Lecture - 20

IIR Filter-Problems

Solved Problems :

Prob. 1 : Obtain direct form I and II realization of a system described by,

$$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)+\frac{1}{2}x(n-1).$$

Soln. :

Calculation of H (Z) :

Given equation is,

$$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2) = x(n)+\frac{1}{2}x(n-1) \quad ...(1)$$

Taking Z-transform of both sides we get,

$$Y(Z)-\frac{3}{4}Z^{-1}Y(Z)+\frac{1}{8}Z^{-2}Y(Z) = X(Z)+\frac{1}{2}Z^{-1}X(Z)$$

$$\therefore \quad Y(Z)\left[1-\frac{3}{4}Z^{-1}+\frac{1}{8}Z^{-2}\right] = X(Z)\left[1+\frac{1}{2}Z^{-1}\right] \quad ...(2)$$

Now the transfer function H (Z) is,

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

Thus from Equation (2) we get,

$$H(Z) = \frac{1+\frac{1}{2}Z^{-1}}{1-\frac{3}{4}Z^{-1}+\frac{1}{8}Z^{-2}} \quad ...(3)$$

We will obtain direct form-I and II structures using cascade connection. This cascade connection is shown in Fig. I-8(a).

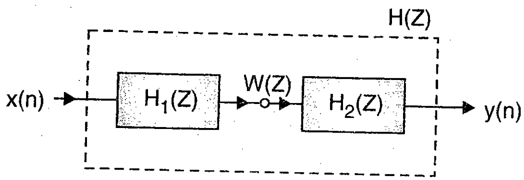


Fig. I-8(a) : $H(Z) = H_1(Z)\cdot H_2(Z)$

Direct form-I structure :

We have, $\quad H(Z) = H_1(Z)\cdot H_2(Z)$

Let $\quad H_1(Z) = 1+\frac{1}{2}Z^{-1}$

But $\quad H_1(Z) = \frac{\text{Output}}{\text{Input}} = \frac{W(Z)}{X(Z)}$

$$\therefore \quad \frac{W(Z)}{X(Z)} = 1+\frac{1}{2}Z^{-1}$$

$$\therefore \quad W(Z) = X(Z)\left[1+\frac{1}{2}Z^{-1}\right]$$

$$\therefore \quad W(Z) = X(Z) + \frac{1}{2} Z^{-1} X(Z) \qquad ...(4)$$

Taking inverse Z-transform (IZT) of Equation (4) we get,

$$w(n) = x(n) + \frac{1}{2} x(n-1) \qquad ...(5)$$

The direct form-I realization of Equation (5) is as shown in Fig. I-8(b).

Now let, $H_2(Z) = \dfrac{1}{1 - \frac{3}{4} Z^{-1} + \frac{1}{8} Z^{-2}}$

But $H_2(Z) = \dfrac{\text{Output}}{\text{Input}} = \dfrac{Y(Z)}{W(Z)}$

$$\therefore \quad \frac{Y(Z)}{W(Z)} = \frac{1}{1 - \frac{3}{4} Z^{-1} + \frac{1}{8} Z^{-2}}$$

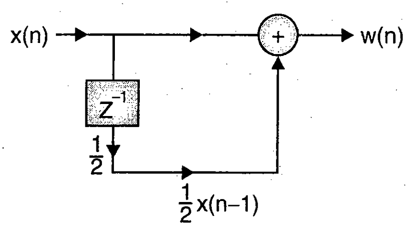


Fig. I-8(b) : Direct form-I realization of $H_1(Z)$

$$\therefore \quad Y(Z)\left[1 - \frac{3}{4} Z^{-1} + \frac{1}{8} Z^{-2}\right] = W(Z)$$

$$Y(Z) - \frac{3}{4} Z^{-1} Y(Z) + \frac{1}{8} Z^{-2} Y(Z) = W(Z)$$

$$\therefore \quad Y(Z) = W(Z) + \frac{3}{4} Z^{-1} Y(Z) - \frac{1}{8} Z^{-2} Y(Z) \qquad ...(6)$$

Taking IZT of Equation (6),

$$y(n) = w(n) + \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) \qquad ...(7)$$

The direct form-I realization of Equation (7) is shown in Fig. I-8(c).

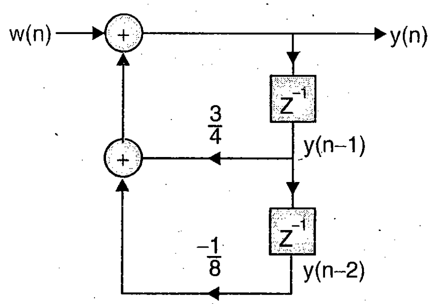


Fig. I-8(c) : Direct form-I realization of $H_2(Z)$

Now direct form-I realization of total transfer function H (Z) is obtained by cascading Fig. I-8(b) and Fig. I-8(c). This is shown in Fig. I-8(d).

Direct form-II realization :

The direct form-II realization is obtained by interchanging the positions of $H_1(Z)$ and $H_2(Z)$. That means by interchanging block-1 and block-2 of Fig. I-8(d). It is shown in Fig. I-8(e).

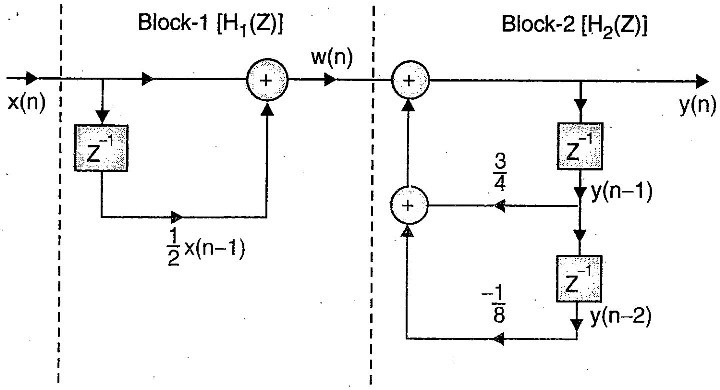


Fig. I-8(d) : Direct form-I realization of H (Z)

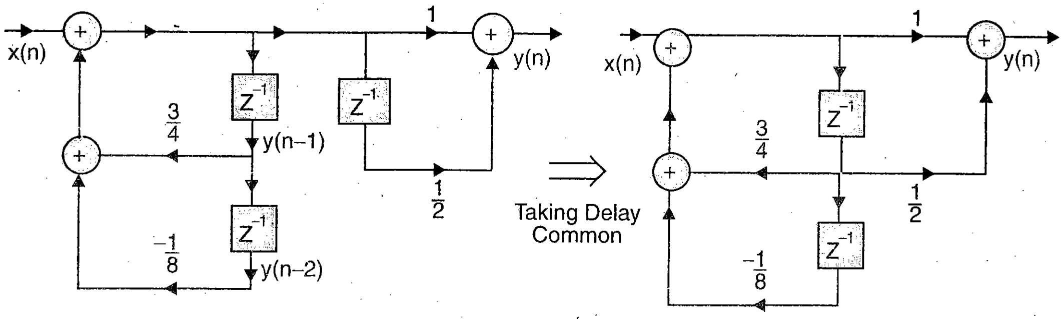


Fig. I-8(e) : Direct form-II realization

Prob. 2 : Consider a causal LTI system with system function,

$$H(Z) = \frac{1+\frac{1}{5}Z^{-1}}{\left(1-0.5\,Z^{-1}+\frac{1}{3}Z^{-2}\right)\left(1+0.25\,Z^{-1}\right)}$$

Draw the signal flow graph for implementation of the system using direct form-II.

Soln. : The given transfer function is,

$$H(Z) = \frac{1+\frac{1}{5}Z^{-1}}{\left(1-0.5\,Z^{-1}+\frac{1}{3}Z^{-2}\right)(1+0.25\,Z^{-1})} \quad ..(1)$$

It can be simplified as follows :

$$H(Z) = \frac{1+\frac{1}{5}Z^{-1}}{1-0.5\,Z^{-1}+\frac{1}{3}Z^{-2}+0.25\,Z^{-1}-0.125\,Z^{-2}+0.0833\,Z^{-3}}$$

$$\therefore \quad H(Z) = \frac{1+\frac{1}{5}Z^{-1}}{1-0.25\,Z^{-1}+0.208\,Z^{-2}+0.0833\,Z^{-3}}$$

For the direct form-II structure, poles are realized first and then the zeros are realized. We can write,

$$H(Z) = H_1(Z) \cdot H_2(Z)$$

Here $H_1(Z)$ = All pole system = $\dfrac{1}{1-0.25\,Z^{-1}+0.208\,Z^{-2}+0.0833\,Z^{-3}}$

and $H_2(Z)$ = All zero system = $1+0.2\,Z^{-1}$

consider $H_1(Z)$ terms.

We have $H_1(Z) = \dfrac{\text{Output}}{\text{Input}} = \dfrac{W(Z)}{X(Z)}$

$$\therefore \quad \frac{W(Z)}{X(Z)} = \frac{1}{1-0.25\,Z^{-1}+0.208\,Z^{-2}+0.0833\,Z^{-3}}$$

$$\therefore \; W(Z)\,[\,1-0.25\,Z^{-1}+0.208\,Z^{-2}+0.0833\,Z^{-3}\,] = X(Z)$$

$$\therefore \; W(Z)-0.25\,Z^{-1}W(Z)+0.208\,Z^{-2}W(Z)+0.0833\,Z^{-3}W(Z) = X(Z)$$

$$\therefore \; W(Z) = X(Z)+0.25\,Z^{-1}W(Z)-0.208\,Z^{-2}W(Z)-0.0833\,Z^{-3}W(Z) \quad ...(2)$$

Taking IZT of Equation (2) we get,

$$w(n) = x(n)+0.25\,w(n-1)-0.208\,w(n-2)-0.0833\,w(n-3) \quad ...(3)$$

The direct form realization of Equation (3) is shown in Fig. I-9(a).

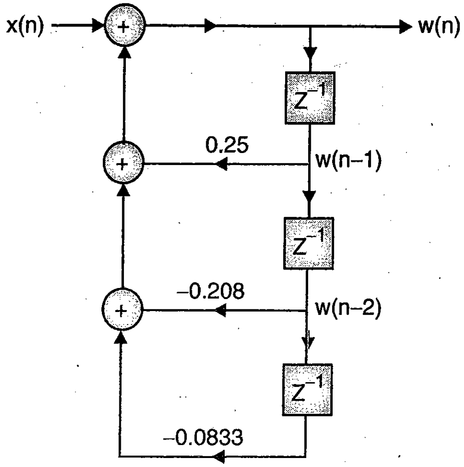


Fig. I-9(a) : Direct form realization of $H_1(Z)$

Now consider $H_2(Z)$ that means all zero system.

$$H_2(Z) = 1 + 0.2\,Z^{-1}$$

But $$H_2(Z) = \frac{\text{Output}}{\text{Input}} = \frac{Y(Z)}{W(Z)}$$

$$\therefore \quad \frac{Y(Z)}{W(Z)} = 1 + 0.2\,Z^{-1}$$

$$\therefore \quad Y(Z) = W(Z) + 0.2\,Z^{-1}\,W(Z) \qquad \text{...(4)}$$

Taking IZT of Equation (4) we get,

$$y(n) = w(n) + 0.2\,w(n-1) \qquad \text{..(5)}$$

The direct form realization of Equation (5) is shown in Fig. I-9(b).

