Fundamental of Block Diagram Algebra

Introduction

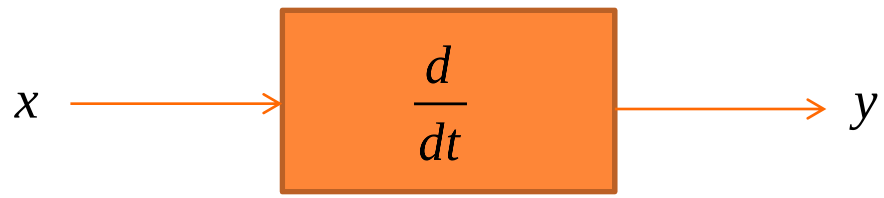
- Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

OR

- A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.

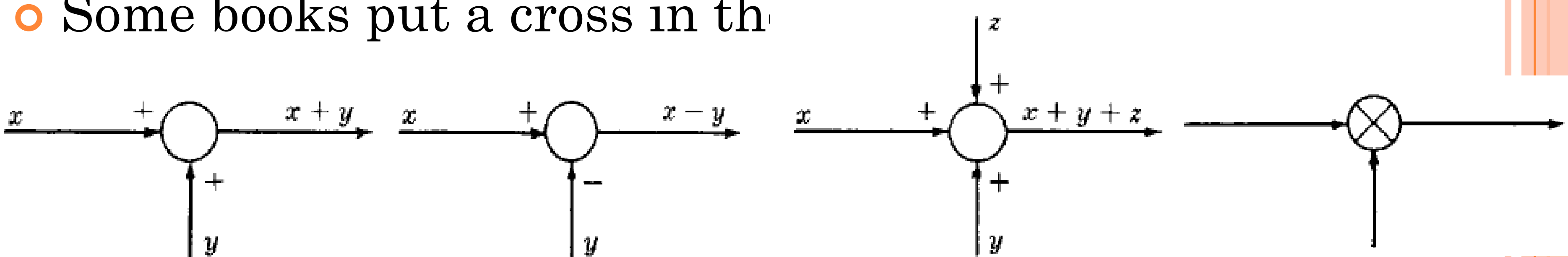
INTRODUCTION

- The simplest form of the block diagram is the single ***block, with one input and one output.***
- The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the output.
- The arrows represent the direction of information or signal flow.



