1.3 Phasor Relationships for R, L and C

Key Words:

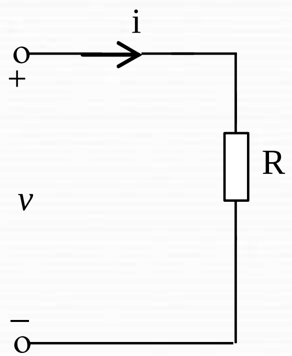
I-V Relationship for R, L and C,

Power conversion

1.3 Phasor Relationships for R, L and C

Resistor

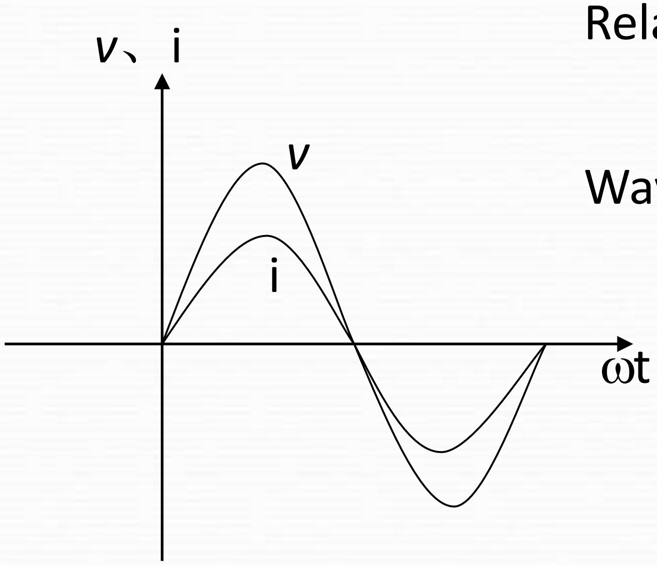
- *v* ~ *i* relationship for a resistor



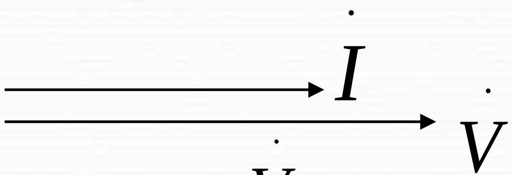
Suppose $v = V_m \sin \omega t$

$$i = \frac{v}{R} = \frac{V_m}{R}\sin \omega t = I_m \sin \omega t$$

Relationship between RMS: $I = \dfrac{V}{R}$



Wave and Phasor diagrams：



$$\dot{I} = \frac{\dot{V}}{R}$$

1.3 Phasor Relationships for R, L and C

Resistor

- Time domain → frequency domain

$$v(t) = V_m \cos(wt + \theta) \quad V(t) = Ri(t)$$

$$i(t) = I_m \cos(wt + \phi)$$

$$V_m e^{j(wt+\theta)} = RI_m e^{j(wt+\phi)}$$

$$V_m e^{j\theta} = RI_m e^{j\phi}$$

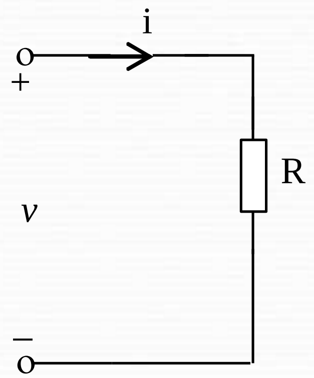
$$V_m \angle\theta = RI_m \angle\phi$$

$$\dot{V} = R\dot{I}$$

With a resistor $\vartheta = \varphi$, $v(t)$ and $i(t)$ are in phase .