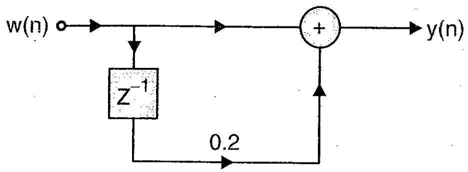


Fig. I-9(b) : Direct form realization of $H_2(Z)$

Now direct form-II realization is obtained by connecting Fig. I-9(a) and Fig. I-9(b) in series (cascade connection). It is shown in Fig. I-9(c).

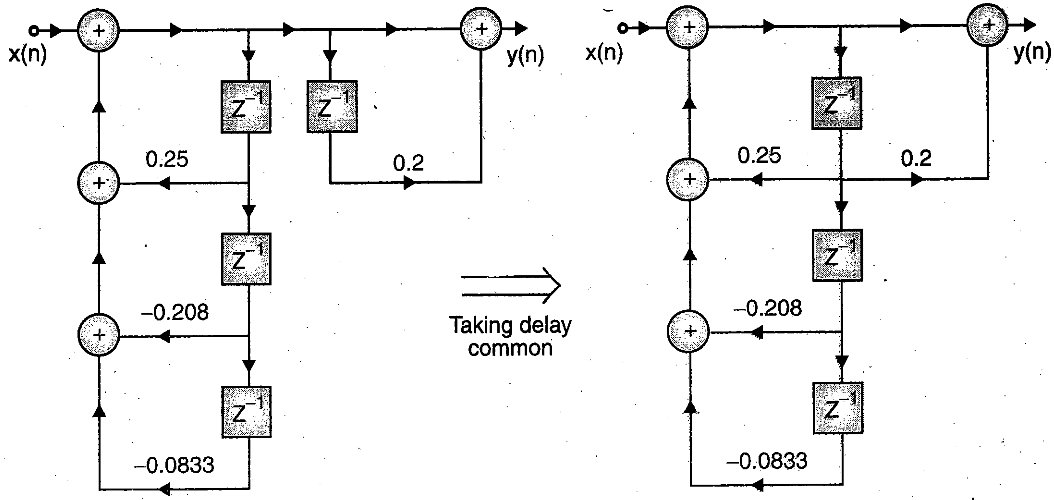


Fig. I-9(c) : Direct form-II realization

Note : We can also obtain direct form-II realization by first drawing direct form-I structure. To draw direct form-I structure take $H_1(Z)$ = all zero system. In this example take $H_1(Z) = 1 + 0.2\,Z^{-1}$ and take $H_2(Z)$ = all pole system, so here $H_2(Z) = \dfrac{1}{1 - 0.25\,Z^{-1} + 0.208\,Z^{-2} + 0.0833\,Z^{-3}}$. Then obtain direct form-I structure. By interchanging the positions of $H_1(Z)$ and $H_2(Z)$; obtain direct form-II structure. But this is tedious method. So, whenever it is asked to obtain direct form-II structure the take $H_1(Z)$ = all pole system and $H_2(Z)$ = all zero system. Then cascade $H_1(Z)$ and $H_2(Z)$ to obtain direct form-II structure.

Now the signal flow graph is as shown in Fig. I-9(d).

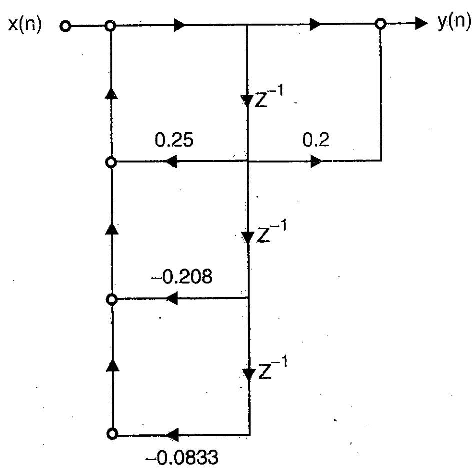


Fig. I-9(d) : Signal flow graph

Shortcut method to draw direct From-II structure :

In problem (1) and (2) we have obtained direct form-II structure by considering the cascade connection. That means by using $H(Z) = H_1(Z) \cdot H_2(Z)$. Now we will study the shortcut method to obtain direct form-II structure. By using this shortcut method we will obtain direct form-II structure for problem (2).

In problem (2) the given H (Z) is,

$$H(Z) = \frac{1 + \frac{1}{5} Z^{-1}}{1 - 0.5\, Z^{-1} + 0.208\, Z^{-2} + 0.083\, Z^{-3}}$$

The different steps are as follows :

Step I :

Write down three terms namely input x (n), middle term w (n) and output y (n) as shown in Fig. I-10(a).

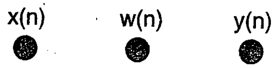


Fig. I-10(a) : Step-I

Step II :

Check the maximum power of Z inverse present in the numerator or denominator. It represents the maximum delay. For the given H (Z) it is '– 3'. This indicates that total '3' delay boxes are required. Draw three Z^{-1} boxes below w (n) as shown in Fig. I-10(b).

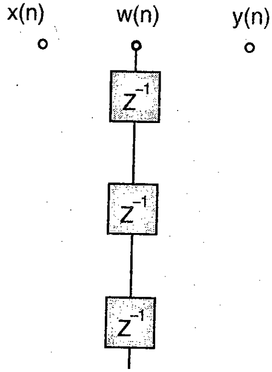


Fig. I-10(b) : Step-II

Step III : Now consider the numerator term. Check the constant multiplier value. It is 0.2 and it is multiplied by Z^{-1}. So write this value at the R.H.S. and below first Z^{-1} box as shown in Fig. I-10(c).

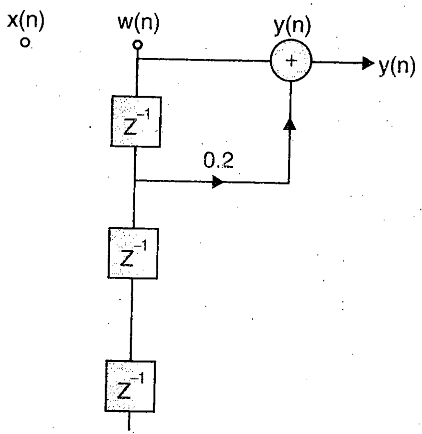


Fig. I-10(c) : Step-III

Then using the adder complete the connection as shown in Fig. I-10(c).

Step IV : Now consider the denominator term. Write down the constant multiplier values at the L.H.S. part of diagram. While writing these values, change the sign. The original constant multipliers are '-0.5' for Z^{-1}, 0.208 for Z^{-2} and 0.083 for Z^{-3}. So change the signs of these values and write down at the L.H.S. near the corresponding delay box. Then by using adders complete the connection as shown in Fig. I-10(d).

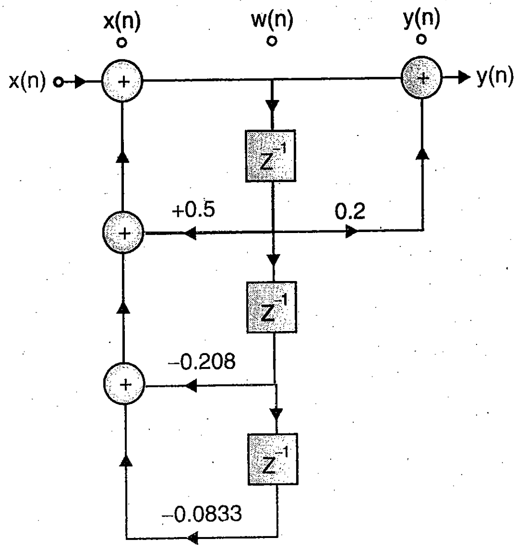


Fig. I-10(d) : Step-IV

This diagram is same as Fig. I-10(c). So by using this simple method; we can directly draw the direct form-II structure.

Now applying this method we will obtain direct form-II structure for problem No. (1). In this problem we have obtained the equation of H (Z) as,

$$H(Z) = \frac{1 + \frac{1}{2} Z^{-1}}{1 - \frac{3}{4} Z^{-1} + \frac{1}{8} Z^{-2}}.$$

Here maximum value of Z inverse is '2'. So two delay elements are required. For the numerator the constant multiplier value is $\frac{1}{2}$. While for the denominator term constant multiplier values are $-\frac{3}{4}$ for Z^{-1} and $\frac{1}{8}$ for Z^{-2}. After changing the signs we get $\frac{3}{4}$ for Z^{-1} and $-\frac{1}{8}$ for Z^{-2}. So we can draw direct form-II structure as shown in Fig. I-10(e).

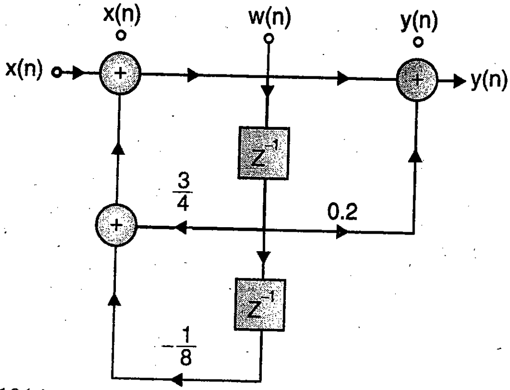


Fig. I-10(e) : Direct form-II structure for problem No. (1)

Prob. 3 : Develop direct form-II realization of the transfer function.

$$H(Z) = \frac{3+3.6\,Z^{-1}+0.6\,Z^{-2}}{1+0.1\,Z^{-1}-0.2\,Z^{-2}}$$

Is it advantageous than direct form-I ?

Soln. : For the direct form-II structure we have to realize the poles first and then zeros.

$$\therefore \quad \text{Let} \quad H_1(Z) = \text{All pole system} = \frac{1}{1+0.1\,Z^{-1}-0.2\,Z^{-2}}$$

$$\text{Now} \quad H_1(Z) = \frac{\text{Output}}{\text{Input}} = \frac{W(Z)}{X(Z)}$$

$$\therefore \quad \frac{W(Z)}{X(Z)} = \frac{1}{1+0.1\,Z^{-1}-0.2\,Z^{-2}}$$

$$\therefore \quad W(Z)[1+0.1\,Z^{-1}-0.2\,Z^{-2}] = X(Z)$$

$$\therefore \quad W(Z)+0.1\,Z^{-1}W(Z)-0.2\,Z^{-2}W(Z) = X(Z)$$

$$\therefore \quad W(Z) = X(Z)-0.1\,Z^{-1}W(Z)+0.2\,Z^{-2}W(Z) \qquad ...(1)$$

Taking IZT of Equation (1),

$$w(n) = x(n)-0.1\,w(n-1)+0.2\,w(n-2) \qquad ...(2)$$

The direct form implementation of Equation (2) is shown in Fig. I-11(a)

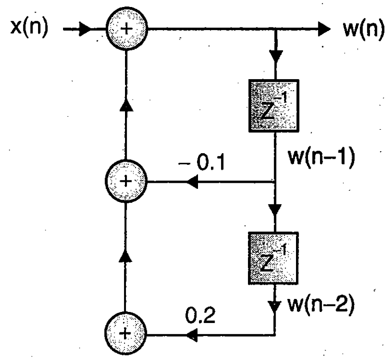


Fig. I-11(a) : Direct form realization of $H_1(Z)$

Now $H_2(Z) = \text{All zero system} = 3 + 3.6\,Z^{-1} + 0.6\,Z^{-2}$

But $H_2(Z) = \dfrac{\text{Output}}{\text{Input}} = \dfrac{Y(Z)}{W(Z)}$

$$\therefore \quad \frac{Y(Z)}{W(Z)} = 3 + 3.6\,Z^{-1} + 0.6\,Z^{-2}$$

$$\therefore \quad Y(Z) = 3\,W(Z) + 3.6\,Z^{-1}\,W(Z) + 0.6\,Z^{-2}\,W(Z) \qquad ...(3)$$

Taking IZT of Equation (3) we get,

$$y(n) = 3\,w(n) + 3.6\,w(n-1) + 0.6\,w(n-2) \qquad ...(4)$$

The direct form implementation of Equation (4) is shown in Fig. I-11(b).

