# Transfer Function

## Modeling of Transfer Function Characteristic of RLC-Circuit

The transfer function is an important factor to consider in the design of any system because with the help of the transfer function one can describes the behavior of the output as a function of the input frequency. Therefore it's easy for us to control the system to achieve the output we wanted. It helps us decide the output of the system for every inputs, and to decide if the system is stable or not, to find out the resonance frequencies of the system, to optimize the system and to determine system behavior with all possible inputs. In other hand, modeling is the process of producing a model, that is a representation of the construction and working of propose system or existing it help in evaluation of the system.

For any control system, there exists a reference input known as excitation or cause which operates through a transfer operation (i.e. the transfer function) to produce an effect resulting in controlled output or response.

Thus the cause and effect relationship between the output and input is related to each other through a **transfer function**.


In a Laplace Transform, if the input is represented by R(s) and the output is represented by C(s), then the transfer function will be:

$$G(s) = \frac{C(s)}{R(s)} \Rightarrow R(s).G(s) = C(s)$$

That is, the transfer function of the system multiplied by the input function gives the output function of the system.

### What is a Transfer Function?

The transfer function of a control system is defined as the ratio of the Laplace transform of the output variable to Laplace transform of the input variable assuming all initial conditions to be zero.

$$G(s) = \frac{C(s)}{R(s)}$$

Procedure for determining the transfer function of a control system is as follows:

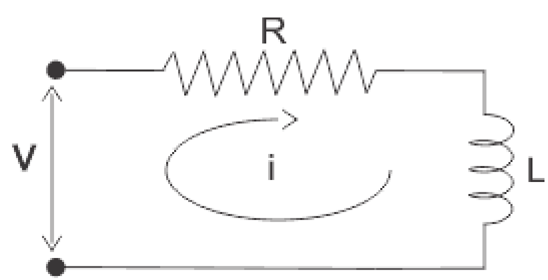
1. We form the equations for the system.
2. Now we take Laplace transform of the system equations, assuming initial conditions as zero.
3. Specify system output and input.
4. Lastly we take the ratio of the Laplace transform of the output and the Laplace transform of the input which is the required transfer function.

Hence a basic block diagram of a control system can be represented as


$$Where,\ R(s) = \pounds r(t),\ C(s) = \pounds c(t)\ \&\ G(s) = \frac{\pounds c(t)}{\pounds r(t)}$$

## Concept of Transfer Function

The transfer function is generally expressed in Laplace Transform and it is nothing but the relation between input and output of a system. Let us consider a system consists of a series connected resistance (R) and inductance (L) across a voltage source (V).




In this circuit, the current 'i' is the response due to applied voltage (V) as cause. Hence the voltage and current of the circuit can be considered as input and output of the system respectively.

From the circuit, we get,

$$V = Ri + L\frac{di}{dt}$$

Now applying Laplace Transform, we get,

Transfer Function

$$V(s) = RI(s) + L\left[sI(s) - i(0^+)\right]$$

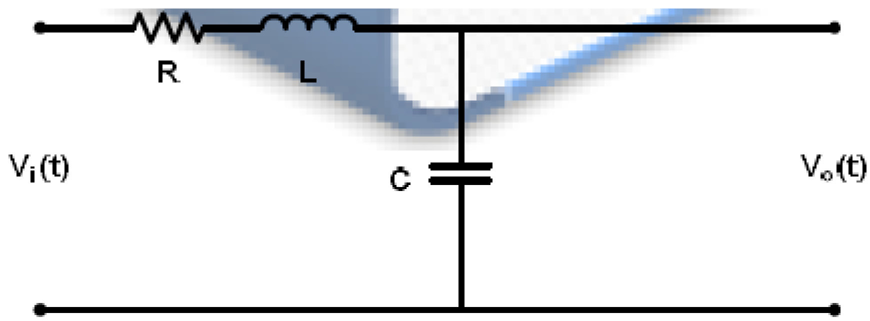
$$\left[\because \textit{Initially inductor behaves as open, hence}, i(0^+) = 0\right]$$

$$\Rightarrow V(s) = I(s)\left[R + Ls\right]$$

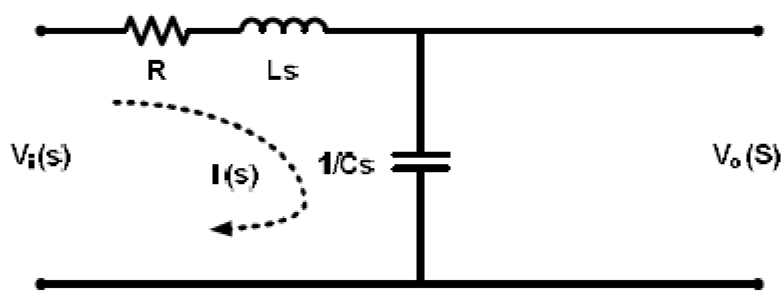
$$\Rightarrow \frac{I(s)}{V(s)} = \frac{1}{R + Ls} = \frac{1/L}{s + R/L}$$

The transfer function of the system, G(s) = I(s)/V(s), the ratio of output to input.

Example: Determine the transfer function of the given fig.