1.3 Phasor Relationships for R, L and C

Resistor

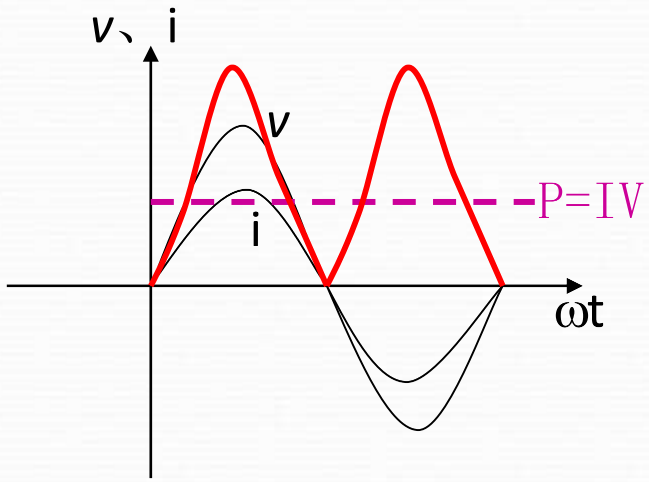


● Power

- Transient Power

$$p = vi = V_m \sin\omega t \cdot I_m \sin\omega t = I_m V_m \sin^2\omega t$$
$$= \frac{I_m V_m}{2}(1-\cos 2\omega t) = IV - IV\cos 2\omega t$$

Note: I and *V* are RMS values. $p>0$



- Average Power

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T VI(1-\cos 2\omega t)dt = VI$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

1.3 Phasor Relationships for R, L and C

Resistor

P1.4 , $v = 311\sin 314t$, R=10Ω， Find i and P。

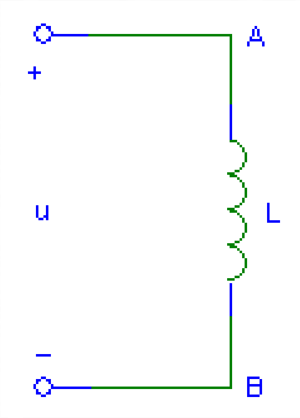
$$V = \frac{V_m}{\sqrt{2}} = \frac{311}{\sqrt{2}} = 220(V)$$

$$I = \frac{V}{R} = \frac{220}{10} = 22(A)$$

$$i = 22\sqrt{2}\sin 314t \qquad P = IV = 220\times 22 = 4840(W)$$

1.3 Phasor Relationships for R, L and C

Inductor



- *v* ~ *i* relationship

$$v = v_{AB} = L\frac{di}{dt}$$

Suppose $i = I_m \sin \omega t$

$$v = L\frac{di}{dt} = L\frac{d\left(I_m \sin \omega t\right)}{dt} = I_m \omega L \cos \omega t$$

$$= I_m \omega L \sin\left(\omega t + 90^\circ\right)$$

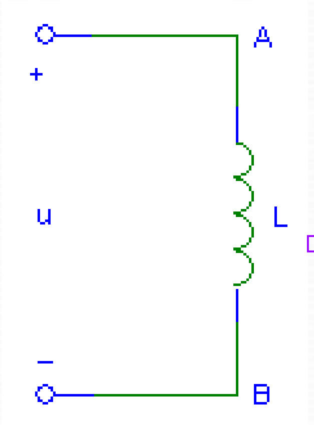
$$= V_m \sin\left(\omega t + 90^\circ\right)$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt = \frac{1}{L}\int_{-\infty}^{0} v\,dt + \frac{1}{L}\int_{0}^{t} v\,dt = i_0 + \frac{1}{L}\int_{0}^{t} v\,dt$$

1.3 Phasor Relationships for R, L and C

Inductor

- *v*~*i* relationship



$$v = L\frac{di}{dt} = I_m\omega L\sin\left(\omega t + 90^\circ\right) = V_m\sin\left(\omega t + 90^\circ\right)$$

$$V_m = I_m\omega L$$

Relationship between RMS: $V = I\omega L$

$$I = \frac{V}{\omega L} \longrightarrow X_L = \omega L = 2\pi f L \quad (\Omega)$$

$$X_L \propto f$$

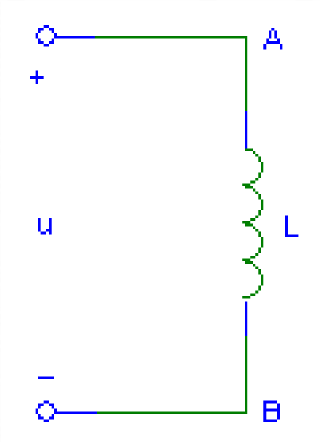
For DC, $f = 0$, $\rightarrow X_L = 0$.