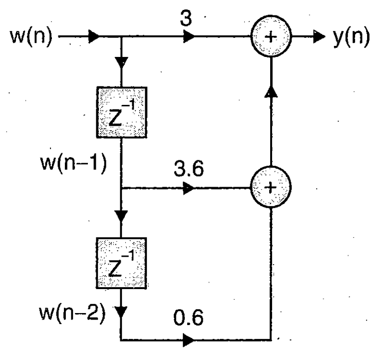


Fig. I-11(b) : Direct form realization of $H_2(Z)$

The direct form-II realization is obtained by cascading $H_1(Z)$ and $H_2(Z)$. It is shown in Fig. I-11(c).

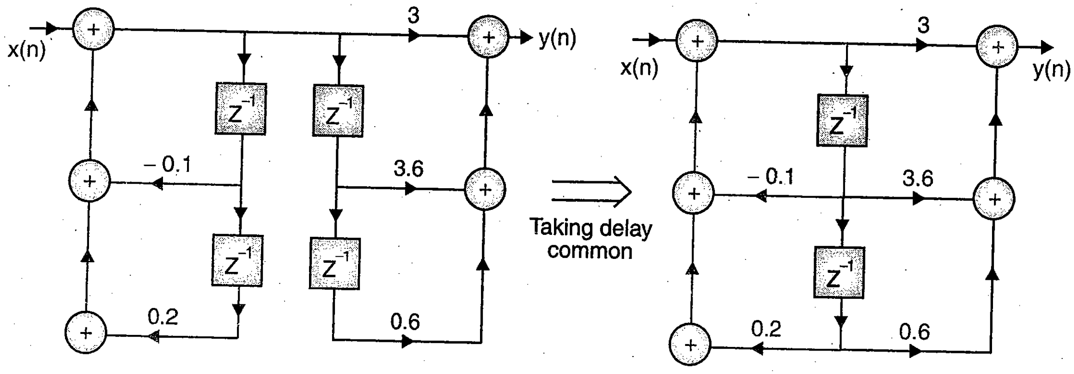


Fig. I-11(c) : Direct form-II realization

Now compared to direct form-I structure; nearly half number of delays are required for direct form-II realization. That means, it requires minimum number of storage elements. So it is advantageous than direct form-I structure.

Quick check :

We can check the answer by using shortcut method to obtain direct form-II realization.

Here maximum delays are 2. The coefficients of numerator are 3.6 for Z^{-1} and 0.6 for Z^{-2}. Now changing the signs, we get the coefficients of denominator as -0.1 for Z^{-1} and $+0.2$ for Z^{-2}. Thus direct form-II realization is shown in Fig. I-11(d).

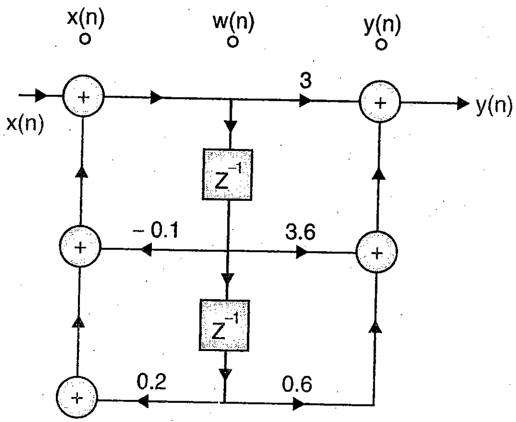


Fig. I-11(d) : Direct form-II realization using shortcut method

Thus Fig. I-11(c) and Fig. I-11(d) are same.

Prob. 4 : Obtain direct form-II implementation of LTI system with transfer function.

$$H(Z) = \frac{1+\frac{5}{6}Z^{-1}+\frac{1}{6}Z^{-2}}{1-\frac{1}{2}Z^{-1}-\frac{1}{2}Z^{-2}}$$

Soln. : For direct form-II structure. All pole system is realized first and then all zero system is realized.

$$\therefore \quad \text{Let} \quad H_1(Z) = \frac{1}{1-\frac{1}{2}Z^{-1}-\frac{1}{2}Z^{-2}}$$

$$\text{But} \quad H_1(Z) = \frac{W(Z)}{X(Z)}$$

$$\therefore \quad \frac{W(Z)}{X(Z)} = \frac{1}{1-\frac{1}{2}Z^{-1}-\frac{1}{2}Z^{-2}}$$

$$\therefore W(Z)-\frac{1}{2}Z^{-1}W(Z)-\frac{1}{2}Z^{-2}W(Z) = X(Z)$$

$$\therefore \quad W(Z) = X(Z)+\frac{1}{2}Z^{-1}W(Z)+\frac{1}{2}Z^{-2}W(Z) \quad \text{...(1)}$$

Taking IZT of Equation (1),

$$w(n) = x(n)+\frac{1}{2}w(n-1)+\frac{1}{2}w(n-2) \quad \text{...(2)}$$

The direct form realization of Equation (2) is shown in Fig. I-12(a).

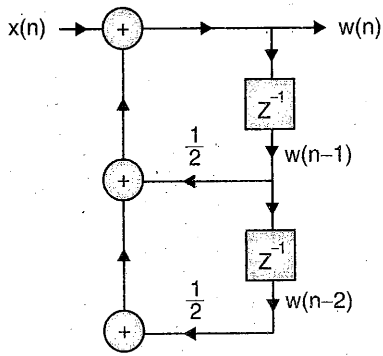


Fig. I-12(a) : Direct form realization of $H_1(Z)$

Now consider all zero system.

$$\therefore \quad H_2(Z) = 1+\frac{5}{6}Z^{-1}+\frac{1}{6}Z^{-2}$$

$$\text{But} \quad H_2(Z) = \frac{\text{Output}}{\text{Input}} = \frac{Y(Z)}{W(Z)}$$

$$\therefore \quad \frac{Y(Z)}{W(Z)} = 1+\frac{5}{6}Z^{-1}+\frac{1}{6}Z^{-2}$$

$$\therefore \quad Y(Z) = W(Z)+\frac{5}{6}Z^{-1}W(Z)+\frac{1}{6}Z^{-2}W(Z) \quad \text{...(3)}$$

Taking IZT of Equation (3),

$$y(n) = w(n)+\frac{5}{6}w(n-1)+\frac{1}{6}w(n-2) \quad \text{...(4)}$$

The direct form implementation of Equation (4) is shown in Fig. I-12(b).

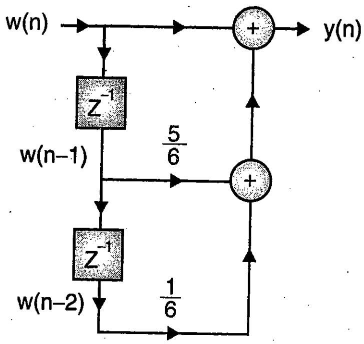


Fig. I-12(b) : Direct form realization of $H_2(Z)$

Now direct form-II realization is obtained by cascading $H_1(Z)$ and $H_2(Z)$ as shown in Fig. I-12(c).

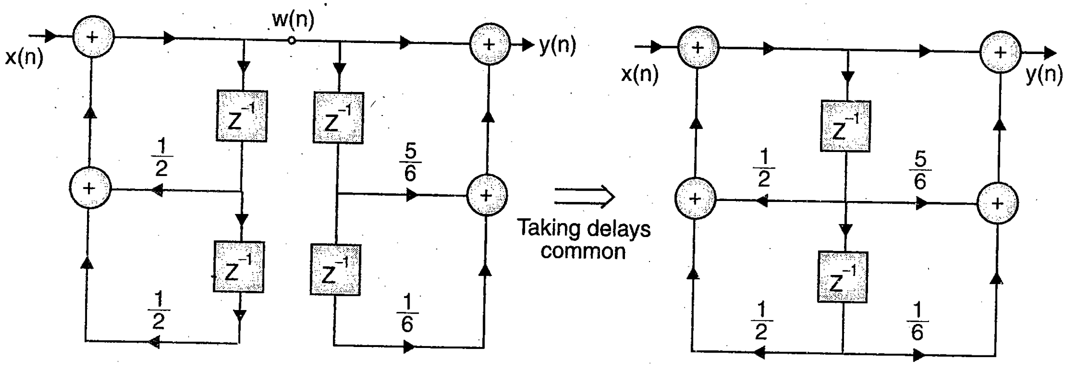


Fig. I-12(c) : Direct form-II realization

Note : We can easily check the answer using shortcut method.

Prob. 5 : A certain discrete time LTI filter has the following data :

Poles are at 0.2 and 0.6.

Zeros are at – 0.4 and origin.

Gain of filter is 5.

Show direct form-II realization and cascade form realization.

Soln. :

Given Zeros $\Rightarrow$ $Z_1 = 0$ and $Z_2 = -0.4$

Poles $\Rightarrow$ $P_1 = 0.2$ and $P_2 = 0.6$

Gain $=$ $k = 5$

The transfer function of system is expressed as,

$$H(Z) = k \cdot \frac{(Z - Z_1)(Z - Z_2)}{(Z - P_1)(Z - P_2)}$$

$$\therefore \quad H(Z) = \frac{5Z(Z + 0.4)}{(Z - 0.2)(Z - 0.6)} \qquad \text{...(1)}$$

$$\therefore \quad H(Z) = \frac{5Z^2 + 2Z}{Z^2 - 0.8Z + 0.12}$$

Converting into negative powers of Z we get,

$$H(Z) = \frac{5 + 2Z^{-1}}{1 - 0.8Z^{-1} + 0.12Z^{-2}} \qquad \text{...(2)}$$

Direct form-II realization :

Using shortcut method, direct form-II realization is shown in Fig. I-13.