Solution: The frequency domain fig is redrawn to:




Applying KVL to loop-1

$$V_i(s) = \left(R + Ls + \frac{1}{Cs}\right)I(s) \qquad -(1)$$

Applying KVL to loop-2

$$V_o(s) = \left(\frac{1}{Cs}\right)I(s) \qquad -(2)$$

Transfer Function

From above eq

$$I(s) = V_o(s) / \left(\frac{1}{Cs}\right) = CsV_o(s)$$

-(3)

Using eq 3 in eq 1:

$$V_i(s) = \left(R + Ls + \frac{1}{Cs}\right)CsV_o(s)$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{\left(R + Ls + \frac{1}{Cs}\right)Cs} = \frac{1}{LCs^2 + RCs + 1}$$

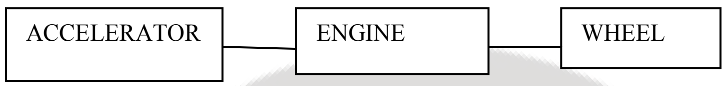
Then transfer function of the given system is

$$G(s) = \frac{1}{LCs^2 + RCs + 1}$$

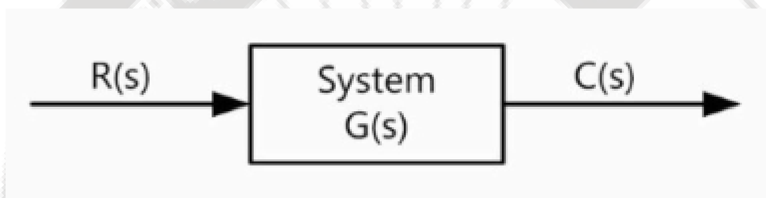
# BLOCK DIAGRAM REDUCTION ALGEBRA FOR TRANSFER FUNCTION

Block diagram is representing each component of a system by its transfer function and then connecting them to mob the system..

Example: Block diagram of an automobile as given below: which is been assembled.




## Open loop control system

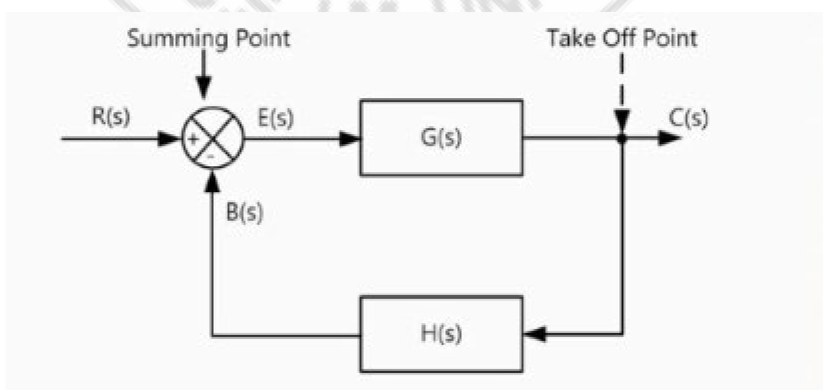



R(s) = Excitation

C(s) = Response

$$G(s) = Transfer\ function = C(s)/R(s)$$

## Close loop control system




G(s) = Forward path
H(s) = Feedback path

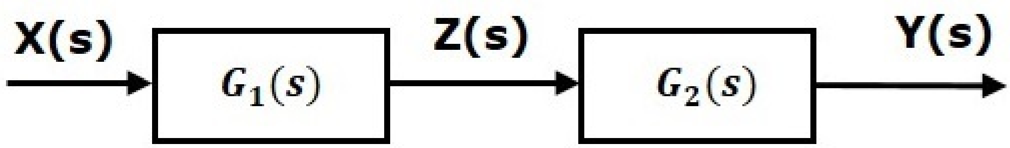
E(s) = Error signal
B(s) = Feedback signal

# Basic Connections for Blocks

There are three basic types of connections between two blocks.

- **Series Connection**

Series connection is also called **cascade connection**. In the following figure, two blocks having transfer function G1(s) and G2(s) are connected in series.