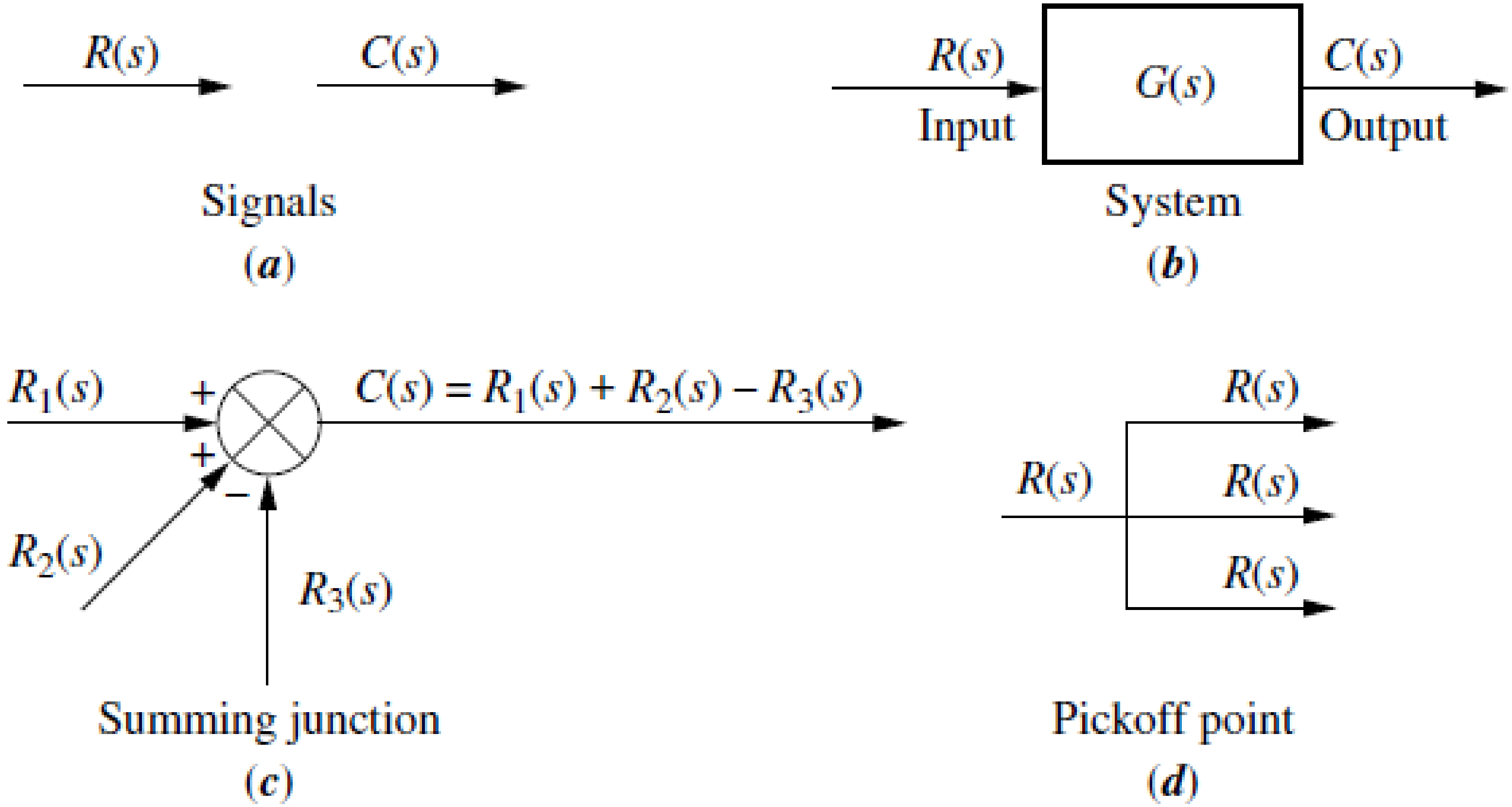
INTRODUCTION

- The operations of addition and subtraction have a special representation.
- The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.
- Any number of inputs may enter a summing point.
- The output is the algebraic sum of the inputs.
- Some books put a cross in th

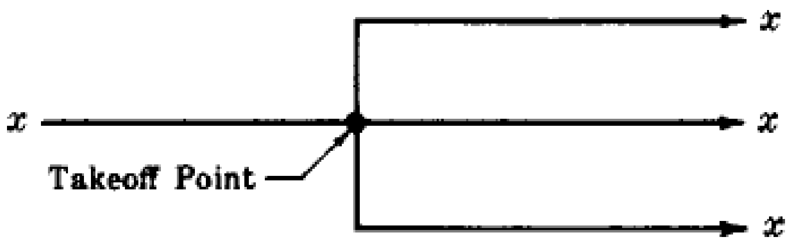


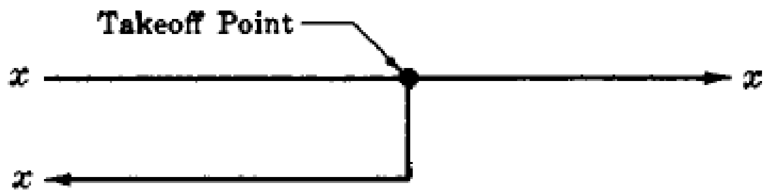
Components of a Block Diagram for a Linear Time Invariant System

- System components are alternatively called elements of the system.
- Block diagram has four components:
  - ***Signals***
  - ***System/ block***
  - ***Summing junction***
  - ***Pick-off/ Take-off point***


R(s)
C(s)
Signals
(a)
R(s)
Input
G(s)
C(s)
Output
System
(b)
R1(s)
+
+
−
R2(s)
R3(s)
C(s) = R1(s) + R2(s) − R3(s)
Summing junction
(c)
R(s)
R(s)
R(s)
R(s)
Pickoff point
(d)

- In order to have the same signal or variable be an input to more than one block or summing point, a takeoff point is used.
- Distributes the input signal, undiminished, to several output points.
- This permits the signal to proceed unaltered along several different paths to several destinations.