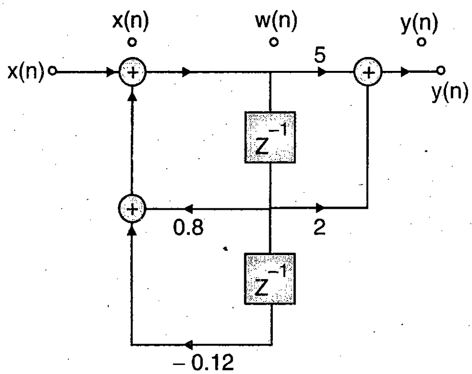


Fig. I-13

Cascade form realization :

According to Equation (1),

$$H(Z) = \frac{5Z(Z + 0.4)}{(Z - 0.2)(Z - 0.6)}$$

$$\therefore \quad H(Z) = \frac{5Z}{Z - 0.2} \cdot \frac{Z + 0.4}{Z - 0.6} \qquad \text{...(3)}$$

Let $H_1(Z) = \frac{5Z}{Z - 0.2} = \frac{5}{1 - 0.2Z^{-1}}$

and $H_2(Z) = \frac{Z + 0.4}{Z - 0.6} = \frac{1 + 0.4Z^{-1}}{1 - 0.6Z^{-1}}$

Using shortcut method $H_1(Z)$ and $H_2(Z)$ are realized as shown in Fig. I-14 and I-15.

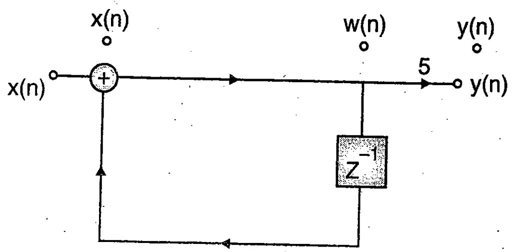


Fig. I-14

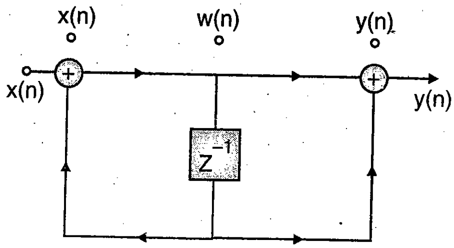


Fig. I-15

The cascade realization is shown in Fig. I-16.

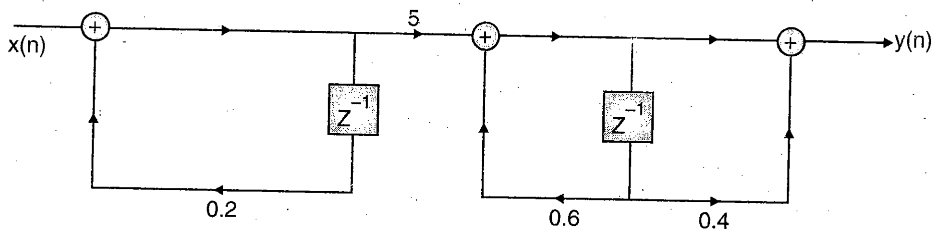


Fig. I-16

Prob. 6 : Determine direct-form-II realization for each of the following LTI system

(i) $2y(n)+y(n-1)-4y(n-3)=x(n)+3x(n-1)$

(ii) $y(n)=x(n)-x(n-1)+2x(n-2)-3x(n-4)$

Soln. :

(i) Given,

$$2y(n)+y(n-1)-4y(n-3) = x(n)+3x(n-1)$$

Taking Z-transform of both sides.

$$2Y(Z)+Z^{-1}Y(Z)-4Z^{-3}Y(Z) = X(Z)+3Z^{-1}X(Z)$$

$$\therefore \quad Y(Z)\left[2+Z^{-1}-4Z^{-3}\right] = X(Z)\left[1+3Z^{-1}\right]$$

$$\therefore \quad \frac{Y(Z)}{X(Z)} = H(Z) = \frac{1+3Z^{-1}}{2+Z^{-1}-4Z^{-3}}$$

Using short-cut method, the direct form-II structure is drawn as shown in Fig. I-17.

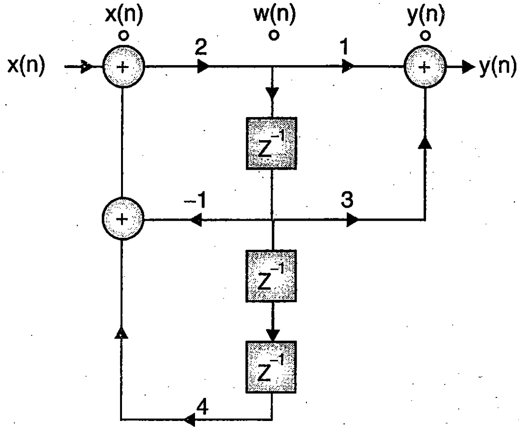


Fig. I-17

(i) We have, $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$

Taking Z-transform of both sides we get,

$$Y(Z) = X(Z) - Z^{-1}X(Z) + 2Z^{-2}X(Z) - 3Z^{-4}X(Z)$$

$$\therefore \quad Y(Z) = X(Z)[1 - Z^{-1} + 2Z^{-2} - 3Z^{-4}]$$

$$\therefore \quad \frac{Y(Z)}{X(Z)} = H(Z) = 1 - Z^{-1} + 2Z^{-2} - 3Z^{-4}$$

Using short-cut method, the direct form-II realization is obtained as shown in Fig. I-18.

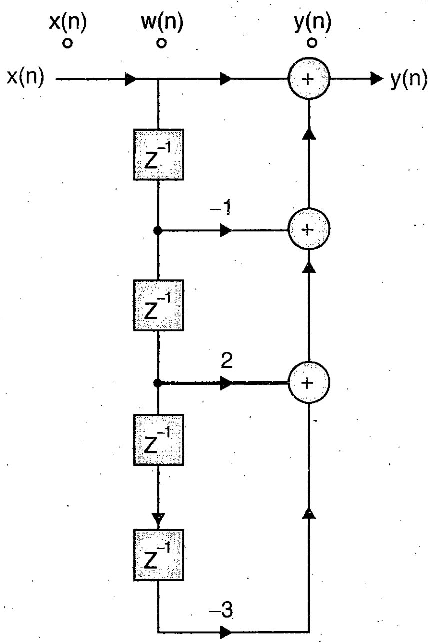


Fig. I-18

Prob. 7 : The transfer function of discrete time causal system is given by,

$$H(Z) = \frac{1 - Z^{-1}}{1 - 0.2\,Z^{-1} - 0.15\,Z^{-2}}$$

Draw cascade and parallel realization.

Soln. : The given transfer function is,

$$H(Z) = \frac{1 - Z^{-1}}{1 - 0.2\,Z^{-1} - 0.15\,Z^{-2}} \quad ..(1)$$

Cascade realization :

First we will obtain the roots of denominator, by using the equation,

$$\text{roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here $a = 1$, $b = -0.2$ and $c = -0.15$

$$\therefore \quad \text{roots} = \frac{0.2 \pm \sqrt{0.04 + 0.6}}{2} = \frac{0.2 \pm 0.8}{2} = 0.5, -0.3$$

Thus $(1 - 0.2\,Z^{-1} - 0.15\,Z^{-2}) = (1 - 0.5\,Z^{-1})(1 + 0.3\,Z^{-1})$

Putting this value in Equation (1) we get,

$$H(Z) = \frac{1 - Z^{-1}}{(1 - 0.5\,Z^{-1})(1 + 0.3\,Z^{-1})} \quad ...(2)$$

For the cascade realization we have to express $H(Z)$ as,

$$H(Z) = H_1(Z) \cdot H_2(Z)$$

Let $$H_1(Z) = \frac{1 - Z^{-1}}{1 - 0.5\,Z^{-1}} \quad ..(3)$$

and let $$H_2(Z) = \frac{1}{1 + 0.3\,Z^{-1}} \quad ...(4)$$

Using shortcut method, we can obtain direct form-II structures for $H_1(Z)$ and $H_2(Z)$ as shown in Fig. I-19(a) and I-19(b) respectively.

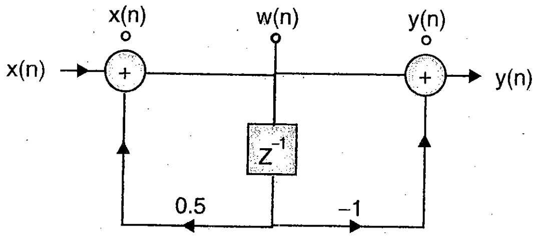


Fig. I-19(a) : Direct form-II realization of $H_1(Z)$

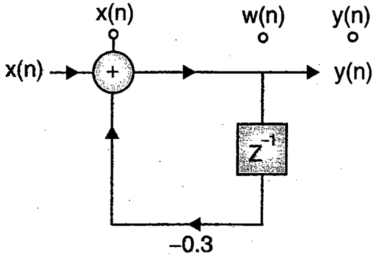


Fig. I-19(b) : Direct form-II realization of $H_2(Z)$

Now for the cascade connection we will have to connect $H_1(Z)$ and $H_2(Z)$ in series. This series connection is shown in Fig. I-19(c).

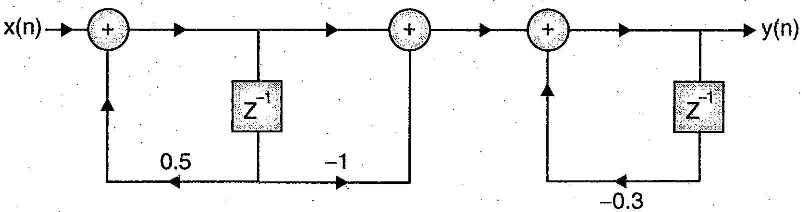


Fig. I-19(c) : Cascade realization

Parallel realization :

For the parallel form realization we have to use partial fraction expansion (PFE). It is similar to PFE which we have studied in Z transformation.

The given transfer function is,

$$H(Z) = \frac{1-Z^{-1}}{1-0.2Z^{-1}-0.15Z^{-2}}$$

To convert this equation in terms of positive powers of Z; multiply numerator and denominator by Z^2.

$$\therefore \quad H(Z) = \frac{Z^2(1-Z^{-1})}{Z^2(1-0.2Z^{-1}-0.15Z^{-2})}$$

$$\therefore \quad H(Z) = \frac{Z^2-Z}{Z^2-0.2Z-0.15} = \frac{Z(Z-1)}{Z^2-0.2Z-0.15}$$

$$\therefore \quad \frac{H(Z)}{Z} = \frac{Z-1}{Z^2-0.2Z-0.15} \qquad ...(5)$$

Equation (5) is in the proper form. The roots of denominator are 0.5 and – 0.3. That means poles are at, $P_1 = 0.5$ and $P_2 = -0.3$

$$\therefore \quad \frac{H(Z)}{Z} = \frac{Z-1}{(Z-0.5)(Z+0.3)} \qquad ..(6)$$

In the partial fraction expansion form Equation (6) can be written as,

$$\frac{H(Z)}{Z} = \frac{A_1}{(Z-0.5)} + \frac{A_2}{(Z+0.3)} \qquad ...(7)$$

Now we will calculate the values of A_1 and A_2.

$$A_1 = (Z-p_1)\frac{H(Z)}{Z}\Bigg|_{Z=p_1}$$

$$\therefore \quad A_1 = \frac{(Z-0.5)(Z-1)}{(Z-0.5)(Z+0.3)}\Bigg|_{Z=0.5}$$

$$\therefore \quad A_1 = \frac{0.5-1}{0.5+0.3}$$

$$\therefore \quad A_1 = -0.625 \qquad ...(8)$$

$$\text{Now} \quad A_2 = (Z-p_2)\frac{H(Z)}{Z}\Bigg|_{Z=p_2}$$

$$\therefore \quad A_2 = \frac{(Z+0.3)(Z-1)}{(Z-0.5)(Z+0.3)}\Bigg|_{Z=-0.3}$$

$$\therefore \quad A_2 = \frac{-0.3-1}{-0.3-0.5}$$

$$\therefore \quad A_2 = 1.625 \qquad ..(9)$$

Putting the values of A_1 and A_2 in Equation (7),

$$\frac{H(Z)}{Z} = \frac{-0.625}{Z-0.5} + \frac{1.625}{Z+0.3}$$

$$\therefore \quad H(Z) = \frac{-0.625\,Z}{Z-0.5} + \frac{1.625\,Z}{Z+0.3} \qquad ..(10)$$