$v(t)$ leads $i(t)$ by 90º, or $i(t)$ lags $v(t)$ by 90º

1.3 Phasor Relationships for R, L and C

Inductor

● *v* ~ *i* relationship



$i(t) = I_m e^{j\omega t}$

$$v(t) = L\frac{di}{dt} = I_m j\omega L e^{j\omega t} = j\omega L i(t)$$

Represent *v*(*t*) and *i*(*t*) as phasors: $\dot{V} = j\omega L\dot{I}$

$$\longrightarrow \dot{I} = \frac{\dot{V}}{j\omega L} = \frac{\dot{V}}{jX_L}$$

• The derivative in the relationship between *v*(*t*) and *i*(*t*) becomes a multiplication by *j*ω in the relationship between $\dot{V}$ and $\dot{I}$.

• The time-domain differential equation has become the algebraic equation in the frequency-domain.

• Phasors allow us to express current-voltage relationships for inductors and capacitors in a way such as we express the current-voltage relationship for a resistor.

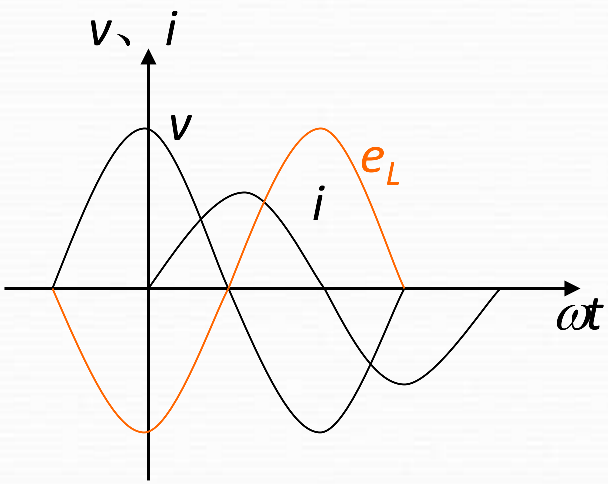
1.3 Phasor Relationships for R, L and C

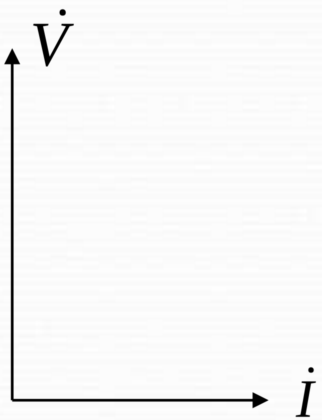
Inductor

● *v* ~ *i* relationship

Wave and Phasor diagrams：

$$\dot{V} = j\dot{I}X_L$$





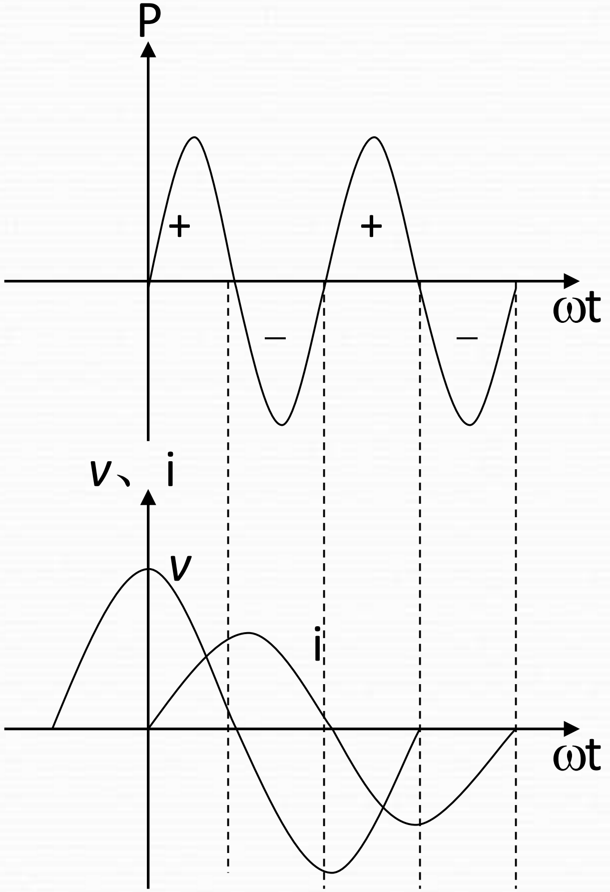
1.3 Phasor Relationships for R, L and C

Inductor

- Power

$$p = vi = V_m \sin(\omega t + 90^\circ) I_m \sin \omega t = V_m I_m \cos \omega t \cdot \sin \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t = VI \sin 2\omega t$$