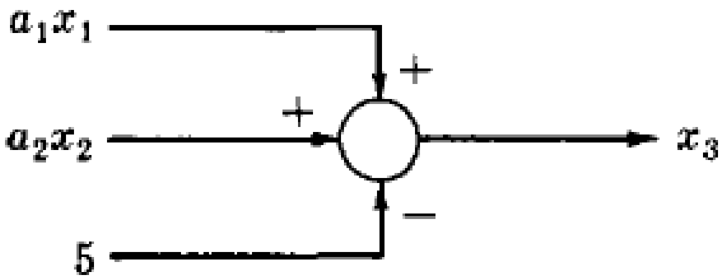


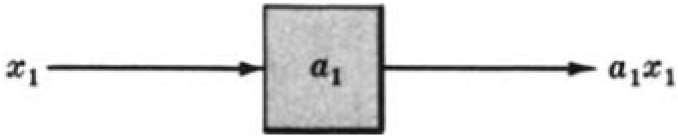


EXAMPLE-1

- Consider the following equations in which x_1, x_2, x_3, are variables, and *a_1, a_2 are* general coefficients or mathematical operators.

$$x_3 = a_1 x_1 + a_2 x_2 - 5$$





EXAMPLE-1

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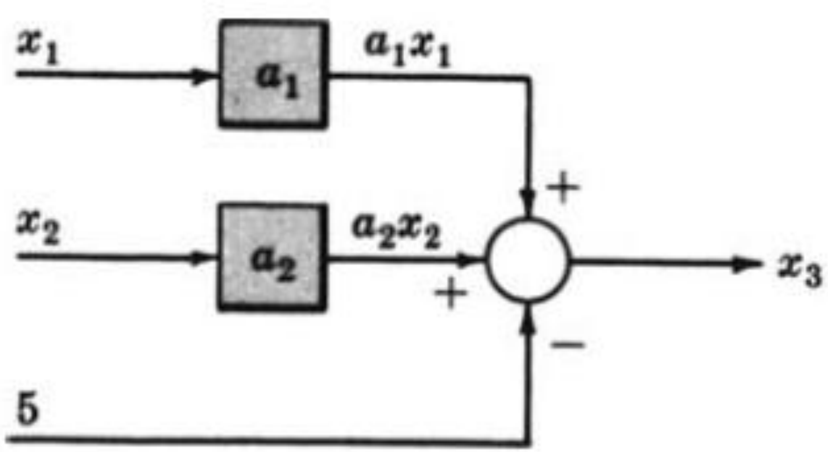
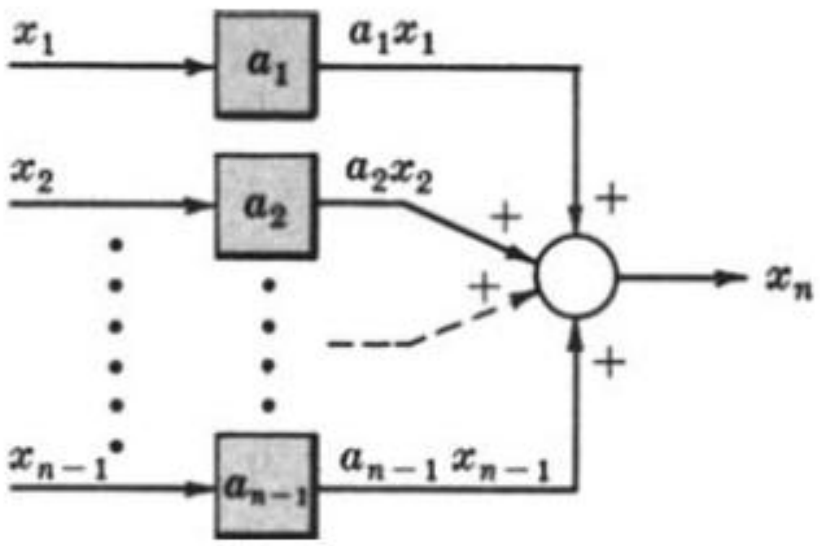


Example-2

- Consider the following equations in which x_1, x_2,. . . , x_n, are variables, and a_1, a_2,. . . , a_n , *are* general coefficients or mathematical operators.

$$x_n = a_1 x_1 + a_2 x_2 + a_{n-1} x_{n-1}$$



Example-3

- Draw the Block Diagrams of the following equations.