Remember that, to realize the given transfer function $H(Z)$; it should be in terms of negative powers of Z. So we will convert Equation (10) in terms of negative powers of Z as follows :

$$H(Z) = \frac{Z^{-1}(-0.625\,Z)}{Z^{-1}(Z-0.5)} + \frac{Z^{-1}(1.625\,Z)}{Z^{-1}(Z+0.3)}$$

$$\therefore \quad H(Z) = \frac{-0.625}{1-0.5\,Z^{-1}} + \frac{1.625}{1+0.3\,Z^{-1}} \qquad ..(11)$$

The general structure of parallel form realization is as shown in Fig. 1-19(d). The equation is.

$$H(Z) = H_1(Z) + H_2(Z).$$

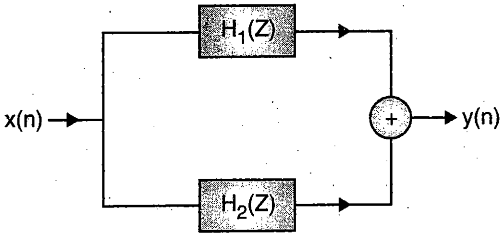
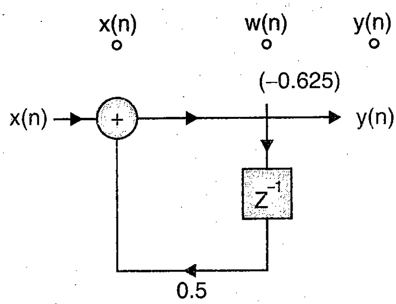


Fig. I-19(d) : General structure of parallel form realization

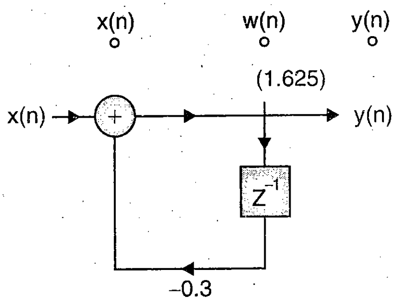
$$\text{Now let} \quad H_1(Z) = \frac{-0.625}{1 - 0.5\,Z^{-1}} \qquad \text{...(12)}$$

$$\text{and} \quad H_2(Z) = \frac{1.625}{1 + 0.3\,Z^{-1}} \qquad \text{...(13)}$$

Using shortcut method; the direct form-II realization of $H_1(Z)$ and $H_2(Z)$ is obtained as shown in Fig. I-19(e) and Fig. I-19(f) respectively.



(e) Direct form-II realization of $H_1(Z)$



(f) : Direct form-II realization of $H_2(Z)$

Fig. I-19

To obtain the parallel form realization; we have to connect Fig. I-19(e) and Fig. I-19(f) in parallel as shown in Fig. I-19(g).

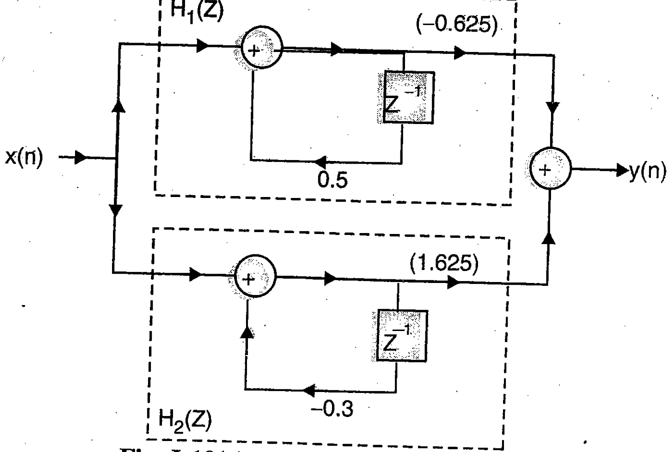


Fig. I-19(g) : Parallel form realization

Prob. 8 : A system has an impulse response,

$h(n) = (0.5)^n u(n) + n(0.2)^n u(n)$

show parallel realization of system.

Soln. : First we will obtain transfer function $H(Z)$.

We have, $$h(n) = (0.5)^n u(n) + n(0.2)^n u(n) \quad ...(1)$$

Recall the standard Z-transform pairs,

$$\alpha^n u(n) \overset{Z}{\longleftrightarrow} \frac{Z}{Z-\alpha} \quad \text{and} \quad n\,a^n u(n) \overset{Z}{\longleftrightarrow} \frac{aZ}{(Z-a)^2}$$

Thus Z-transform of Equation (1) can be written as,

$$H(Z) = \frac{Z}{Z-0.5} + \frac{0.2\,Z}{(Z-0.2)^2} \quad ...(2)$$

Let $$H_1(Z) = \frac{Z}{Z-0.5} = \frac{1}{1-0.5\,Z^{-1}}$$

The direct form-II realization of $H_1(Z)$ is shown in Fig. I-20(a).

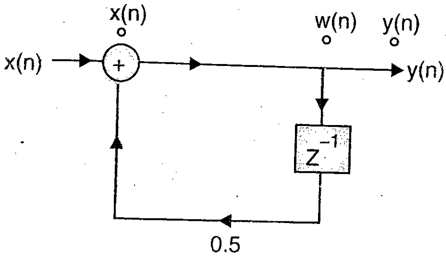


Fig. I-20(a)

Let $H_2(Z) = \dfrac{0.2\,Z}{(Z-0.2)^2}$

$$= \frac{0.2\,Z}{Z^2 - 0.4\,Z + 0.04}$$

Multiplying numerator and denominator by Z^{-2} we get,

$$H_2(Z) = \frac{0.2\,Z^{-1}}{1 - 0.4\,Z^{-1} + 0.04\,Z^{-2}}$$

The direct form-II realization of $H_2(Z)$ is shown in Fig. I-20(b).

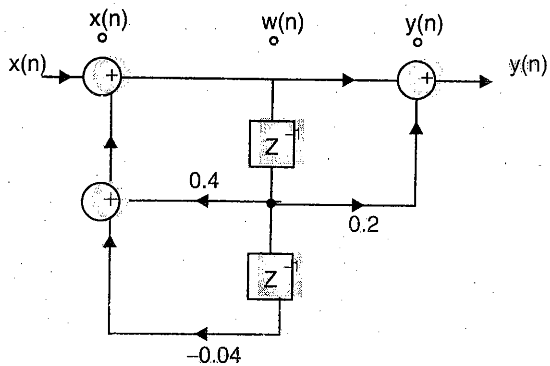


Fig. I-20(b)

The general structure parallel form realization is shown in Fig. I-20(c).

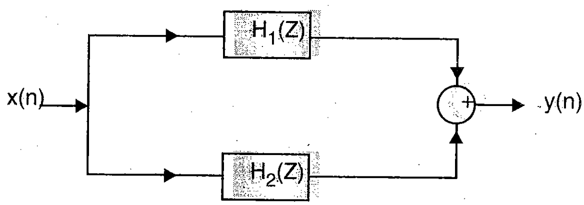


Fig. I-20(c)

Thus using Fig. I-20(a) and Fig. I-20(b); we can draw the parallel form realization of $H(Z)$ as shown in Fig. I-20(d).

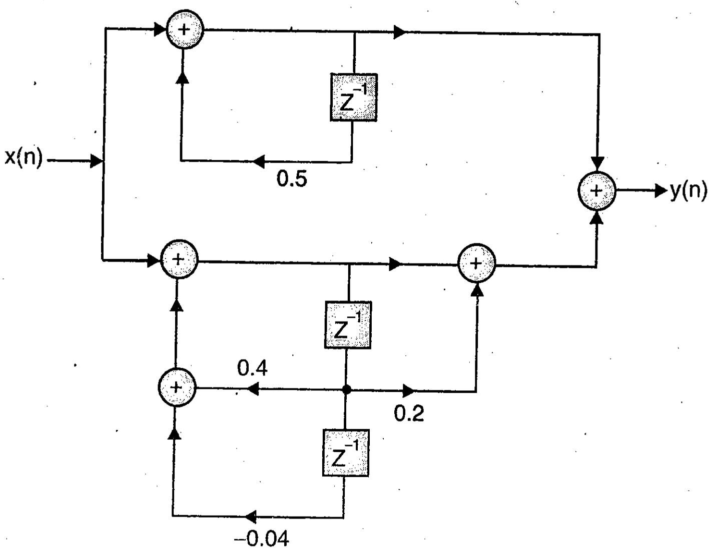


Fig. I-20(d)

Prob. 9 : Develop parallel form realization for the digital filter with transfer function.

$$H(Z) = \frac{1 + 2Z^{-1} + Z^{-2}}{1 - 0.75Z^{-1} + 0.125Z^{-2}}$$

Using first order systems.

Soln. : To obtain the parallel form realization we have to use partial fraction expansion.

The given transfer function is,

$$H(Z) = \frac{1 + 2Z^{-1} + Z^{-2}}{1 - 0.75Z^{-1} + 0.125Z^{-2}} \quad \text{...(1)}$$

To convert if in terms of positive power of Z; multiply numerator and denominator by Z^2.

$$\therefore \quad H(Z) = \frac{Z^2 + 2Z + 1}{Z^2 - 0.75Z + 0.125}$$

$$\therefore \quad \frac{H(Z)}{Z} = \frac{Z^2 + 2Z + 1}{Z(Z^2 - 0.75Z + 0.125)} \quad \text{...(2)}$$

Now we will obtain roots of $Z^2 - 0.75Z + 0.125$

Here $a = 1$, $b = -0.75$ and $c = 0.125$

$$\therefore \quad \text{roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad \text{roots} = \frac{0.75 \pm \sqrt{(-0.75)^2 - (4 \times 0.125)}}{2} = \frac{0.75 \pm 0.25}{2}$$

$$\therefore \quad \text{roots} = 0.5,\ 0.25$$

Thus Equation (2) becomes,

$$\frac{H(Z)}{Z} = \frac{Z^2+2Z+1}{Z(Z-0.5)(Z-0.25)} \quad ..(3)$$

Thus poles are at $p_1 = 0$, $p_2 = 0.5$ and $p_3 = 0.25$

In the partial fraction expansion form, Equation (3) can be written as,

$$\frac{H(Z)}{Z} = \frac{A_1}{Z} + \frac{A_2}{Z-0.5} + \frac{A_3}{Z-0.25} \quad ...(4)$$