Energy stored: $W = \int_0^t vidt = \int_0^i Lidi = \frac{1}{2} Li^2$

$$W_{\max} = \frac{1}{2} L I_m^2 = L I^2$$

Average Power $P = \frac{1}{T}\int_0^T pdt = \frac{1}{T}\int_0^T VI \sin 2\omega t dt = 0$

Reactive Power $Q = IV = I^2 X_L = \frac{V^2}{X_L}$ （Var）

1.3 Phasor Relationships for R, L and C

Inductor

P1.5， L = 10mH， v = 100sinωt， Find i_L when f = 50Hz and 50kHz.

$$X_L = 2\pi f L = 2\pi \times 50 \times 10 \times 10^{-3} = 3.14(\Omega)$$

$$I_{50} = \frac{V}{X_L} = \frac{100/\sqrt{2}}{3.14} = 22.5(A)$$

$$i_L(t) = 22.5\sqrt{2}\sin\left(\omega t - 90^\circ\right)A$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 10^3 \times 10 \times 10^{-3} = 3140(\Omega)$$

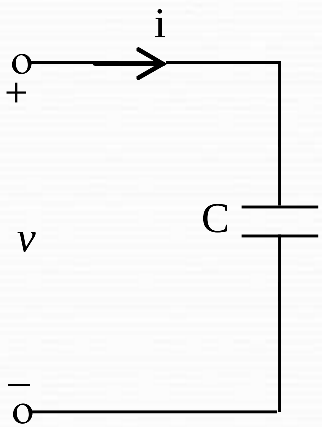
$$I_{50k} = \frac{V}{X_L} = \frac{100/\sqrt{2}}{3.14} = 22.5(mA)$$

$$i_L(t) = 22.5\sqrt{2}\sin\left(\omega t - 90^\circ\right)mA$$

1.3 Phasor Relationships for R, L and C

Capacitor

● *v* ~ *i* relationship



$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

Suppose: $v = V_m \sin \omega t$

$I_m = \omega C V_m$

$$i = \omega C V_m \cos \omega t = \omega C V_m \sin\left(\omega t + 90^\circ\right) = I_m \sin\left(\omega t + 90^\circ\right)$$

$$v = \frac{1}{c}\int_{-\infty}^{t} i dt = \frac{1}{c}\int_{-\infty}^{0} i dt + \frac{1}{c}\int_{0}^{t} i dt = v_0 + \frac{1}{c}\int_{0}^{t} i dt$$