(1) $$x_2 = a_1 \frac{dx_1}{dt} + \frac{1}{b} \int x_1 dt$$

(2) $$x_3 = a_1 \frac{d^2 x_2}{dt^2} + 3 \frac{dx_1}{dt} - bx_1$$

Topologies

- We will now examine some common topologies for interconnecting subsystems and derive the single transfer function representation for each of them.
- These common topologies will form the basis for reducing more complicated systems to a single block.

CASCADE

- Any finite number of blocks in series may be algebraically combined by multiplication of transfer functions.
- That is, *n components or blocks with transfer functions* G_1 *,* G_2*, . . . ,* G_n*, connected in cascade* are equivalent to a single element G with a transfer function given by

$$G = G_1 \cdot G_2 \cdot G_3 \cdots G_n = \prod_{i=1}^{n} G_i$$

Example

- Multiplication of transfer functions is *commutative; that is,*

$$GiGj = GjGi$$

for any i or j .

Cascade:

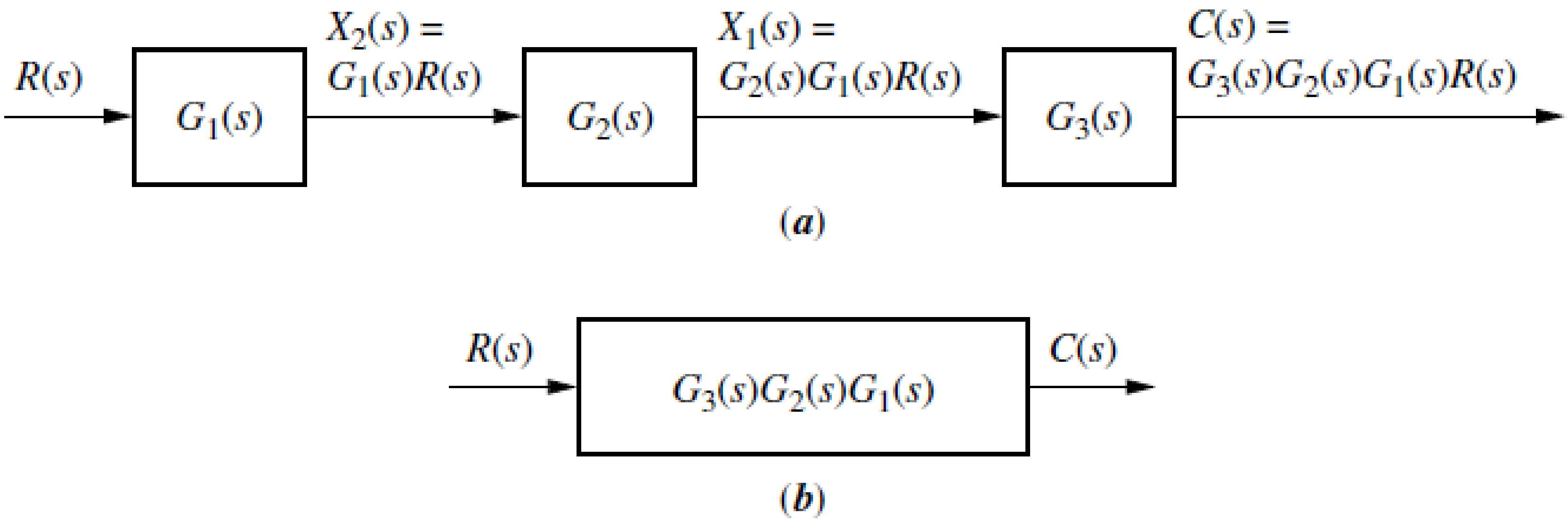


Figure:
a) Cascaded Subsystems.
b) Equivalent Transfer Function.

The equivalent transfer function is

$$G_e(s) = G_3(s)G_2(s)G_1(s)$$

Parallel Form:

- Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.