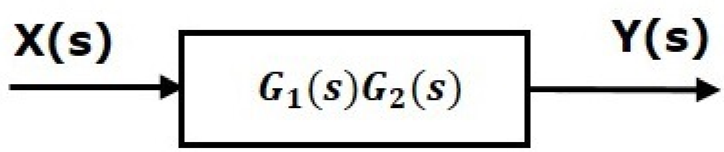

For this combination, we will get the output Y(s) as

$$Y(s) = G2(s)Z(s)$$

Where,

$$Z(s) = G1(s)X(s)$$
$$\Rightarrow Y(s) = \{G1(s)G2(s)\}X(s)$$

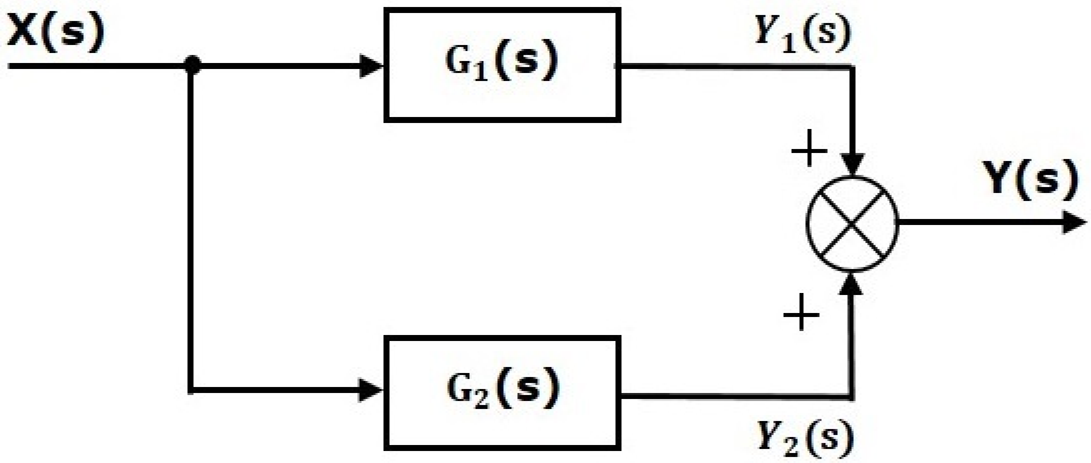
That means we can represent the **series connection** of two blocks with a single block. The transfer function of this single block is the **product of the transfer functions** of those two blocks. The equivalent block diagram is shown below.




Similarly, you can represent series connection of 'n' blocks with a single block. The transfer function of this single block is the product of the transfer functions of all those 'n' blocks.

- **Parallel Connection**

The blocks which are connected in **parallel** will have the **same input**. In the following figure, two blocks having transfer functions G1(s) and G2(s) are connected in parallel. The outputs of these two blocks are connected to the summing point.



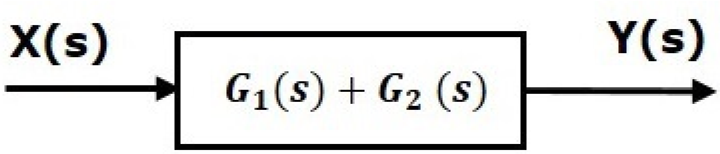

For this combination, we will get the output Y(s) as

$$Y(s) = Y1(s) + Y2(s)$$

$$Where,\ Y1(s) = G1(s)X(s)\ and\ Y2(s) = G2(s)X(s)$$

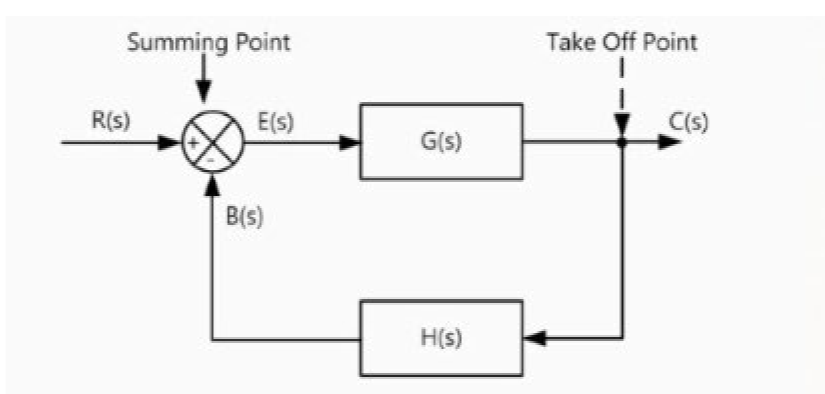
$$\Rightarrow Y(s) = \{G1(s) + G2(s)\}X(s)$$

That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.




Similarly, you can represent parallel connection of 'n' blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those 'n' blocks.

- <u>**FeedBack Connection**</u>



$$B(S) = C(s)H(s)$$

$$E(s) = R(s) - B(s)$$

$$C(s) = E(s)G(s) = [R(s) - B(s)]G(s) = [R(s) - C(s)H(s)]G(s)$$

$$C(s)/R(s) = G(s)/1 + G(s)H(s)$$

# Block Diagram Algebra for Summing Points

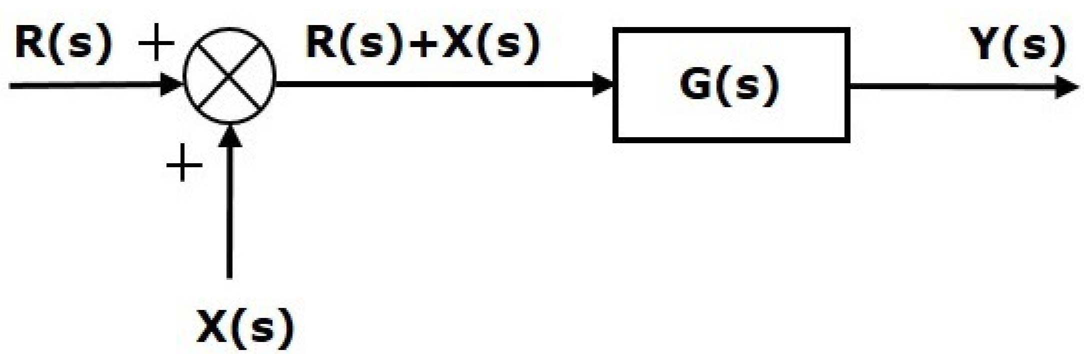
There are two possibilities of shifting summing points with respect to blocks −

- Shifting summing point after the block
- Shifting summing point before the block

Let us now see what kind of arrangements need to be done in the above two cases one by one.

- **Shifting Summing Point After the Block**

Consider the block diagram shown in the following figure. Here, the summing point is present before the block.

