## BLOCK DIAGRAM REDUCTION TECHNIQUES

## Reduction techniques

1. Combining blocks in cascade

2. Combining blocks in parallel


## Reduction techniques

3. Moving a summing point behind a block


## Reduction techniques

3. Moving a summing point ahead of a block

4. Moving a pickoff point behind a block

5. Moving a pickoff point ahead of a block


## Reduction techniques

6. Eliminating a feedback loop

7. Swap with two neighboring summing points


## Block Diagram Transformation

 Theorems|  | Transformation | Equation | Block Diagram | Equivalent Block Diagram |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Combining Blocks in Cascade | $Y=\left(P_{1} P_{2}\right) X$ |  |  |
| 2 | Combining Blocks in Parallel; or Eliminating a Forward Loop | $Y=P_{1} X \pm P_{2} X$ |  | $\xrightarrow{X} P_{1} \pm P_{2} \xrightarrow{Y}$ |
| 3 | Removing a Block from a Forward Path | $Y=P_{1} X \pm P_{2} X$ |  |  |
| 4 | Eliminating a Feedback Loop | $Y=P_{1}\left(X \mp P_{2} Y\right)$ |  |  |
| 5 | Removing a Block from a Feedback Loop | $Y=P_{1}\left(X \mp P_{2} Y\right)$ |  |  |

The letter $\boldsymbol{P}$ is used to represent any transfer function, and $W, X$, Y, Z denote any transformed signals.

## Transformation Theorems Continue:

|  | Transformation | Equation | Block Diagram | Equivalent Block Diagram |
| :---: | :---: | :---: | :---: | :---: |
| 6a | Rearranging Summing Points | $Z=W \pm X \pm Y$ |  |  |
| 6b | Rearranging Summing Points | $Z=W \pm X \pm Y$ |  |  |
| 7 | Moving a Summing Point Ahead of a Block | $Z=P X \pm Y$ |  |  |
| 8 | Moving a Summing Point Beyond a Block | $Z=P[X \pm Y]$ |  |  |

## Transformation Theorems Continue:

|  | Transformation | Equation | Block Diagram | Equivalent Block Diagram |
| :---: | :---: | :---: | :---: | :---: |
| 9 | Moving a Takeoff Point Ahead of a Block | $Y=P X$ |  |  |
| 10 | Moving a Takeoff Point Beyond a Block | $Y=P X$ |  |  |
| 11 | Moving a Takeoff Point Ahead of a Summing Point | $Z=X \pm Y$ |  |  |
| 12 | Moving a Takeoff Point Beyond a Summing Point | $Z=X \pm Y$ |  |  |

## Reduction of Complicated Block DIAGRAMS:

The block diagram of a practical feedback control system is often quite complicated. It may include several feedback or feedforward loops, and multiple inputs. By means of systematic block diagram reduction, every multiple loop linear feedback system may be reduced to canonical form.

The following general steps may be used as a basic approach in the reduction of complicated block diagrams.

Step 1: Combine all cascade blocks using Transformation 1.
Step 2: Combine all parallel blocks using Transformation 2.
Step 3: Eliminate all minor feedback loops using Transformation 4.
Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12.
Step 5: Repeat Steps $\sqrt{ }$ to 4 until the canonical form has been achieved for a particular input.
Step 6: Repeat Steps 1 to 5 for each input, as required.

## Example-4: Reduce the Block Diagram to

 Canonical Form.

Step 1: Combine all cascade blocks using Transformation 1.


Step 2: Combine all parallel blocks using Transformation 2.


## EXAMPLE-4: CONTINUE.

Step 3: Eliminate all minor feedback loops using Transformation 4.


Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12. However in this example step-4 does not apply.

Step 5: Repeat Steps $\sqrt{ }$ to 4 until the canonical form has been achieved for a particular input.


Step 6: Repeat Steps 1 to 5 for each input, as required. However in this example step-6 does not apply.

## Example-5: Simplify the Block Diagram.



By moving the summing point of the negative feedback loop containing $\mathrm{H}_{2}$ outside the positive feedback loop containing $H_{1}$, we obtain Figure


## Example-5: Continue.

Eliminating the positive feedback loop, we have


The elimination of the loop containing $H_{2} / G_{1}$ gives


Finally, eliminating the feedback loop results in


## Example-6: Reduce the Block Diagram.



First, to eliminate the loop $G_{3} G_{4} H_{1}$, we move $H_{2}$ behind block $G_{4}$


Eliminating the loop $G_{3} G_{4} H_{1}$ we obtain

## EXAMPLE-6: CONTINUE.



Then, eliminating the inner loop containing $H_{2} / G_{4}$, we obtain


Finally, by reducing the loop containing $H_{3}$, we obtain


## Example-7: Reduce the Block Diagram.



First, the three summing junctions can be collapsed into a single summing junction,


## Example-7: CONTINUE.

Second, recognize that the three feedback functions, $H_{1}(s), \mathrm{H}_{2}(s)$, and $H_{3}(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Alsorecognize that $G_{2}(s)$ and $G_{3}(s)$ are connected in cascade.


Finally, the feedback system is reduced and multiplied by $G_{1}(s)$ to yield the equivalent transfer function shown in Figure