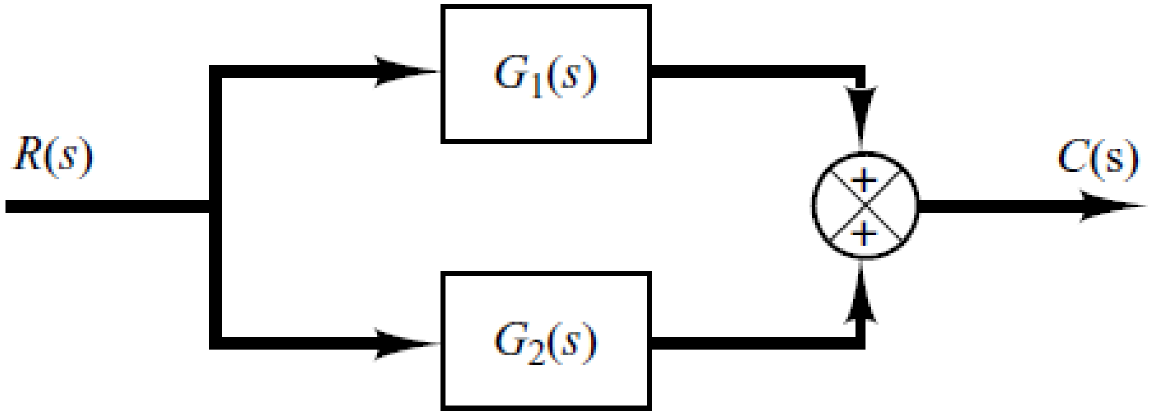


Figure: Parallel Subsystems.

PARALLEL FORM:

