Lecture : 14

Long-run production continued....

Elasticity of substitution (σ):

The concept of elasticity of substitution is a numerical measure that can help us to describe a firm's opportunities for input substitution. σ measures how quickly the marginal rate of technical substitution of labour for capital changes as we move along the isoquant.

$$\sigma = \frac{\partial (K/L) / K/L}{\partial MRTS_{(L,K)} / MRTS_{(L,K)}}$$

For linear production function

$\sigma = \infty$ (L & K are substitutes)

For fixed proportion production function

$\sigma = 0$ (L & K are complementary)

Now, let us discuss a special type of production function which comes between linear production function and fixed proportion production function. It is called Cobb-Douglas production function for which $\sigma = 1$

Find σ for C-D production function

$$Q = b_0 L^{b_1} K^{b_2} \qquad (b_0, b_1 \text{ \& } b_2 \text{ are positive constants})$$

Solution:

$$MP_L = \frac{\partial Q}{\partial L} = b_0 b_1 L^{b_1 - 1} K^{b_2} = b_1 b_0 L^{b_1} K^{b_2} / L$$

or $$MP_L = \frac{b_1 Q}{L}$$

$$MP_K = \frac{\partial Q}{\partial K} = b_0 L^{b_1} b_2 K^{b_2 - 1} = b_2 b_0 L^{b_1} K^{b_2} / K$$

or $$MP_K = \frac{b_2 Q}{K}$$

$$MRTS(L, K) = \frac{MP_L}{MP_K} = \frac{b_1 Q}{L} \Big/ \frac{b_2 Q}{K}$$

$$= \frac{b_1 \cancel{Q}}{L} \times \frac{K}{b_2 \cancel{Q}} = \frac{b_1}{b_2} \cdot \frac{K}{L}$$

$$\sigma = \frac{\partial (K/L) / K/L}{\partial (MRTS_{L,K}) / MRTS_{L,K}}$$

Substituting the values we get

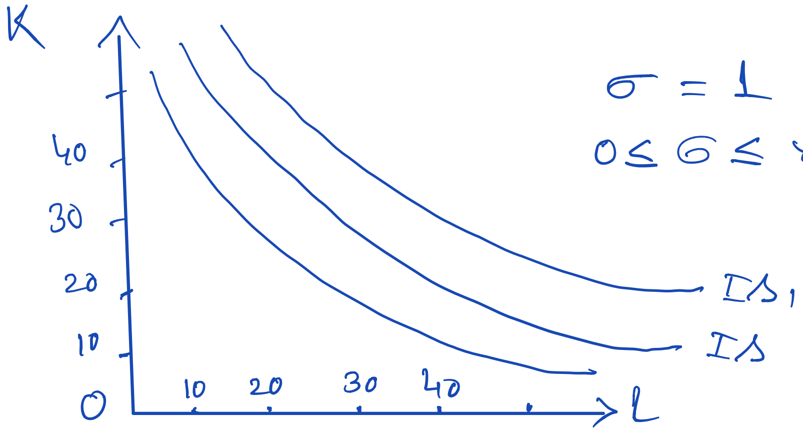
$$\sigma = \frac{\partial(K/L)\,/\,K/L}{\partial\left(\frac{b_1}{b_2}\cdot K/L\right)\Big/\left(\frac{b_1}{b_2}\cdot\frac{K}{L}\right)}$$

$$= \frac{\partial(\cancel{K}/L)\,/\,(K/\cancel{L})}{\frac{\cancel{b_1}}{b_2}(\partial \cancel{K/L})\,/\,\frac{\cancel{b_1}}{b_2}\cdot \cancel{K/L}}$$

$$= 1$$

With cobb-Douglas production function L & K can be substituted in variable proportions.

The isoquants for cobb-Douglas production function are non-linear downward sloping curves.



$\sigma = 1$

$0 \le \sigma \le \infty$

Laws of returns to scale:

Now we will describe the effect of proportional increase in all the inputs on the level of output.

For e.g if a firm changes all the inputs used in production, how much change will happen to output.

The answer to this question depends on the concept of returns to scale.

In the long-run

$$Q = f(L, K)$$ L & K are variable inputs.

The effect of change in L & K on Q can be discussed in three ways.