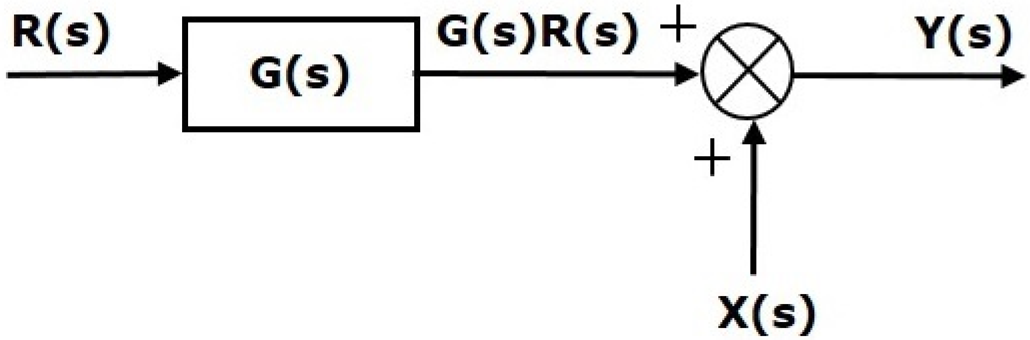



Summing point has two inputs R(s) & X(s) The output of it is $\{R(s) + X(s)\}$.
So, the input to the block $G(s)$ is $\{R(s) + X(s)\}$ and the output of it is –

$$Y(s) = G(s)\{R(s) + X(s)\}$$

$$\Rightarrow Y(s) = G(s)R(s) + G(s)X(s) \quad \textbf{(Equation 1)}$$

Now, shift the summing point after the block. This block diagram is shown in the following figure.



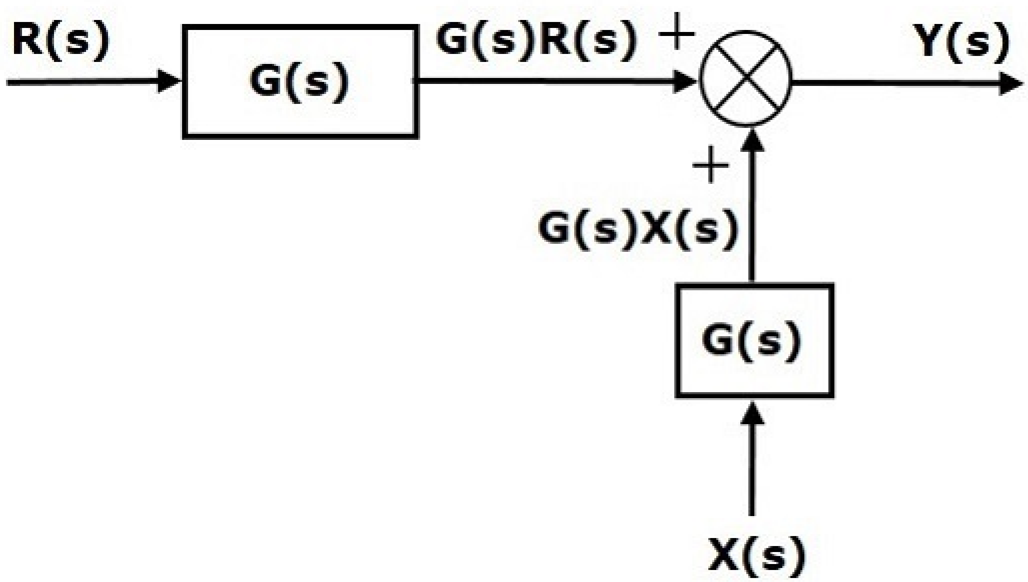

Output of the block $G(s)$ is $G(s)R(s)$.

The output of the summing point is

$$Y(s) = G(s)R(s) + X(s) \quad \textbf{(Equation 2)}$$

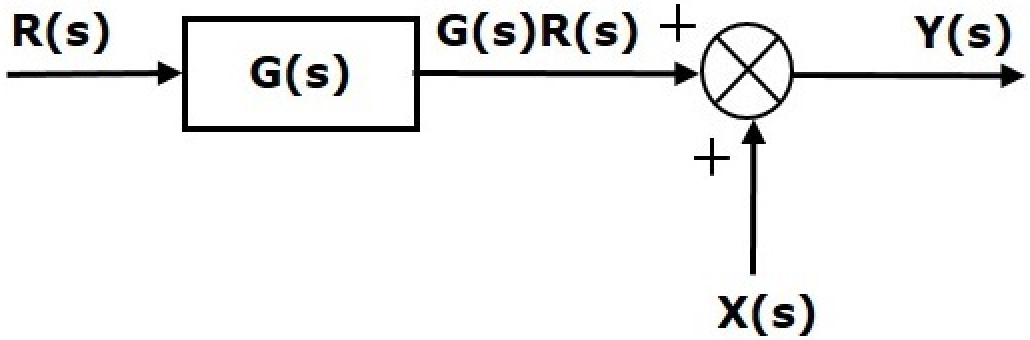
Compare Equation 1 and Equation 2.

The first term $G(s)R(s)$ is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block $G(s)$. It is having the input $X(s)$ and the output of this block is given as input to summing point instead of $X(s)$. This block diagram is shown in the following figure.




