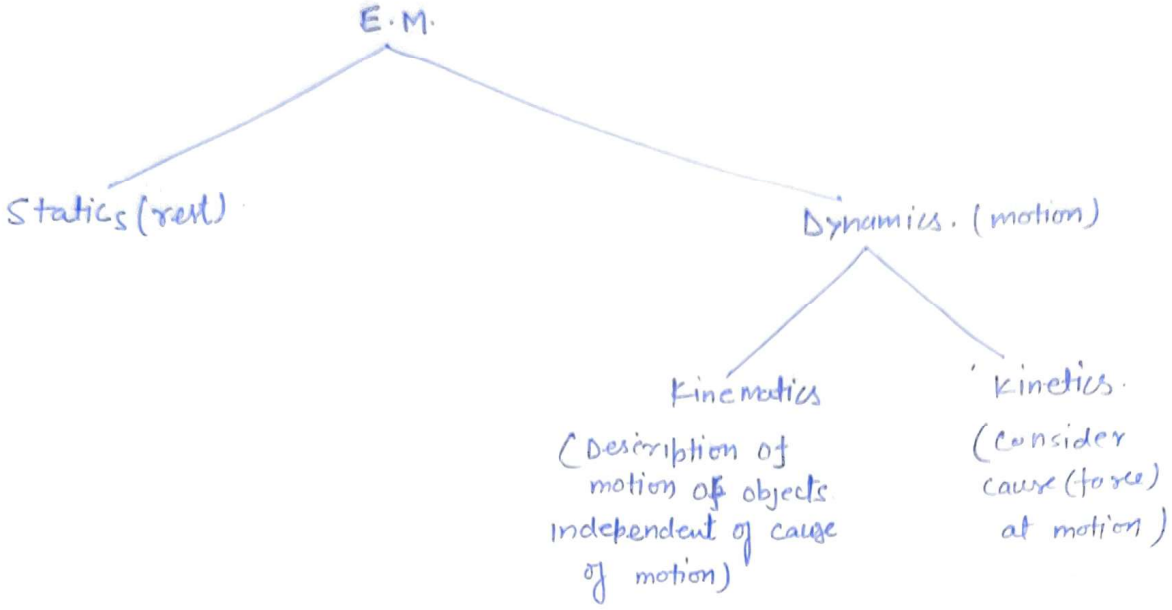
Engineering Mechanics.

Engineering mechanics is a basic subject which describes and predicts the effect of forces on rigid bodies.



Matter – Matter is anything that occupies space, possess mass and offer resistance to any external force. ex – iron stone wood etc.

Particle – A particle is an object that has infinitely small volume (occupies negligible space) but has a mass which can be considered to be a concentrated at a point.

Body – A body has a definite shape and consists of number of particles.

Rigid body – A body in which the distance between two particles remains constant. (shape & size do not change) under the action of external force.

Deformable body – A body that changes its shape / volume under the action of external force (

Newton's law of motion! - (1) First Law -

Every body continues in its state of rest or of uniform motion in a straight line if there is no unbalanced force acting upon it. [concept of inertia]

Second law - The rate of change of linear momentum is directly proportional to the impressed force.

Third law - To every action, there is equal & opposite reaction.

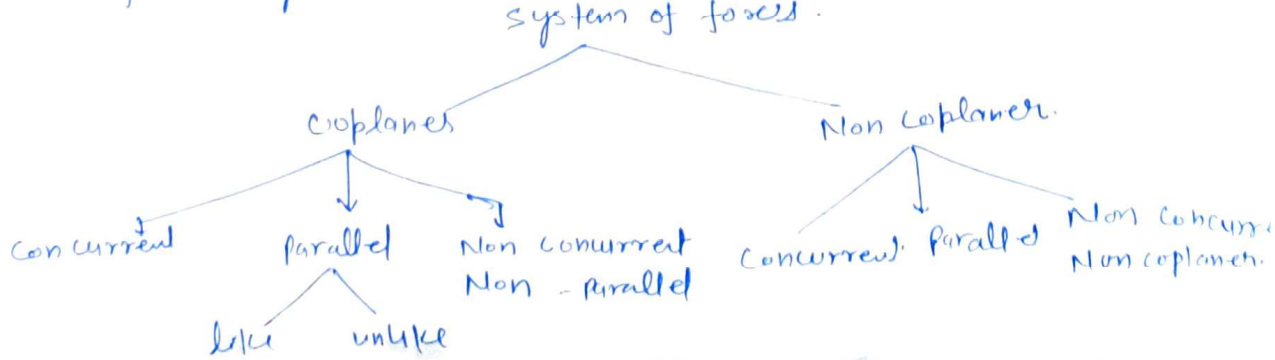
Quantities →
- scalar (completely defined by its magnitude alone ex - mass, length, volume, time, temperature etc)
- vector. [defined by magnitude & direction ex. displacement, velocity, acceleration, momentum, force etc.].

Force - Force is an external agent which tends to change the position of rest or of uniform motion of a body.