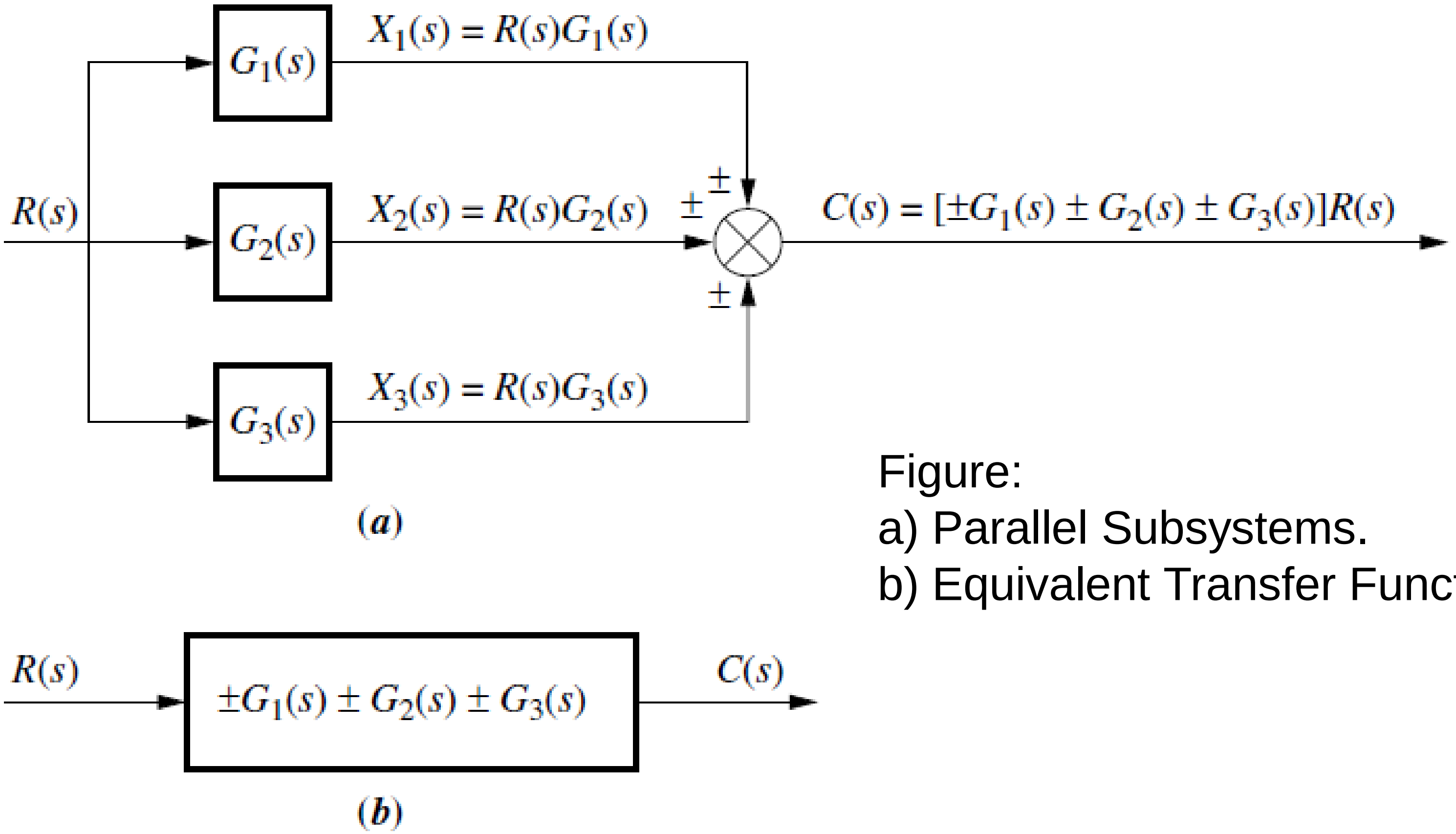


Figure:
a) Parallel Subsystems.
b) Equivalent Transfer Function.

The equivalent transfer function is

$$G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$

Feedback Form:

- The third topology is the feedback form. Let us derive the transfer function that represents the system from its input to its output. The typical feedback system, shown in figure:

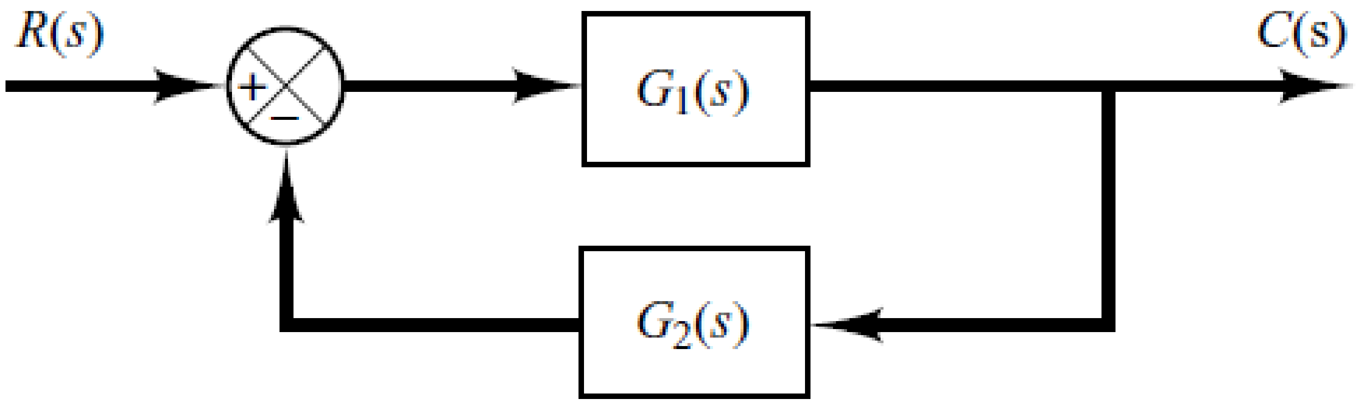
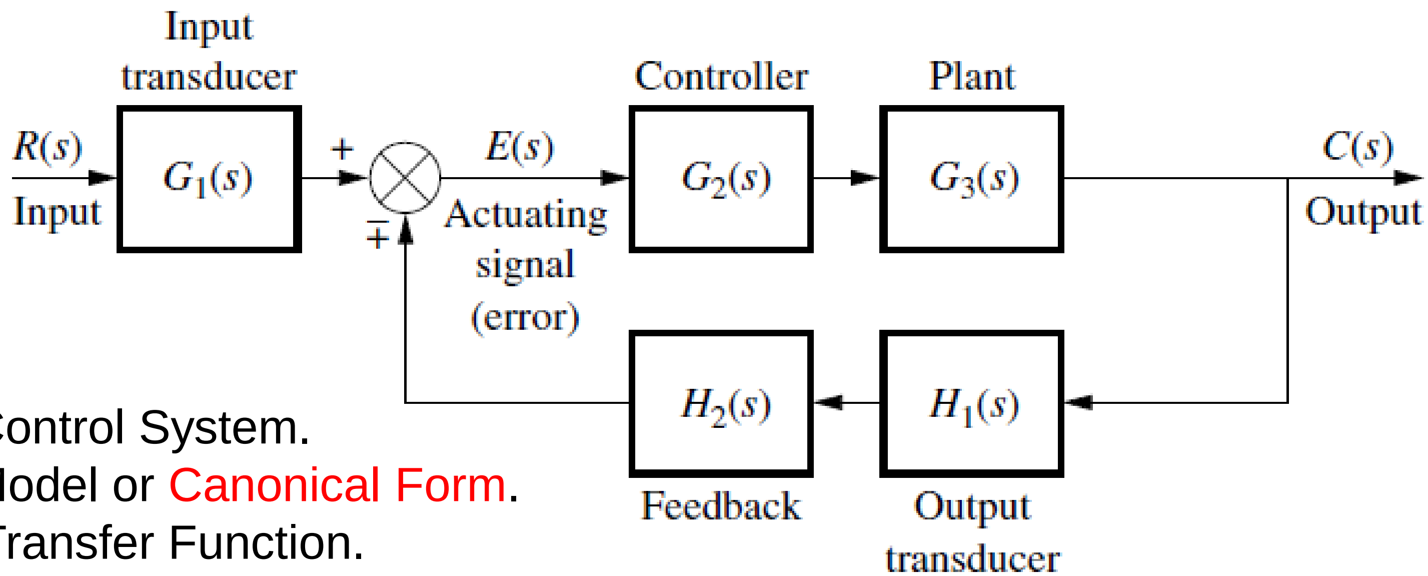