- **<u>Shifting Summing Point Before the Block</u>**

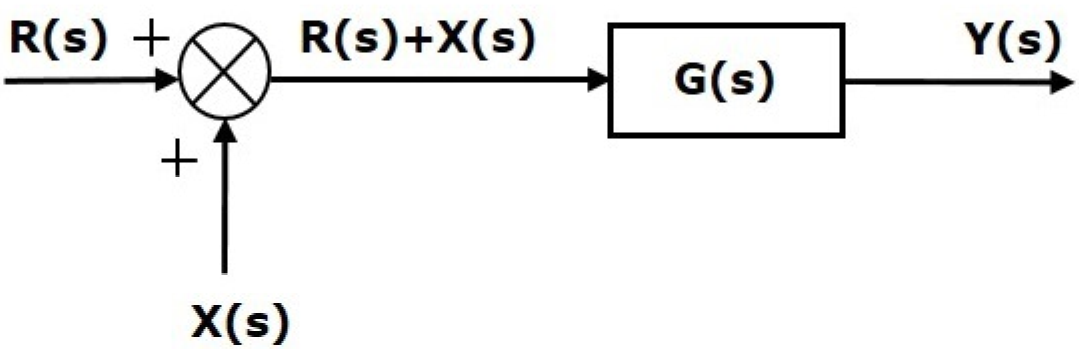
Consider the block diagram shown in the following figure. Here, the summing point is present after the block.




Output of this block diagram is -

$$Y(s) = G(s)R(s) + X(s) \quad \textbf{(Equation 3)}$$

Now, shift the summing point before the block. This block diagram is shown in the following figure.

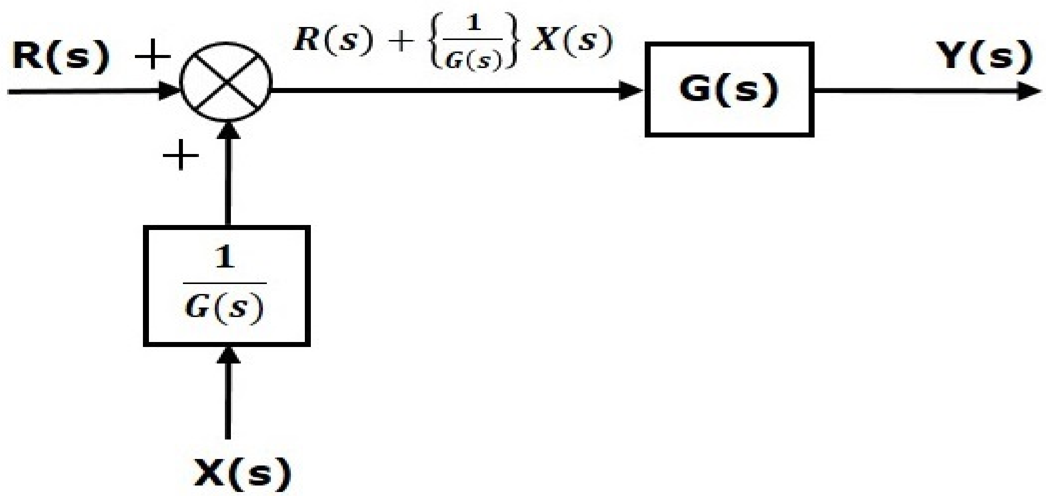



Output of this block diagram is -

$$Y(S) = G(s)R(s) + G(s)X(s) \quad \textbf{(Equation 4)}$$

Compare Equation 3 and Equation 4,

The first term $G(s)R(s)$ is same in both equations. But, there is difference in the second term. In order to get the second term also same, we require one more block $1/G(s)$. It is having the input $X(s)$ and the output of this block is given as input to summing point instead of $X(s)$. This block diagram is shown in the following figure.



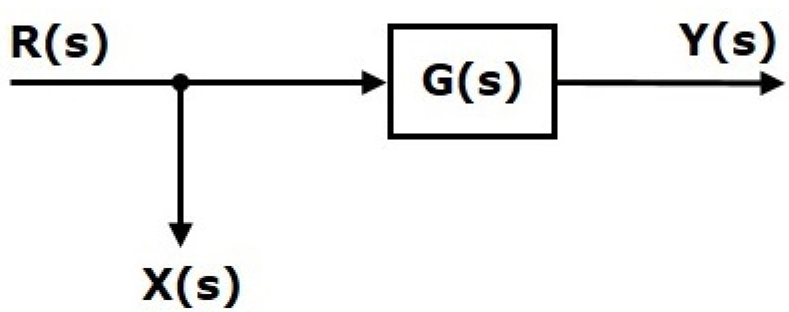

# Block Diagram Algebra for Take-off Points

There are two possibilities of shifting the take-off points with respect to blocks −

- Shifting take-off point after the block
- Shifting take-off point before the block

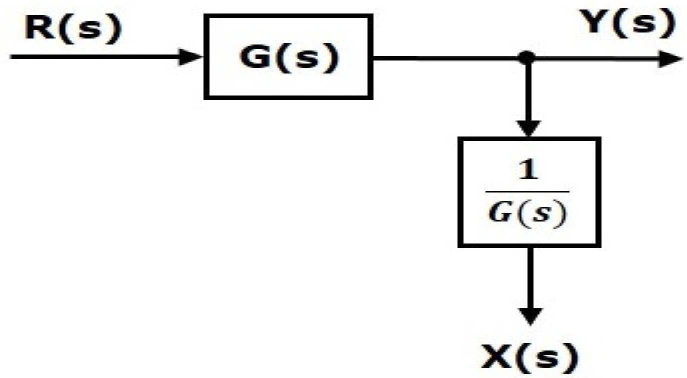
- **Shifting Take-off Point After the Block**

Consider the block diagram shown in the following figure. In this case, the take-off point is present before the block.