Now $A_1 = \left.\frac{H(Z)}{Z} \times (Z-p_1)\right|_{Z=p_1} = \left.\frac{Z^2+2Z+1}{Z(Z-0.5)(Z-0.25)} \times Z\right|_{Z=0}$

$$\therefore \quad A_1 = \frac{1}{(-0.5)(-0.25)}$$

$$\therefore \quad A_1 = 8 \quad ...(5)$$

$$A_2 = \left.\frac{H(Z)}{Z} \times (Z-p_2)\right|_{Z=p_2} = \left.\frac{Z^2+2Z+1}{Z(Z-0.5)(Z-0.25)} \times (Z-0.5)\right|_{Z=0.5}$$

$$\therefore \quad A_2 = \left.\frac{Z^2+2Z+1}{Z(Z-0.25)}\right|_{Z=0.5} \quad \therefore \quad A_2 = \frac{(0.5)^2+2(0.5)+1}{0.5(0.5-0.25)}$$

$$\therefore \quad A_2 = 18 \quad ...(6)$$

and $A_3 = \left.\frac{H(Z)}{Z} \times (Z-p_3)\right|_{Z=p_3}$

$$\therefore \quad A_3 = \left.\frac{Z^2+2Z+1}{Z(Z-0.5)(Z-0.25)} \times (Z-0.25)\right|_{Z=0.25}$$

$$\therefore \quad A_3 = \frac{(0.25)^2+2(0.25)+1}{0.25(0.25-0.5)}$$

$$\therefore \quad A_3 = -25 \quad ...(7)$$

Putting the values of A_1, A_2 and A_3 in Equation (4) we get,

$$\frac{H(Z)}{Z} = \frac{8}{Z} + \frac{18}{(Z-0.5)} - \frac{25}{(Z-0.25)}$$

$$\therefore \quad H(Z) = 8 + \frac{18Z}{Z-0.5} - \frac{25Z}{Z-0.25} \quad ...(8)$$

To realize H (Z); we want the expression in terms of negative powers of Z.

$$\therefore \quad H(Z) = 8 + \frac{Z^{-1}(18Z)}{Z^{-1}(Z-0.5)} - \frac{Z^{-1}(25Z)}{Z^{-1}(Z-0.25)}$$

$$\therefore \quad H(Z) = 8 + \frac{18}{1-0.5Z^{-1}} - \frac{25}{1-0.25Z^{-1}} \qquad ..(9)$$

Let $H_1(Z) = 8$

$$H_2(Z) = \frac{18}{1-0.5Z^{-1}}$$

and $$H_3(Z) = \frac{-25}{1-0.25Z^{-1}}$$

Using shortcut method; direct form-II realization of $H_2(Z)$ and $H_3(Z)$ is obtained as shown in Fig. I-21(a) and I-21(b) respectively.

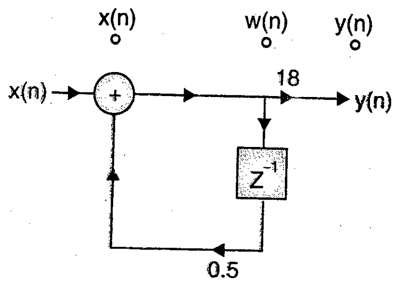


Fig. I-21(a) : Direct form-II realization of $H_2(Z)$

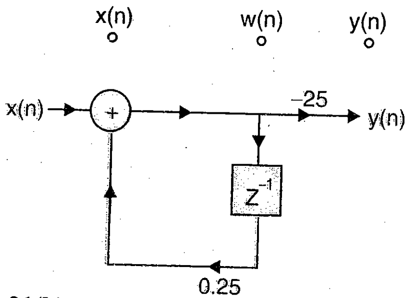


Fig. I-21(b) : Direct form-II realization of H_3 (Z)

Now $H_1(Z) = 8$; which is constant. So it can be expressed as,

$$H_1(Z) = \frac{Y_1(Z)}{X_1(Z)}$$

$$\therefore \quad Y_1(Z) = H_1(Z) \cdot X_1(Z) = 8\,X_1(Z)$$

Taking IZT of both sides we get,

$$y_1(n) = 8\,x_1(n) \qquad ...(10)$$

Using Fig. I-21(a), Fig. I-21(b) and Equation (10); the parallel form realization is drawn as shown in Fig. I-21(c).

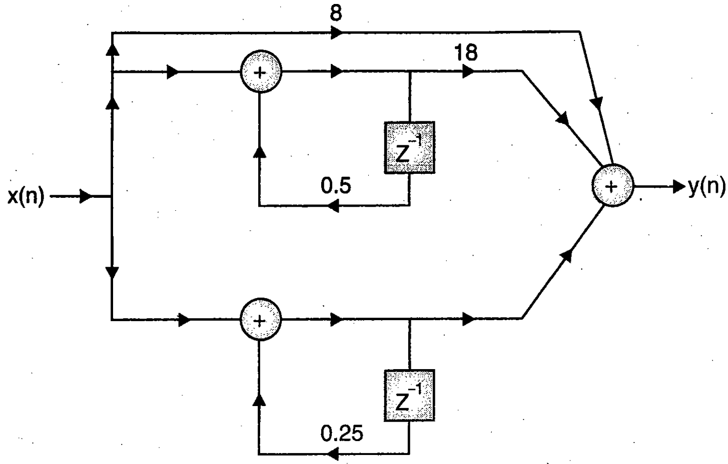


Fig. I-21(c) : Parallel form realization

Prob. 10 : System has transfer function as shown in Fig. I-22.

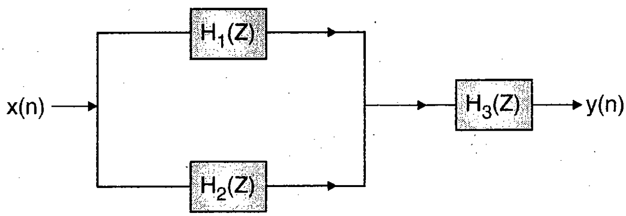


Fig. I-22

$H_1(Z)$ and $H_2(Z)$ has poles at $+1$ and -1.

$H_1(Z)$ has two zeros at origin, $H_2(Z)$ has only one zero at origin. The transfer $H_3(Z)$ is given by,

$H_3(Z) = 1 + 1.5\,Z^{-1} - 1.5\,Z^{-2} - Z^{-3}$

Show realization diagram of overall transfer function.

Soln. : First we will obtain expressions of $H_1(Z)$ and $H_2(Z)$.

For $H_1(Z)$:

$$\text{Zeros} \Rightarrow Z_1 = 0 \text{ and } Z_2 = 0;\ \text{two zeros at origin}$$

$$\text{Poles} \Rightarrow P_1 = 1 \text{ and } P_2 = -1$$

$$\text{Now } H_1(Z) = \frac{(Z - Z_1)(Z - Z_2)}{(Z - P_1)(Z - P_2)}$$

$$\therefore \quad H_1(Z) = \frac{Z^2}{(Z-1)(Z+1)} \quad \text{...(1)}$$

For $H_2(Z)$:

Zeros $\Rightarrow Z_1 = 0$

Poles $\Rightarrow P_1 = 1$ and $P_2 = -1$

$$\text{Now} \quad H_2(Z) = \frac{(Z-Z_1)}{(Z-P_1)(Z-P_2)}$$

$$\therefore \quad H_2(Z) = \frac{Z}{(Z-1)(Z+1)} \quad \text{...(2)}$$

Realization of $H_1(Z)$:

$$\text{We have} \quad H_1(Z) = \frac{Z^2}{(Z-1)(Z+1)} = \frac{Z^2}{Z^2-1}$$

$$\therefore \quad H_1(Z) = \frac{1}{1-Z^{-2}}$$

Direct form-II realization of $H_1(Z)$ is shown in Fig. I-23(a).

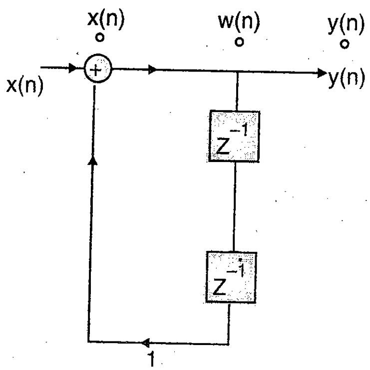


Fig. I-23(a)

Realization of $H_2(Z)$:

$$\text{We have,} \quad H_2(Z) = \frac{Z}{(Z-1)(Z+1)} = \frac{Z}{Z^2-1}$$

$$\therefore \quad H_2(Z) = \frac{Z^{-1}}{1-Z^{-2}}$$

Direct form-II realization of $H_2(Z)$ is shown in Fig. I-23(b).

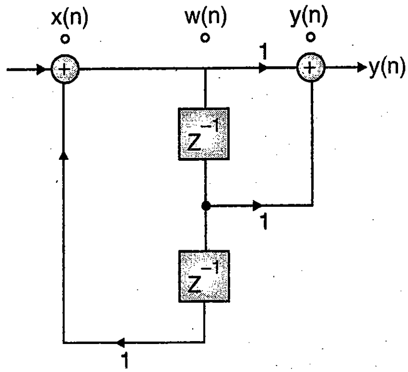


Fig. I-23(b)

Realization of $H_3(Z)$:

We have, $$H_3(Z) = 1 + 1.5Z^{-1} - 1.5Z^{-2} - Z^{-3}$$

Direct form-II realization of $H_3(Z)$ is shown in Fig. I-23(c).

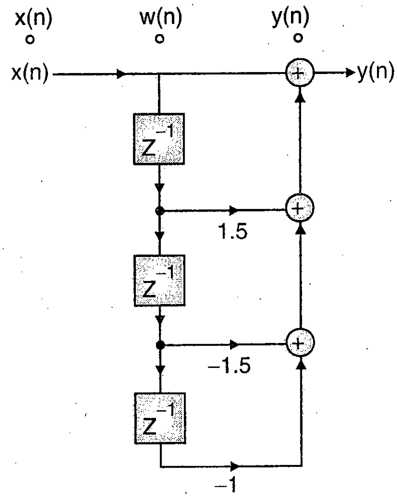


Fig. I-23(c)

Realization of overall transfer function :

As shown in Fig. I-22, $H_1(Z)$ and $H_2(Z)$ are connected in parallel and this parallel combination is cascaded with $H_3(Z)$. Thus realization of overall transfer function is shown in Fig. I-23(d).