Figure: Feedback (Closed Loop) Control System.

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

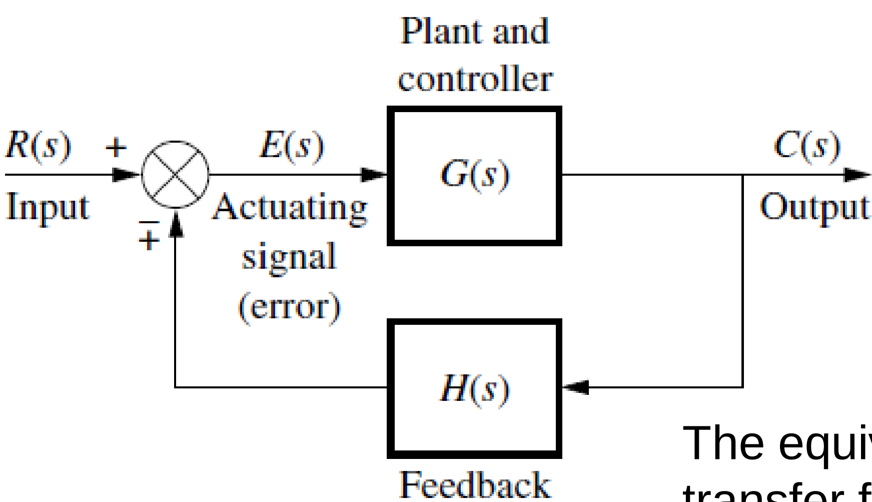
FEEDBACK FORM:



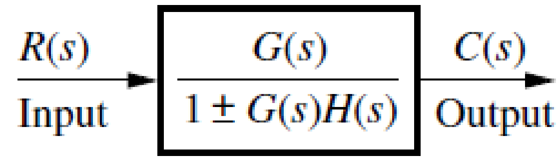
(*a*)

Figure:

a) Feedback Control System.
b) Simplified Model or Canonical Form.
c) Equivalent Transfer Function.



(*b*)



(*c*)

The equivalent or closed-loop transfer function is

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Characteristic Equation

- The control ratio is the closed loop transfer function of the system.

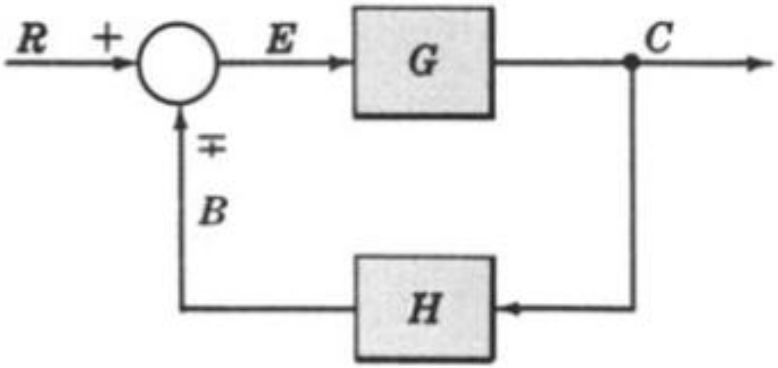
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

- The denominator of closed loop transfer function determines the characteristic equation of the system.