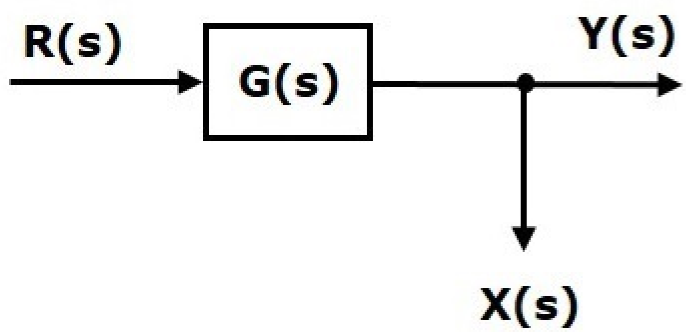

Here, $X(s) = R(s)$
and $Y(s) = G(s)R(s)$

When you shift the take-off point after the block, the output Y(s) will be same. But, there is difference in X(s) value. So, in order to get the same X(s) value, we require one more block $1/G(s)$. It is having the input Y(s) and the output is X(s). This block diagram is shown in the following figure.



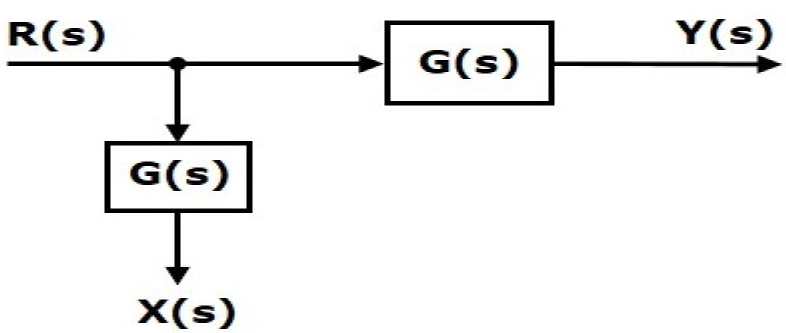

- **<u>Shifting Take-off Point Before the Block</u>**

Consider the block diagram shown in the following figure. Here, the take-off point is present after the block.



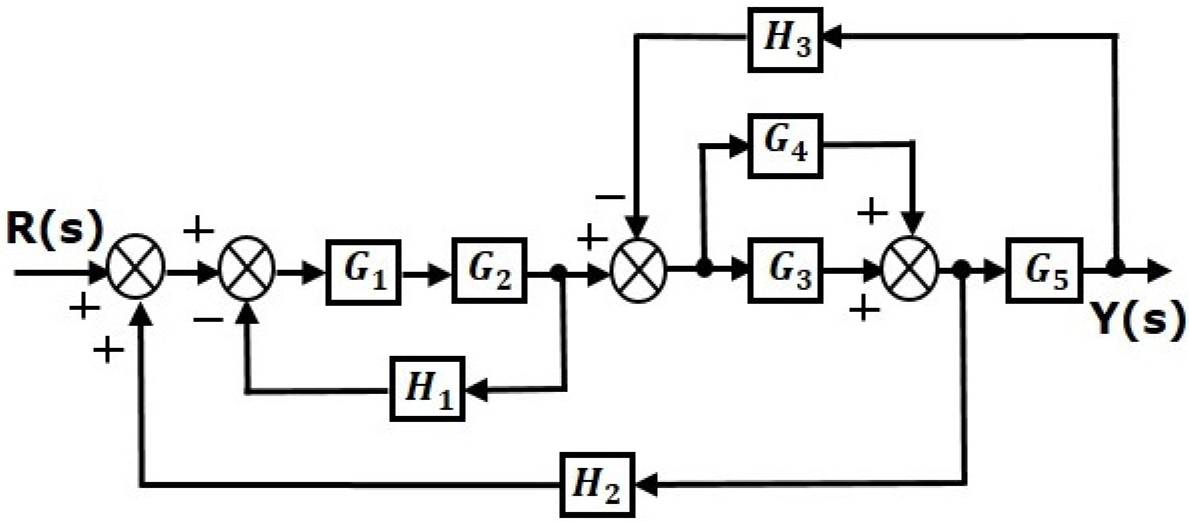

Here, $X(s) = Y(s) = G(s)R(s)$

When you shift the take-off point before the block, the output Y(s) will be same. But, there is difference in X(s) value. So, in order to get same X(s) value, we require one more block G(s). It is having the input R(s) and the output is X(s). This block diagram is shown in the following figure.