Relationship between RMS: $I = \omega C V = \frac{V}{1/\omega C} = \frac{V}{X_C}$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad (\Omega)$$

$$X_C \propto \frac{1}{f}$$

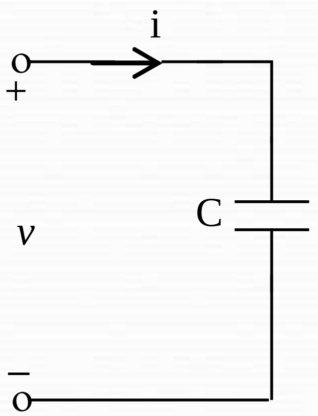
For DC, $f = 0$, $\rightarrow X_C \rightarrow \infty$

i(*t*) leads *v*(*t*) by 90º, or *v*(*t*) lags *i*(*t*) by 90º

1.3 Phasor Relationships for R, L and c

Capacitor

● *v* ~ *i* relationship



$v(t) = V_m\, e^{j\omega t}$

$$i(t) = C\frac{dv(t)}{dt} = C\frac{dV_m e^{j\omega t}}{dt} = j\omega C V_m e^{j\omega t}$$

Represent *v*(*t*) and *i*(*t*) as phasors: $\dot{I} = j\omega C\dot{V} = \dfrac{\dot{V}}{jX_C}$

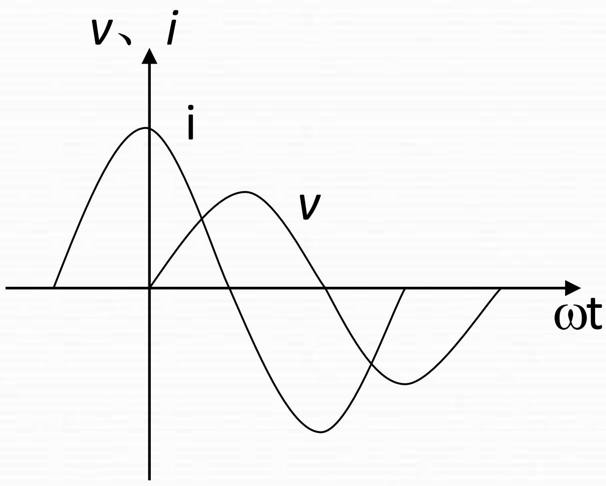
- The derivative in the relationship between *v*(*t*) and *i*(*t*) becomes a multiplication by $j\omega$ in the relationship between $\dot{V}$ and $\dot{I}$.
- The time-domain differential equation has become the algebraic equation in the frequency-domain.
- Phasors allow us to express current-voltage relationships for inductors and capacitors much like we express the current-voltage relationship for a resistor.

