1.2 Phasors

A sinusoidal voltage/current at a given frequency , is characterized by only two parameters : amplitude and phase

Key Words:

Complex Numbers

Rotating Vector

Phasors

1.2 Phasors

E.g. voltage response

Time domain

$$v(t) = V_m \cos(\omega t + \varphi)$$

$R_e\{v(t)\}$

Complex form: $v(t) = V_m e^{j(\omega t + \varphi)}$

Angular frequency ω is known in the circuit.

Phasor form: $\dot{V} = |V_m| \angle \varphi$

Frequency domain

A sinusoidal *v*/*i*

$$v(t) = V_m \cos(\omega t + \varphi)$$

Complex transform

Phasor transform

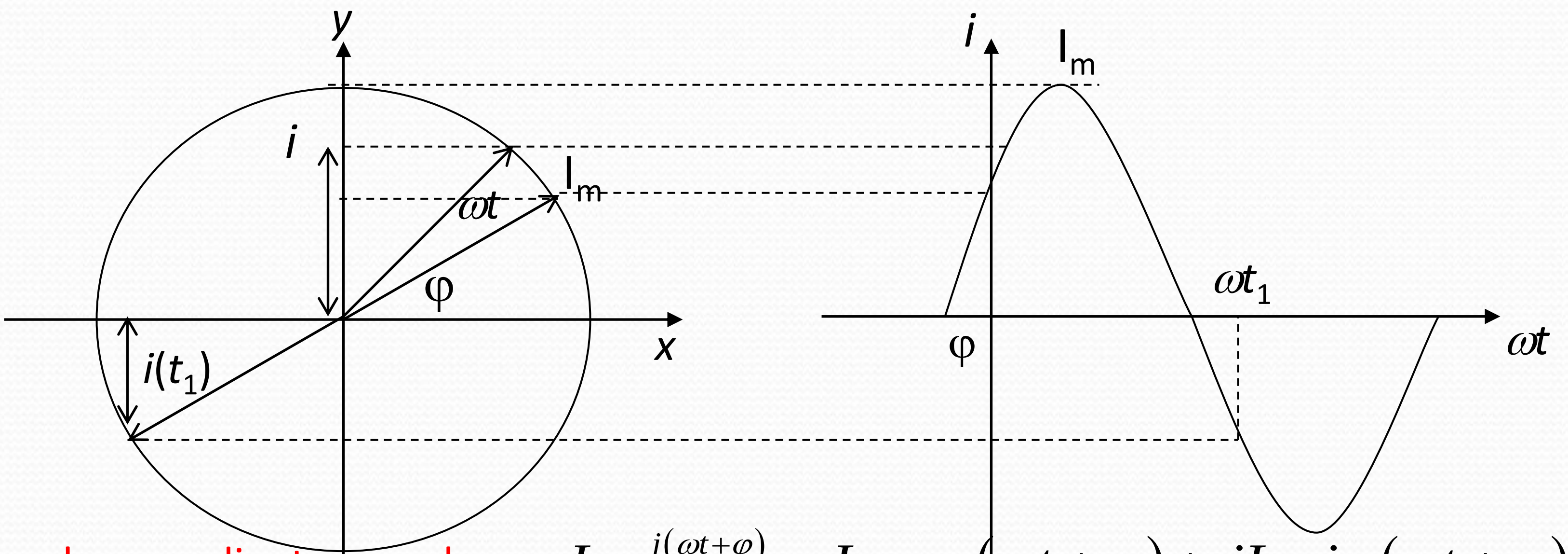
$$\dot{V} = |V_m| \angle \varphi$$

By knowing angular frequency ω *rads/s.*

1.2 Phasors

Rotating Vector

$$i(t) = I_m \sin(\omega t + \varphi)$$



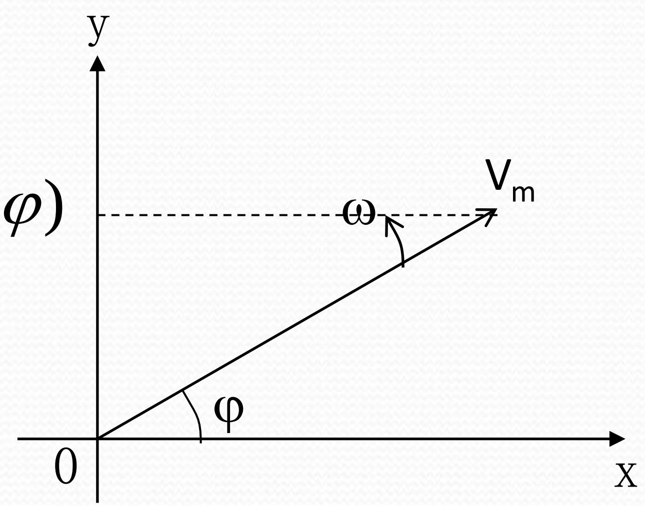
A complex coordinates number: $I_m e^{j(\omega t+\varphi)} = I_m \cos(\omega t + \varphi) + jI_m \sin(\omega t + \varphi)$

Real value: $i(t) = I_m \sin(\omega t + \varphi) = \text{Imag}\left(I_m e^{j(\omega t+\varphi)}\right)$

