Electrical Analogies of Mechanical System

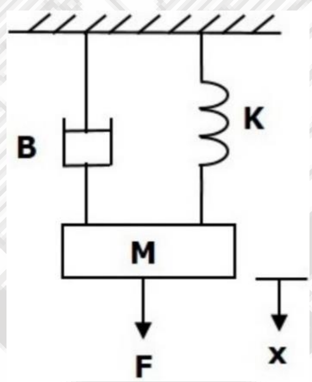
Two systems are said to be analogous to each other if the following two conditions are satisfied.

- The two systems are physically different.
- Differential equation modelling of those two systems are same.

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

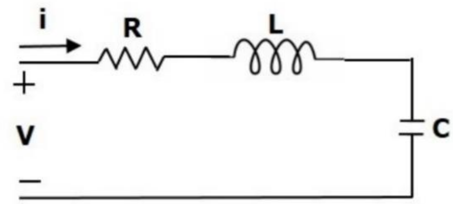
In force voltage analogy, the mathematical equations of translational mechanical system are compared with mesh equations of the electrical system.
Consider the following translational mechanical system shown in the following figure.



$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \qquad \textbf{(Equation 1)}$$

Consider the following electrical system as shown in the following figure. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series. The input voltage applied to this circuit is V volts and the current flowing through the circuit is i Amps.



Mesh equation for this circuit is

$$V = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{c}\int idt$$ **(Equation 2)**

$$i = \frac{\mathrm{d}q}{\mathrm{d}t}$$

$$V = R\frac{\mathrm{d}q}{\mathrm{d}t} + L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + \frac{q}{C}$$

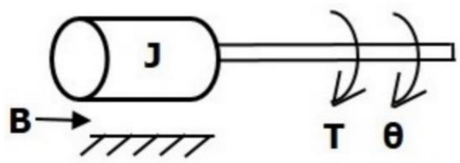
$$\Rightarrow V = L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \left(\frac{1}{c}\right)q$$ **(Equation 3)**

By comparing Equation 1 and Equation 3, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance $\left(\frac{1}{c}\right)$
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

<u>Torque Voltage Analogy</u>

In this analogy, the mathematical equations of rotational mechanical system are compared with mesh equations of the electrical system.
Rotational mechanical system is shown in the following figure.



the torque balanced equation is

$$T = T_j + T_b + T_k$$

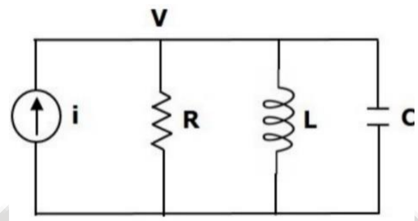
$$\Rightarrow T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k\theta \qquad \textbf{(Equation 4)}$$

By comparing Equation 4 and Equation 3, we will get the analogous quantities of the rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance $(\frac{1}{c})$
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy

In force current analogy, the mathematical equations of the translational mechanical system are compared with the nodal equations of the electrical system.
Consider the following electrical system as shown in the following figure. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.



The nodal equation is

$$i = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{\mathrm{d}V}{\mathrm{d}t} \qquad \textbf{(Equation 5)}$$

$$V = \frac{\mathrm{d}\Psi}{\mathrm{d}t}$$

$$i = \frac{1}{R}\frac{\mathrm{d}\Psi}{\mathrm{d}t} + \left(\frac{1}{L}\right)\Psi + C\frac{\mathrm{d}^2\Psi}{\mathrm{d}t^2}$$

$$\Rightarrow i = C\frac{\mathrm{d}^2\Psi}{\mathrm{d}t^2} + \left(\frac{1}{R}\right)\frac{\mathrm{d}\Psi}{\mathrm{d}t} + \left(\frac{1}{L}\right)\Psi \qquad \textbf{(Equation 6)}$$

By comparing Equation 1 and Equation 6, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance $\left(\frac{1}{R}\right)$
Spring constant(K)	Reciprocal of Inductance $\left(\frac{1}{L}\right)$
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Torque Current Analogy

In this analogy, the mathematical equations off the rotational mechanical system are compared with the nodal mesh equations off the electrical system.

By comparing Equation 4 and Equation 6, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance $\left(\frac{1}{R}\right)$
Torsional spring constant(K)	Reciprocal of Inductance $\left(\frac{1}{L}\right)$
Angular displacement(θ)	Magnetic flux(ψ)
Angular velocity(ω)	Voltage(V)