1.3 Phasor Relationships for R, L and C

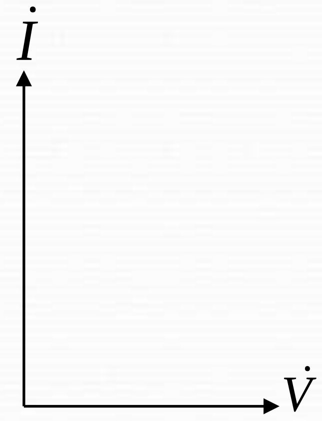
Capacitor

● $v \sim i$ relationship

Wave and Phasor diagrams：



$$\dot{V} = -j\dot{I}X_C$$



1.3 Phasor Relationships for R, L and C

Capacitor

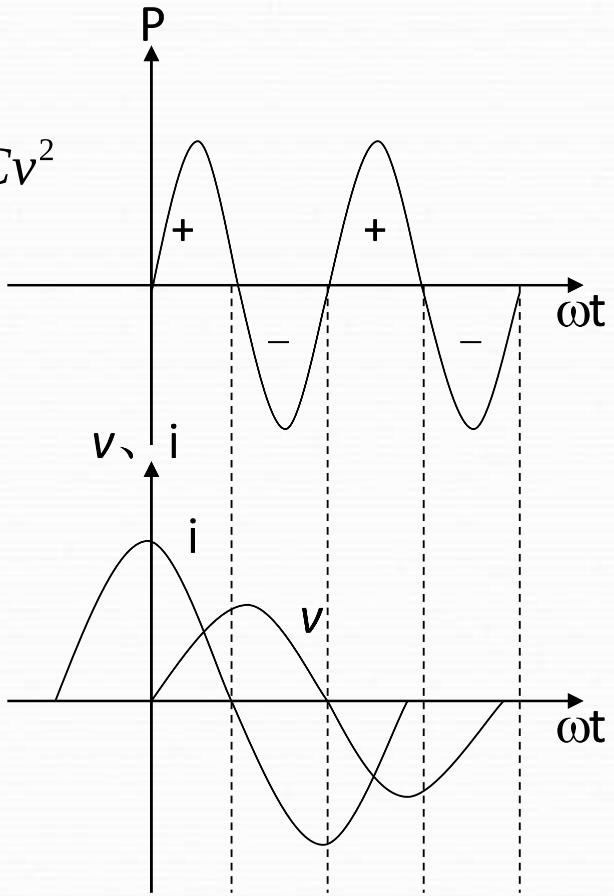
● Power

$$p = vi = V_m \sin \omega t \cdot I_m \sin\left(\omega t + 90^\circ\right) = \frac{V_m I_m}{2} \sin 2\omega t = VI \sin 2\omega t$$

Energy stored:

$$W = \int_0^t vidt = \int_0^v v \cdot C \cdot \frac{dv}{dt} \cdot dt = \int_0^v Cvdv = \frac{1}{2} Cv^2$$

$$W_{\max} = \frac{1}{2} CV_m^2 = CV^2$$



Average Power: P=0

Reactive Power $Q = IV = I^2 X_C = \frac{V^2}{X_C}$ （Var）

1.3 Phasor Relationships for R, L and C

Capacitor

P1.7， Suppose C=20μF， AC source v=100sinωt， Find X_C and I for f = 50Hz and 50kHz。

$$f = 50\text{Hz} \rightarrow X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 159\Omega$$

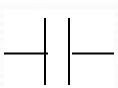
$$I = \frac{V}{X_c} = \frac{V_m}{\sqrt{2} X_c} = 1.38\text{A}$$

$$f = 50\text{KHz} \rightarrow X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 0.159(\Omega)$$

$$I = \frac{V}{X_c} = \frac{V_m}{\sqrt{2} X_c} = 1380(\text{A})$$

1.3 Phasor Relationships for R, L and C

Review (*v-I relationship*)

	Time domain	Frequency domain
 R	$v = R \cdot i$	$\dot{V} = R \cdot \dot{I}$, *v* and *i* are in phase.
 L	$v_L = L \dfrac{di}{dt}$	$\dot{V} = j\omega L \cdot \dot{I}$, $X_L = \omega L$, *v* leads *i* by 90° .
 C	$v_C = C \dfrac{dv}{dt}$	$\dot{V} = \dfrac{1}{j\omega C} \cdot \dot{I}$, $X_C = \dfrac{1}{\omega C}$, *v* lags *i* by 90° .

1.3 Phasor Relationships for R, L and C

Summary

- R: $X_R = R$ $\Delta\varphi = 0$

  L: $X_L = \omega L = 2\pi f L \propto f$ $\Delta\varphi = \varphi_v - \varphi_i = \frac{\pi}{2}$

  C: $X_C = \frac{1}{\omega c} = \frac{1}{2\pi f c} \propto \frac{1}{f}$ $\Delta\varphi = \varphi_v - \varphi_i = -\frac{\pi}{2}$

- $V = IX$

- Frequency characteristics of an Ideal Inductor and Capacitor:

  A capacitor is an open circuit to DC currents;

  A Inducter is a short circuit to DC currents.