- Which is usually determined as:

$$1 \pm G(s)H(s) = 0$$

Canonical Form of a Feedback Control System



$G \equiv$ direct transfer function $\equiv$ forward transfer function

$H \equiv$ feedback transfer function

$GH \equiv$ loop transfer function $\equiv$ open-loop transfer function

$C/R \equiv$ closed-loop transfer function $\equiv$ control ratio $\quad \frac{C}{R} = \frac{G}{1 \pm GH}$

$E/R \equiv$ actuating signal ratio $\equiv$ error ratio $\quad \frac{E}{R} = \frac{1}{1 \pm GH}$

$B/R \equiv$ primary feedback ratio $\quad \frac{B}{R} = \frac{GH}{1 \pm GH}$

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

1. Open loop transfer function $\frac{B(s)}{E(s)} = G(s)H(s)$

2. Feed Forward Transfer function $\frac{C(s)}{E(s)} = G(s)$

3. control ratio $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

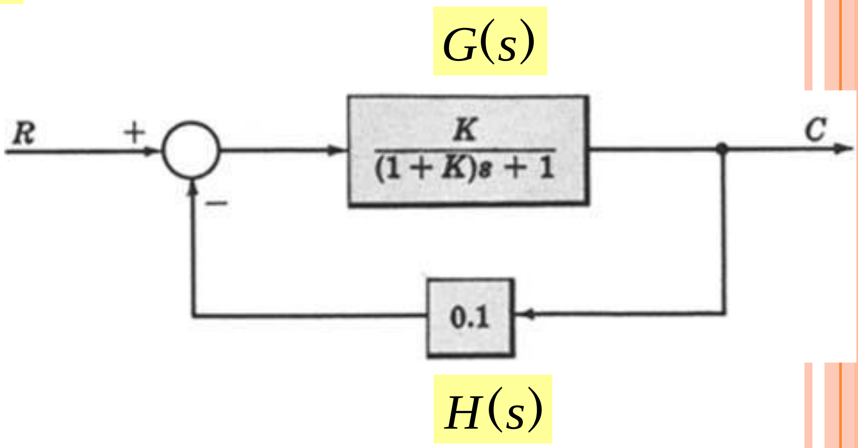
4. feedback ratio $\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$

5. error ratio $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$

6. closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

7. characteristic equation $1+G(s)H(s) = 0$

8. closed loop poles and zeros if K=10.



Characteristic Equation

$C/R \equiv$ closed-loop transfer function $\equiv$ control ratio

$$\frac{C}{R} = \frac{G}{1 \pm GH}$$

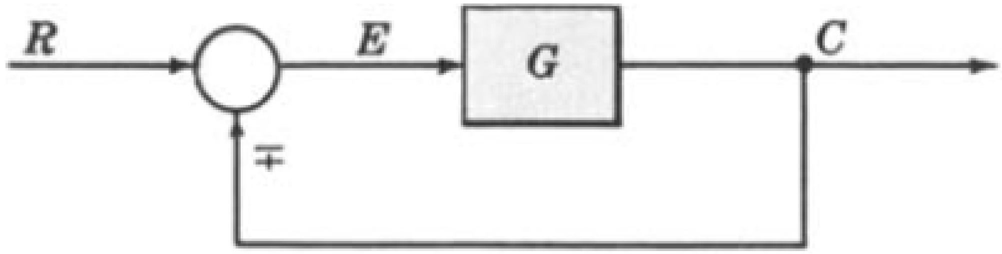
The denominator of C/R determines the *characteristic equation* of the system, which is usually determined from $1 \pm GH = 0$ or, equivalently,

$$D_{GH} \pm N_{GH} = 0$$

where D_{GH} is the denominator and N_{GH} is the numerator of GH

UNITY FEEDBACK SYSTEM

A **unity feedback system** is one in which the primary feedback b is identically equal to the controlled output $H = 1$ for a linear, unity feedback system



Any feedback system with only linear time-invariant elements can be put into the form of a unity feedback system by using Transformation 5.