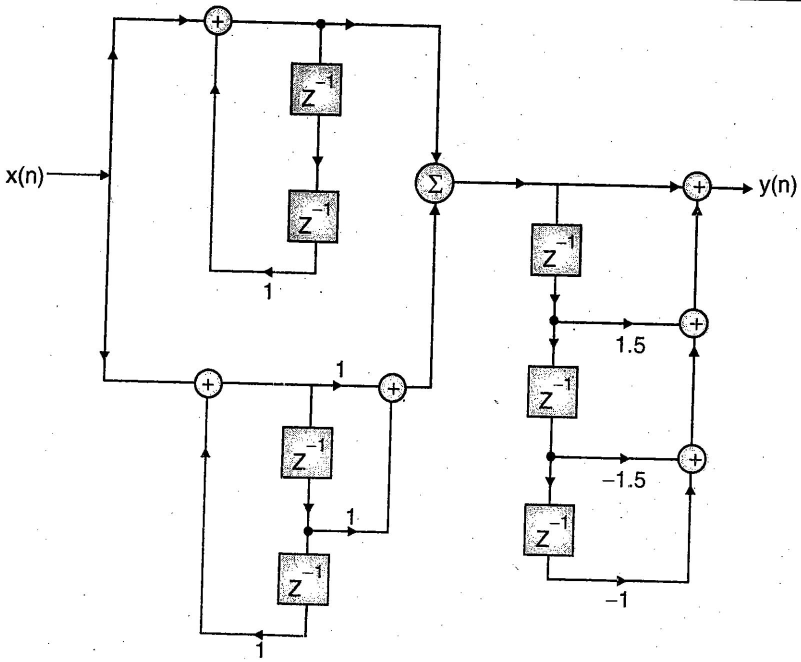
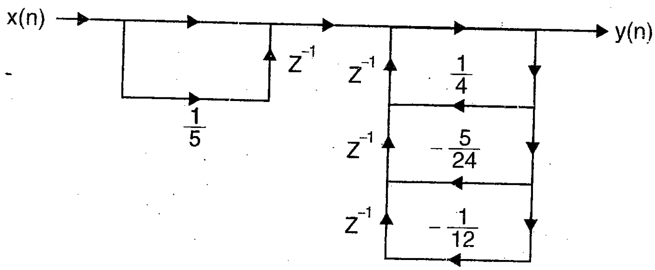
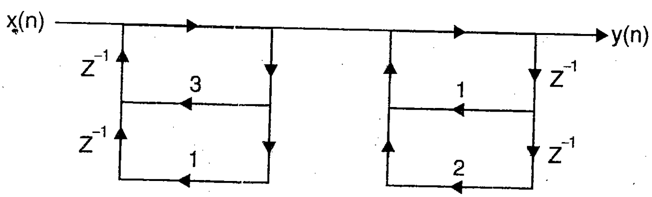


Fig. I-23(d)

Prob. 11 : For the graphs of Figs. I-24(a) and (b) write difference equation and system function.



(a) Given graph



(b) Given graph
Fig. I-24

Soln. :

For graph (a) :

By looking at the graph; we can easily conclude that it is a cascade connection. We know that in direct form-II structure poles are realized first and then zeros are realized. That means first part of Fig. I-24(a) indicates all pole system and second part indicates all zero system.

We can easily write the difference equation for the graph shown in Fig. I-24(a). It is,

$$y(n) = x(n)+\frac{1}{5}x(n-1)+\frac{1}{4}y(n-1)-\frac{5}{24}y(n-2)-\frac{1}{12}y(n-3) \qquad ...(1)$$

This is the difference equation for the given graph. Now system transfer function is denoted by $H(Z)$. It is obtained by taking Z transform of Equation (1).

$$\therefore \quad Y(Z) = X(Z)+\frac{1}{5}Z^{-1}X(Z)+\frac{1}{4}Z^{-1}Y(Z)-\frac{5}{24}Z^{-2}Y(Z)-\frac{1}{12}Z^{-3}Y(Z)$$

$$\therefore \quad Y(Z) = X(Z)\left[1+\frac{1}{5}Z^{-1}\right]+Y(Z)\left[\frac{1}{4}Z^{-1}-\frac{5}{24}Z^{-2}-\frac{1}{12}Z^{-3}\right]$$

$$\therefore \quad Y(Z)-Y(Z)\left[\frac{1}{4}Z^{-1}-\frac{5}{24}Z^{-2}-\frac{1}{12}Z^{-3}\right] = X(Z)\left[1+\frac{1}{5}Z^{-1}\right]$$

$$\therefore \quad Y(Z)\left[1-\frac{1}{4}Z^{-1}-\frac{5}{24}Z^{-2}-\frac{1}{12}Z^{-3}\right] = X(Z)\left[1+\frac{1}{5}Z^{-1}\right] \qquad ...(2)$$

The system transfer function $H(Z)$ is given by,

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

Thus from Equation (2) we get,

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1+\frac{1}{5}Z^{-1}}{1-\frac{1}{4}Z^{-1}-\frac{5}{24}Z^{-2}-\frac{1}{12}Z^{-3}} \qquad ...(3)$$

For graph (b) :

Now consider the graph shown in Fig. I-24(b). From this graph we cannot directly write the difference equation because first part of Fig. I-24(b) indicates the feedback operation (observe the directions of arrows). So we have to write the equations by considering $w(n)$.

$$\therefore \quad w(n) = x(n)+3\,w(n-1)+1\,w(n-2) \qquad ...(4)$$

$$\text{and} \quad y(n) = w(n)+1\,y(n-1)+2y(n-2) \qquad ...(5)$$

Taking Z transform of Equation (4),

$$W(Z) = X(Z)+3Z^{-1}W(Z)+Z^{-2}W(Z)$$

$$\therefore \quad W(Z)-3Z^{-1}W(Z)-Z^{-2}W(Z) = X(Z)$$

$$\therefore \quad W(Z)\left[1-3Z^{-1}-Z^{-2}\right] = X(Z)$$

$$\therefore \quad \frac{W(Z)}{X(Z)} = \frac{1}{1-3Z^{-1}-Z^{-2}} \qquad ...(6)$$

Now taking Z transform of Equation (5),

$$Y(Z) = W(Z)+Z^{-1}Y(Z)+2Z^{-2}Y(Z)$$

$$\therefore \quad Y(Z)-Z^{-1}Y(Z)-2Z^{-2}Y(Z) = W(Z)$$

$$\therefore \quad Y(Z)\left[1-Z^{-1}-2Z^{-2}\right] = W(Z)$$

$$\therefore \quad \frac{Y(Z)}{W(Z)} = \frac{1}{1-Z^{-1}-2Z^{-2}} \qquad ...(7)$$

Now system transfer function H (Z) can be expressed as,

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{Y(Z)}{W(Z)} \times \frac{W(Z)}{X(Z)}$$

$$\therefore \quad H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1}{(1-Z^{-1}-2Z^{-2})(1-3Z^{-1}-Z^{-2})}$$

$$\therefore \quad H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1}{1-3Z^{-1}-Z^{-2}-Z^{-1}+3Z^{-2}+Z^{-3}-2Z^{-2}+6Z^{-3}+2Z^{-4}}$$

$$\therefore \quad H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1}{1-4Z^{-1}+7Z^{-3}+2Z^{-4}} \qquad ...(8)$$

Equation (8) gives the transfer function of overall system. To obtain the difference equation of overall system; we will take IZT of Equation (8). Rearranging Equation (8) we get,

$$\therefore \quad Y(Z)(1-4Z^{-1}+7Z^{-3}+2Z^{-4}) = X(Z)$$

$$\therefore \quad Y(Z)-4Z^{-1}Y(Z)+7Z^{-3}Y(Z)+2Z^{-4}Y(Z) = X(Z)$$

$$\therefore \quad Y(Z) = X(Z)+4Z^{-1}Y(Z)-7Z^{-3}Y(Z)-2Z^{-4}Y(Z)$$

Now taking IZT of both sides,

$$y(n) = x(n)+4y(n-1)-7y(n-3)-2y(n-4) \qquad ...(9)$$

Equation (9) represents the difference equation of overall system.

Prob. 12 : Using first order sections, obtain cascade realization for

$$H(Z) = \frac{\left(1+\frac{1}{2}Z^{-1}\right)\left(1+\frac{1}{4}Z^{-1}\right)}{\left(1-\frac{1}{2}Z^{-1}\right)\left(1-\frac{1}{4}Z^{-1}\right)\left(1-\frac{1}{8}Z^{-1}\right)}$$

Soln. : We know that for cascade realization the given transfer function can be expressed as,

$$H(Z) = H_1(Z)\cdot H_2(Z)\cdot H_3(Z)...$$

Thus the given transfer function can be expressed as,

$$H(Z) = \frac{\left(1+\frac{1}{2}Z^{-1}\right)}{\left(1-\frac{1}{2}Z^{-1}\right)}\cdot\frac{\left(1+\frac{1}{4}Z^{-1}\right)}{\left(1-\frac{1}{4}Z^{-1}\right)}\cdot\frac{1}{\left(1-\frac{1}{8}Z^{-1}\right)}$$

Here $H_1(Z) = \dfrac{1+\frac{1}{2}Z^{-1}}{1-\frac{1}{2}Z^{-1}} = \dfrac{Y_1(Z)}{X(Z)}$

$$\therefore \quad Y_1(Z)\left(1-\frac{1}{2}Z^{-1}\right) = X(Z)\left(1+\frac{1}{2}Z^{-1}\right)$$

$$\therefore \quad Y_1(Z)-\frac{1}{2}Z^{-1}Y_1(Z) = X(Z)+\frac{1}{2}Z^{-1}X(Z)$$

$$\therefore \quad Y_1(Z) = X(Z)+\frac{1}{2}Z^{-1}X(Z)+\frac{1}{2}Z^{-1}Y_1(Z)$$

Taking IZT of both sides,

$$y_1(n) = x(n)+\frac{1}{2}x(n-1)+\frac{1}{2}y_1(n-1) \quad ...(1)$$

It is realized as shown in Fig. I-25(a).

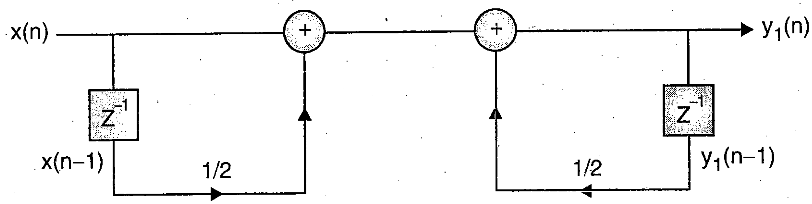


Fig. I-25(a)

Its direct form-II realization is as shown in Fig. I-25(b).

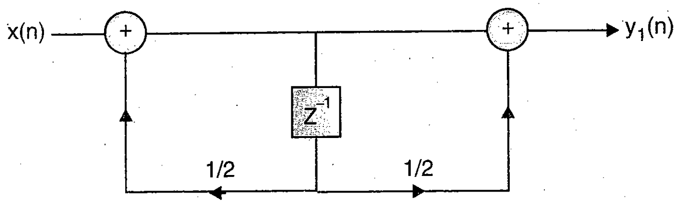


Fig. I-25(b) : Direct form-II

Now $H_2(Z) = \dfrac{1+\frac{1}{4}Z^{-1}}{1-\frac{1}{4}Z^{-1}} = \dfrac{Y_2(Z)}{X_2(Z)}$

$$\therefore \quad Y_2(Z)\left(1-\frac{1}{4}Z^{-1}\right) = X_2(Z)\left(1+\frac{1}{4}Z^{-1}\right)$$

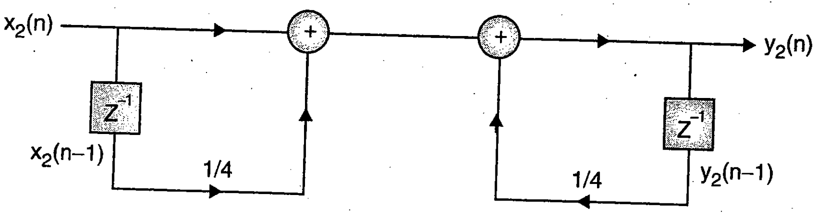
$$\therefore \quad Y_2(Z)-\frac{1}{4}Z^{-1}Y_2(Z) = X_2(Z)+\frac{1}{4}Z^{-1}X_2(Z)$$

$$\therefore \quad Y_2(Z) = X_2(Z)+\frac{1}{4}Z^{-1}X_2(Z)+\frac{1}{4}Z^{-1}Y_2(Z)$$

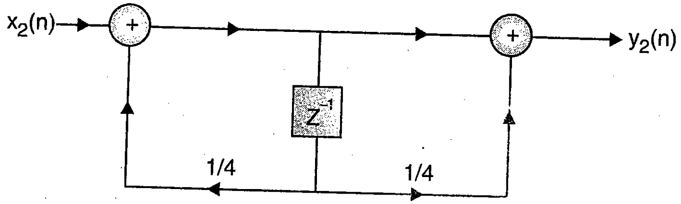
Taking IZT of both sides,

$$y_2(n) = x_2(n) + \frac{1}{4}x_2(n-1) + \frac{1}{4}y_2(n-1) \qquad \text{...(2)}$$

Its realization is as shown in Figs. I-25(c) and I-25(d).