① **The law of constant returns to scale:**

Condition in which all inputs are increased in some proportion and output also increases in the same proportion. In other words, the proportionate change in output is equal to the proportionate change in inputs.

$$Q = f(l, K)$$

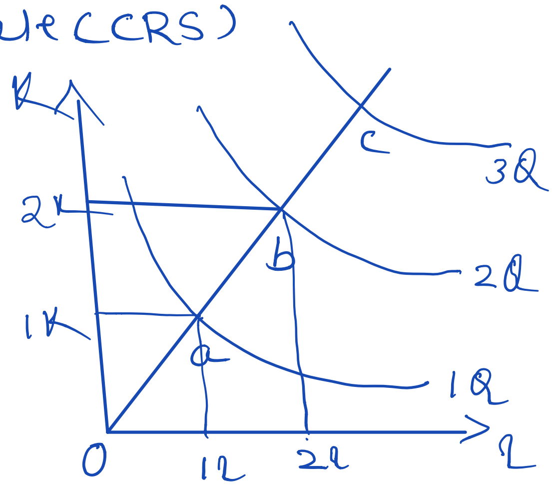
Let inputs increase by a constant 'c' and proportionate change in output is 'z'

$$f(cl, cK) = zQ$$

If $z = c$ it's a case of constant returns to scale (CRS)

In case of C.R.S, isoquants are equidistant from each other.

Here $Oa = ab = bc$



② Increasing returns to scale: condition in which when inputs are increased in some proportion, output increases in a larger proportion. In other words, the proportionate change in output is more than proportionate change in inputs.

$$Q = f(l, K)$$

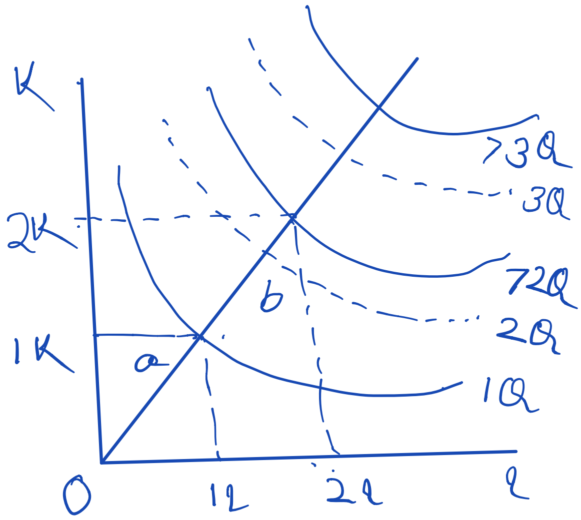
l & K change by 'c' & Q changes by 'z'

$$f(cl, cK) = zQ$$

In case of increasing returns to scale

$$z > c$$

When l & K are doubled, Q is more than doubled.



③ Decreasing returns to scale: Condition in which when all inputs are increased in some proportion, output increases by less than that proportion.

In other words, the proportionate change in output is less than proportionate change in inputs.

$Q = f(L, K)$

L & K change by the proportion 'C'

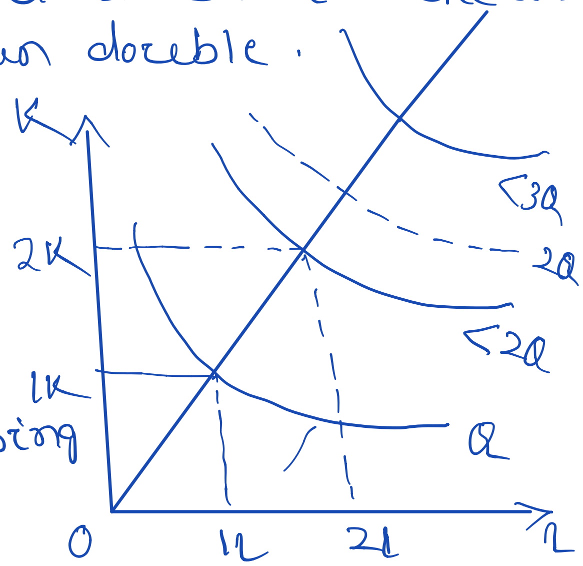
& Q changes by 'Z'

$f(CL, CK) = ZQ$

$Z < C$

When L & K are doubled, Q increases but it's less than double.

It requires more than 2L & 2K to produce 2Q in case of decreasing returns to scale.



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