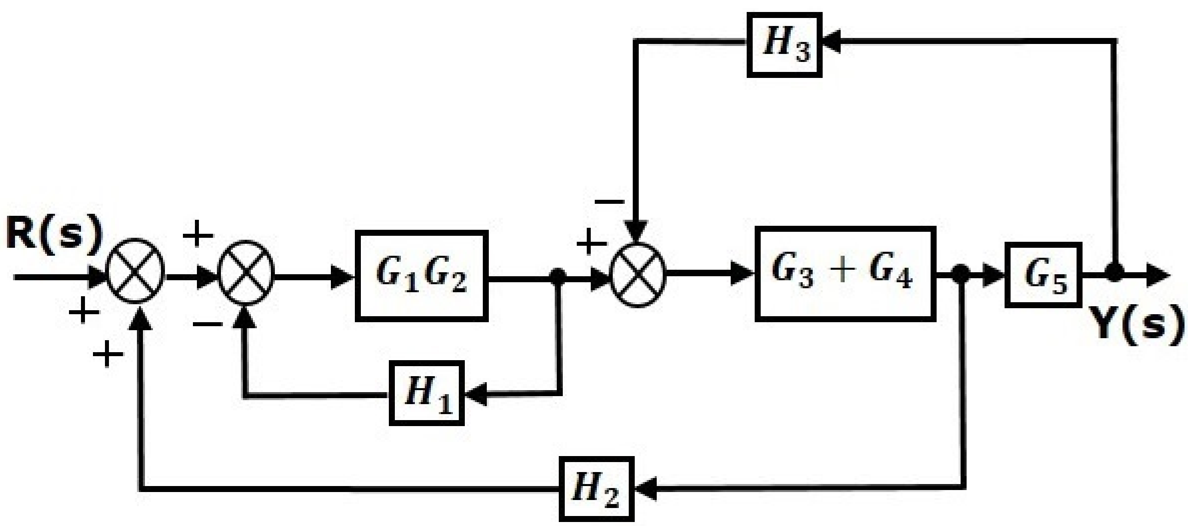


## Example

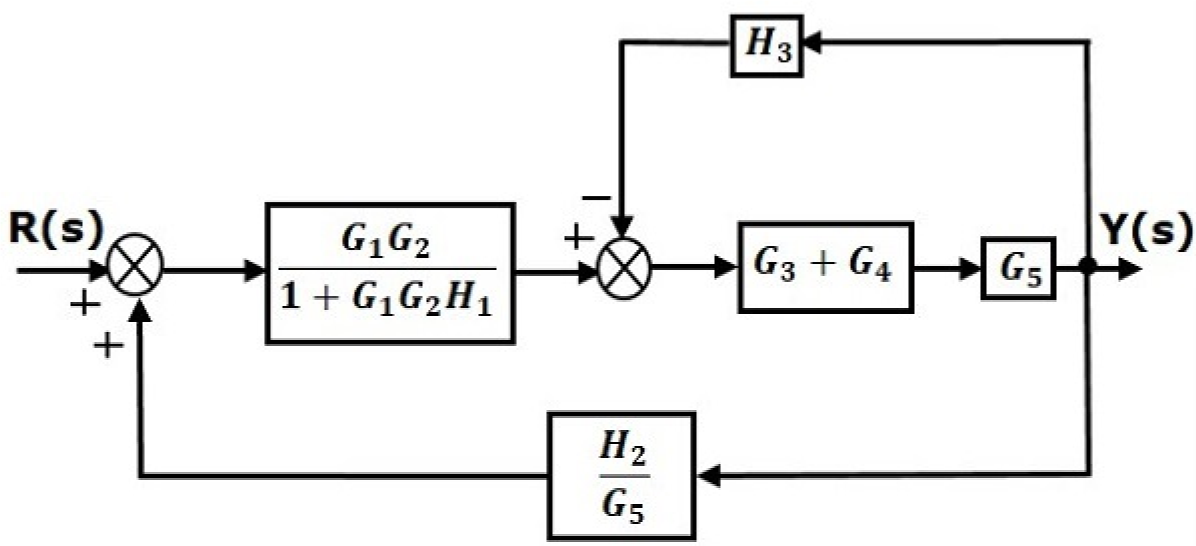
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



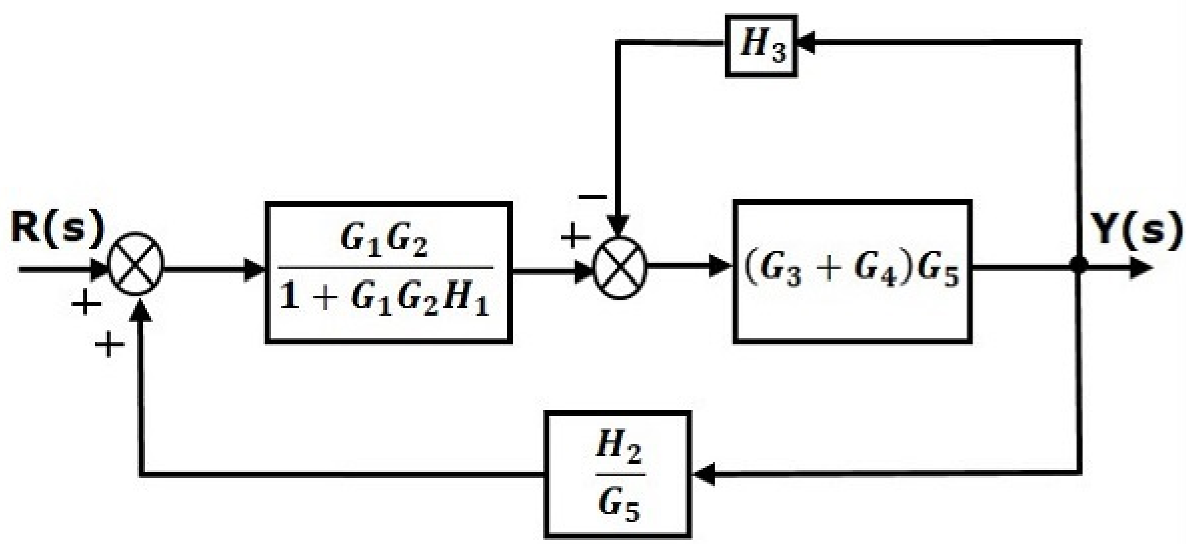

**Step 1** − Use Rule 1 for blocks $G_1$ and $G_2$. Use Rule 2 for blocks $G_3$ and $G_4$. The modified block diagram is shown in the following figure.