1.2 Phasors

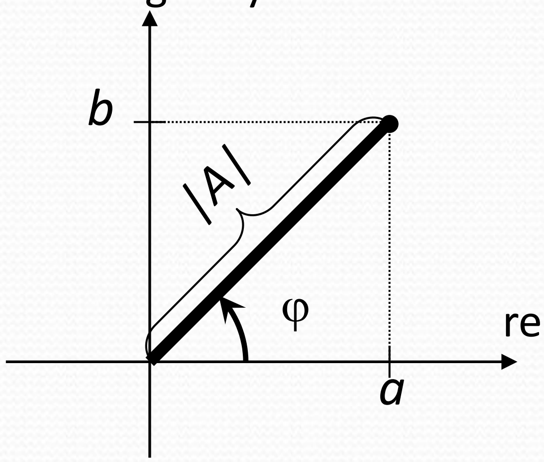
Rotating Vector

$$v = V_m \sin(\omega t + \varphi)$$



1.2 Phasors

Complex Numbers



$A = a + jb$ — Rectangular Coordinates

$A = |A|(\cos\varphi + j\sin\varphi)$

$A = |A|e^{j\varphi}$— Polar Coordinates

conversion：

$$A = a + jb \rightarrow A = |A|e^{j\varphi} \quad \begin{cases} |A| = \sqrt{a^2 + b^2} \\ \varphi = arctg\,\frac{b}{a} \end{cases}$$

$$|A|e^{j\varphi} \rightarrow a + jb \quad \begin{cases} a = |A|\cos\varphi \\ b = |A|\sin\varphi \end{cases}$$

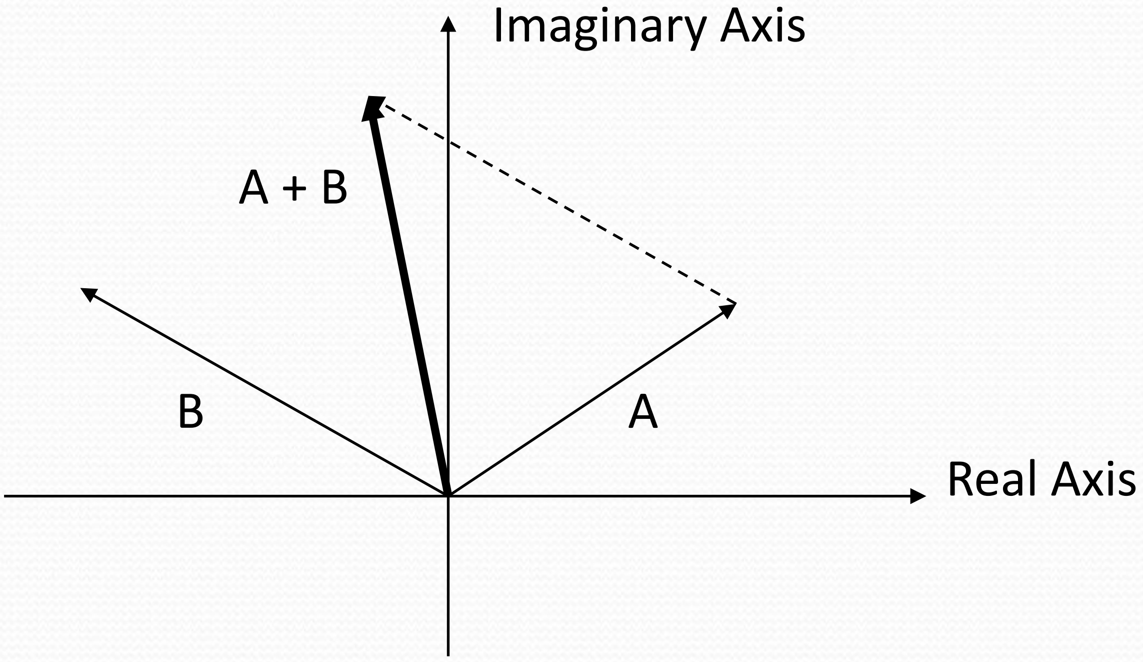
$$e^{\pm j90^\circ} = \cos 90^\circ \pm j\sin 90^\circ = 0 \pm j = \pm j$$

1.2 Phasors

Complex Numbers

Arithmetic With Complex Numbers

Addition: $A = a + jb$, $B = c + jd$, $A + B = (a + c) + j(b + d)$



1.2 Phasors

Complex Numbers

Arithmetic With Complex Numbers

Multiplication : $A = A_m \angle \varphi_A, \; B = B_m \angle \varphi_B$

$$A \times B = (A_m \times B_m) \angle (\varphi_A + \varphi_B)$$

Division: $A = A_m \angle \varphi_A , \; B = B_m \angle \varphi_B$

$$A / B = (A_m / B_m) \angle (\varphi_A - \varphi_B)$$

1.2 Phasors

Phasors

A phasor is a complex number that represents the magnitude and phase of a sinusoid:

$$i_m \cos(\omega t + \varphi) \Longleftrightarrow \dot{I} = I_m \angle \varphi$$

Phasor Diagrams

- A phasor diagram is just a graph of several phasors on the complex plane (using real and imaginary axes).
- A phasor diagram helps to visualize the relationships between currents and voltages.

1.2 Phasors

Complex Exponentials

$$A = |A| e^{j\varphi}$$

$$Ae^{j\omega t} = |A| e^{j(\omega t+\varphi)} = |A| \cos(\omega t + \varphi) + j|A| \sin(\omega t + \varphi)$$

$$\mathrm{Re}\{Ae^{j\omega t}\} = |A| \cos(\omega t + \varphi)$$

- A real-valued sinusoid is the real part of a complex exponential.
- Complex exponentials make solving for AC steady state an algebraic problem.