→ define by — Magnitude, direction & line of action.

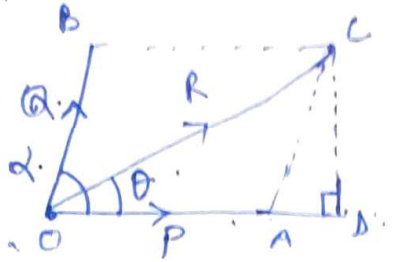
effect - linear displacement, turning / rotating moment.

System of forces! - When two or more (several) forces of different magnitude and direction act upon a body. They constitute a force system.



Law of parallelogram of forces :-

If two forces, acting at a point be represented in magnitude and dirn at a point in a plane by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude & dirn by the diagonal of parallelogram passing through that point.



$\angle DAC = \angle AOB \Rightarrow$ (Corresponding Angle)

$AC = Q$.

$AD = Q\cos\alpha$, $CD = Q\sin\alpha$

In $\triangle OCD$.

$OC^2 = OD^2 + DC^2$, $OC = R$, $OA = P$, $AD = Q\cos\alpha$

$R^2 = (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$

$= P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$ (*)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

Dirn of Resultant force :- (θ)

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}, \quad \theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

(good)

I case. P & Q act at right angles – $\alpha = 90°$.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90}$$

$\cos 90 = 0$ (6)

$$R = \sqrt{P^2 + Q^2}$$

$$\& \quad \theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right) = \tan^{-1}\left(\frac{Q \times 1}{P}\right) = \tan^{-1}\left(\frac{Q}{P}\right).$$

II nd – P & Q are equal & angle α.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}, \quad P = Q$$

$$R = \sqrt{2P^2 + 2P^2\cos\alpha}.$$

$$R = \sqrt{2P^2(1 + \cos\alpha)}$$

$$R = \sqrt{2P^2 \times 2\cos^2\frac{\alpha}{2}} = 2P\cos\frac{\alpha}{2}$$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1$

$\cos\alpha + 1 = 2\cos^2\frac{\alpha}{2}$ — (i)

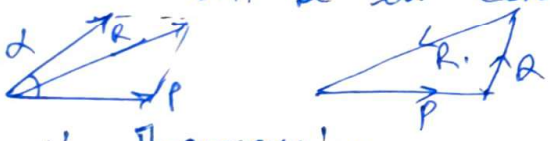
$\sin\alpha = 2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$ — (ii)

$$\& \quad \theta, \tan^{-1}\frac{Q\sin\alpha}{P + Q\cos\alpha} = \tan^{-1}\left(\frac{P\sin\alpha}{P + P\cos\alpha}\right) = \tan^{-1}\left(\frac{\sin\alpha}{2\cos^2\frac{\alpha}{2}}\right) = \tan^{-1}\left(\frac{2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}\right)$$

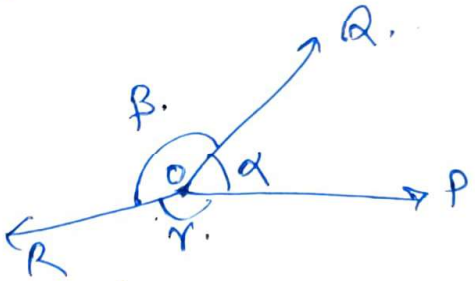
$$= \tan^{-1}\left(\tan\frac{\alpha}{2}\right) = \left(\frac{\alpha}{2}\right)$$

* X

Law of triangles of forces – If three forces acting at a point be represented in magnitude & dirn by three sides of a triangle, taken in order. They will be in equilibrium.



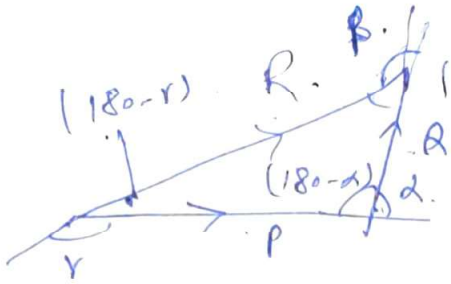
Lami's Theorem' ⇒



If three forces acting at a point are in equilibrium each force will be proportional to the sine of the angle blw the other two forces.

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\beta}}$$

Proof



Using triangle law of forces.

Now Applying Sine Rule.

$$\frac{P}{\sin(180-\beta)} = \frac{Q}{\sin(180-\gamma)} = \frac{R}{\sin(180-\alpha)}$$

$$\boxed{\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}}$$

[dir^{n} of forces should be towards or the pointer away from the point]

2. – Two forces of magnitude 10N & 8N are acting at a point. If the angle b/w the forces is 60°. determine the magnitude of the resultant force.

$$R = \sqrt{100 + 64 + 2\times10\times8\times\cos 60} \quad \text{1/2}$$

$$= \sqrt{164 + 80}$$

$$= \sqrt{244} = 15.62N.$$

(2) – P, P, $\alpha = 60$, $R = 20\sqrt{3}$ N