(c) Direct form-I



(d) Direct form-II
Fig. I-25

Now $$H_3(Z) = \frac{1}{1-\frac{1}{8}Z^{-1}} = \frac{Y(Z)}{X_3(Z)}$$

$$\therefore \quad Y(Z)\left(1-\frac{1}{8}Z^{-1}\right) = X_3(Z)$$

$$\therefore \quad Y(Z)-\frac{1}{8}Z^{-1}Y(Z) = X_3(Z)$$

$$\therefore \quad Y(Z) = X_3(Z)+\frac{1}{8}Z^{-1}Y_3(Z)$$

Taking IZT of both sides,

$$y(n) = x_3(n)+\frac{1}{8}y(n-1) \qquad \text{...(3)}$$

Its realization is as shown in Fig. I-25(e).

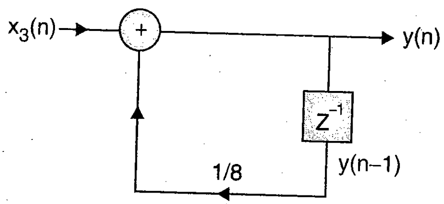


Fig. I-25(e)

Now cascade realization is obtained by cascading the direct form- II structures of $H_1(Z)$, $H_2(Z)$ and $H_3(Z)$ as shown in Fig. I-25(f).

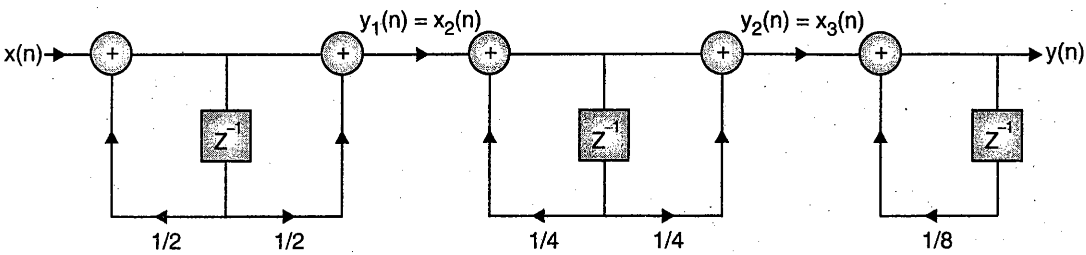


Fig. I-25(f)

Prob. 13 : Determine the parallel realization of the IIR digital filter transfer functions.

(i) $H(Z) = \dfrac{3(2Z^2+5Z+4)}{(2Z+1)(Z+2)}$

(ii) $H(Z) = \dfrac{3Z(5Z-2)}{\left(Z+\frac{1}{2}\right)(3Z-1)}$

Soln. :

(i) Given $\quad H(Z) = \dfrac{3(2Z^2-5Z+4)}{(2Z+1)(Z+2)}$

Multiplying numerator and denominator by $\frac{1}{2}$ we get,

$$H(Z) = \frac{\frac{3}{2}(2Z^2+5Z+4)}{\left(Z+\frac{1}{2}\right)(Z+2)} \qquad \therefore \quad H(Z) = \frac{3Z^2+\frac{15}{2}Z+12}{\left(Z+\frac{1}{2}\right)(Z+2)}$$

$$\therefore \quad \frac{H(Z)}{Z} = \frac{3Z^2+\frac{15}{2}Z+12}{Z\left(Z+\frac{1}{2}\right)(Z+2)}$$

In the partial fraction expansion form we can write,

$$\frac{H(Z)}{Z} = \frac{A_1}{Z}+\frac{A_2}{Z+\frac{1}{2}}+\frac{A_3}{Z+2} \qquad \text{Here} \quad A_1 = \left.\frac{Z\left(3Z^2+\frac{15}{2}Z+12\right)}{Z\left(Z+\frac{1}{2}\right)(Z+2)}\right|_{Z=0}$$

$$\therefore \quad A_1 = \frac{12}{\frac{1}{2}(2)} = \mathbf{1}$$

$$A_2 = \left.\frac{\left(Z+\frac{1}{2}\right)\cdot\left(3Z^2+\frac{15}{2}Z+12\right)}{Z\left(Z+\frac{1}{2}\right)(Z+2)}\right|_{Z=-\frac{1}{2}}$$

$$\therefore \quad A_2 = \frac{3\left(\frac{1}{4}\right)+\left(-\frac{15}{4}\right)+12}{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}+2\right)} = \frac{\frac{3}{3}-\frac{15}{4}+12}{\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)}$$

$$\therefore \quad A_2 = \frac{-\frac{12+48}{4}}{-3/4} = \frac{36}{-3} = -12$$

$$\text{and} \quad A_3 = \frac{(Z+2)\left(3Z^2+\frac{15}{2}Z+12\right)}{Z\left(Z+\frac{1}{2}\right)(Z+2)}\Bigg|_{Z=-2}$$

$$\therefore \quad A_3 = \frac{(12-15+12)}{(-2)\left(-2+\frac{1}{2}\right)} = \frac{9}{(-2)\left(-\frac{3}{2}\right)}$$

$$\therefore \quad A_3 = \frac{9}{3} = 3$$

$$\therefore \quad \frac{H(Z)}{Z} = \frac{12}{Z} - \frac{12}{Z+\frac{1}{2}} + \frac{3}{Z+2} \qquad \therefore \quad H(Z) = 12 - \frac{12Z}{Z+\frac{1}{2}} + \frac{3Z}{Z+2}$$

$$\therefore \quad H(Z) = 12 - \frac{12}{1+\frac{1}{2}Z^{-1}} + \frac{3}{1+2Z^{-1}}$$

In the general form we can write,

$$H(Z) = H_1(Z) + H_2(Z) + H_3(Z)$$

Here $H_1(Z) = 12$ $\qquad H_2(Z) = -\dfrac{12}{1+\frac{1}{2}Z^{-1}}$

and $H_3(Z) = \dfrac{3}{1+\frac{1}{2}Z^{-1}}$

The general structure of parallel form realization is shown in Fig. I-26(a).

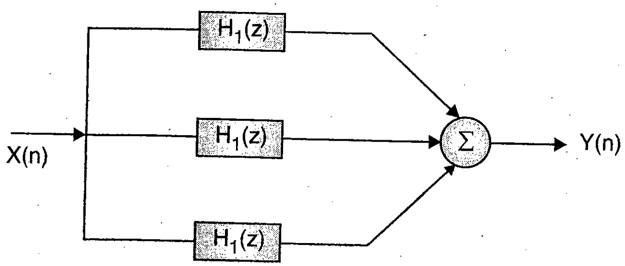


Fig. I-26(a)

Using short cut method, it can be realised as shown in Fig. I-26(b).

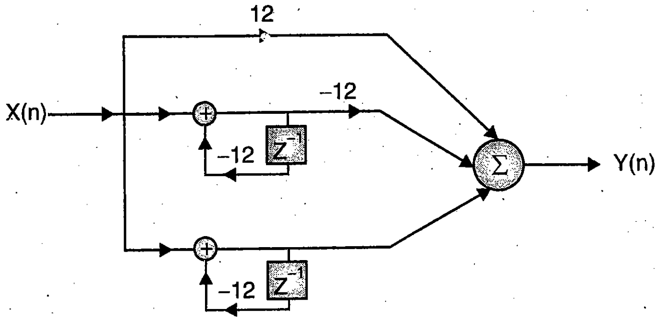


Fig. I-26(b)

(ii)
$$H(Z) = \frac{3Z(5Z-2)}{\left(Z+\frac{1}{2}\right)(3Z-1)} \qquad \therefore \frac{H(Z)}{Z} = \frac{3(5Z-2)}{\left(Z+\frac{1}{2}\right)(3Z-1)}$$

$$\therefore \frac{H(Z)}{Z} = \frac{15Z-6}{\left(Z+\frac{1}{2}\right)(3Z-1)}$$

Multiplying numerator and denominator by $\frac{1}{3}$ we get,

$$\frac{H(Z)}{Z} = \frac{5Z-2}{\left(Z+\frac{1}{2}\right)\left(Z-\frac{1}{3}\right)}$$

In partial fraction expansion from we can write,

$$\frac{H(Z)}{Z} = \frac{A_1}{Z+\frac{1}{2}} + \frac{A_2}{Z-\frac{1}{3}}$$

Here $$A_1 = \left(Z+\frac{1}{2}\right) \cdot \frac{(5Z-2)}{\left(Z+\frac{1}{2}\right)\left(Z-\frac{1}{3}\right)}\Bigg|_{Z=-1/2}$$

$$\therefore A_1 = \frac{-5/2-2}{-\frac{1}{2}-\frac{1}{3}} = \frac{-\frac{9}{2}}{-\frac{5}{6}} = \frac{27}{5}$$

and $$A_2 = \left(Z-\frac{1}{3}\right) \frac{(5Z-2)}{\left(Z+\frac{1}{2}\right)\left(Z-\frac{1}{3}\right)}\Bigg|_{Z=1/3}$$

$$\therefore A_2 = \frac{5/3-2}{\frac{1}{3}+\frac{1}{2}} \qquad \therefore A_2 = \frac{-\frac{1}{3}}{5/6} = -2/5$$

$$\therefore \frac{H(Z)}{Z} = \frac{27/5}{Z+\frac{1}{2}} - \frac{2/5}{Z-\frac{1}{3}}$$

$$\therefore \quad H(Z) = \frac{(27/5)Z}{Z+\frac{1}{2}} - \frac{(2/5)Z}{Z-\frac{1}{3}} \qquad \therefore \quad H(Z) = \frac{27/5}{1+\frac{1}{2}Z^{-1}} - \frac{2/5}{1-\frac{1}{3}Z^{-1}}$$

In the general form we can write,

$$H(Z) = H_1(Z) + H_2(Z)$$

Using short cut method it can be realized as shown in Fig. I-26(c).

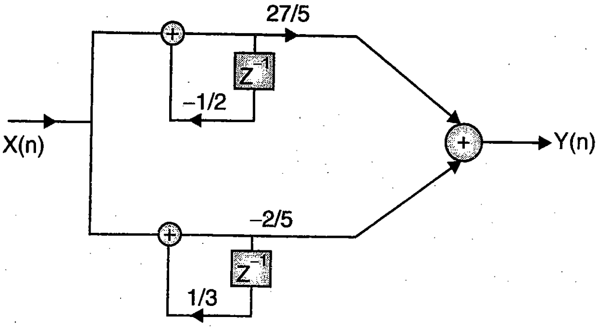


Fig. I-26(c)