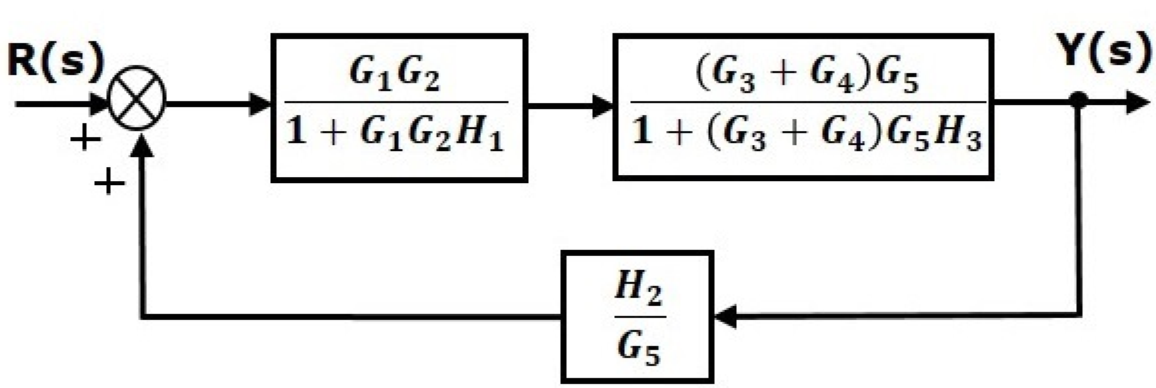

**Step 2** − Use Rule 3 for blocks $G_1G_2$ and $H_1$. Use Rule 4 for shifting take-off point after the block $G_5$. The modified block diagram is shown in the following figure.



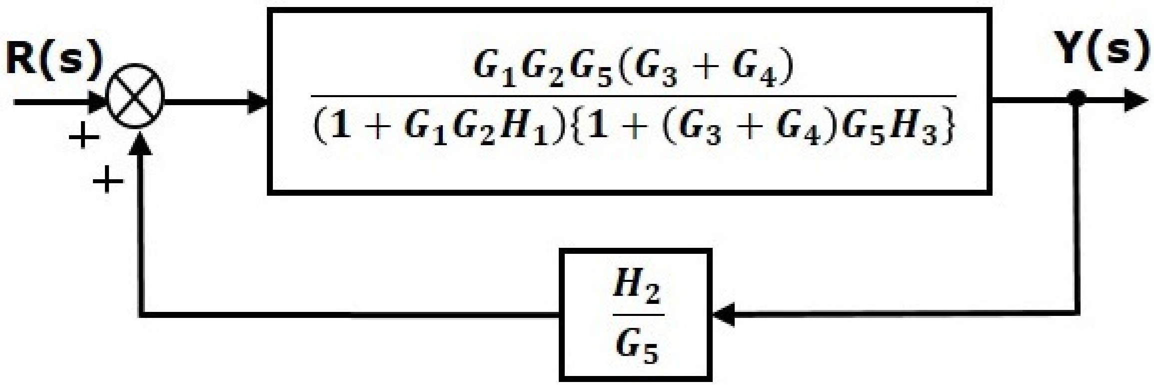

**Step 3** − Use Rule 1 for blocks ($G_3$+$G_4$) and $G_5$. The modified block diagram is shown in the following figure.



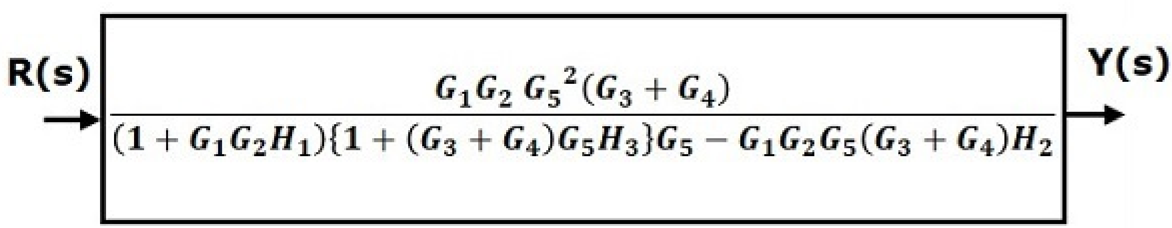

**Step 4** − Use Rule 3 for blocks ($G_3$+$G_4$)$G_5$ and $H_3$. The modified block diagram is shown in the following figure.



**Step 5** − Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.




**Step 6** − Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.




Therefore, the transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2{G_5}^2(G_3 + G_4)}{(1 + G_1G_2HS_1)\{1 + (G_3 + G_4)G_5H_3\}G_5 - G_1G_2G_5(G_3 + G_4)H_2}$$