1.4 Impedance

Key Words:

complex currents and voltages.

Impedance

Phasor Diagrams

1.4 Impedance

Complex voltage， Complex current， Complex Impedance

- AC steady-state analysis using phasors allows us to express the relationship between current and voltage using a formula that looks likes Ohm's law:

$$\dot{V} = \dot{I}Z$$

Z is called impedance. measured in ohms (Ω)

$$\dot{V} = V_m e^{j\varphi_v} = V_m \angle \varphi_v$$

$$\dot{I} = I_m e^{j\varphi_i} = I_m \angle \varphi_i$$

$$Z = \frac{\dot{V}}{\dot{I}} = \frac{V_m}{I_m} e^{j(\varphi_v - \varphi_i)} = |Z| e^{j\varphi} = |Z| \angle \varphi$$

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Complex Impedance

$$Z = \frac{\dot{V}}{\dot{I}} = \frac{V_m}{I_m} e^{j(\varphi_v - \varphi_i)} = |Z| e^{j\varphi} = |Z| \angle \varphi$$

- Complex impedance describes the relationship between the voltage across an element (expressed as a phasor) and the current through the element (expressed as a phasor)
- Impedance is a complex number and is not a phasor (why?).
- Impedance depends on frequency

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Complex Impedance

Resistor——The impedance is *R*

$Z_R = R \quad \Delta\varphi = 0$; or $Z_R = R\angle 0$

Capacitor——The impedance is $1/j\omega C$

$$Z_c = \frac{1}{\omega C} e^{-j\frac{\pi}{2}} = \frac{-j}{\omega C} = -jX_c \quad \text{or} \quad Z_C = \frac{1}{\omega C}\angle -90^\circ$$

$$(\Delta\varphi = \varphi_v - \varphi_i = -\frac{\pi}{2})$$

Inductor——The impedance is $j\omega L$

$$Z_L = \omega L e^{j\frac{\pi}{2}} = j\omega L = jX_L \quad \text{or} \quad Z_L = \omega L\angle 90^\circ$$

$$(\Delta\varphi = \varphi_v - \varphi_i = \frac{\pi}{2})$$

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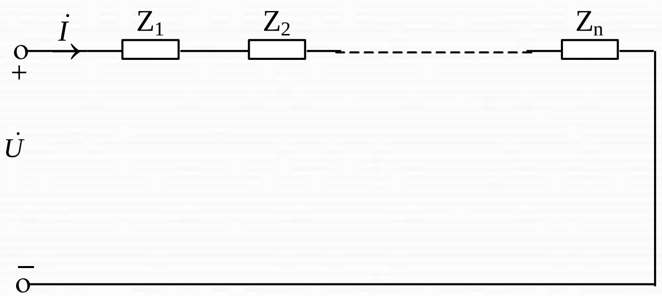
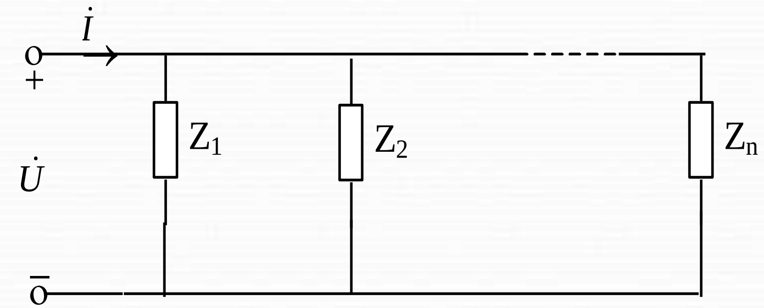
Complex Impedance

Impedance in series/parallel can be combined as resistors.





$$Z = Z_1 + Z_2 + \ldots + Z_n = \sum_{k=1}^{n} Z_k$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n} = \sum_{k=1}^{n} \frac{1}{Z_k}$$

Voltage divider:

$$\dot{V}_i = \dot{V} \frac{Z_i}{\sum_{k=1}^{n} Z_k}$$

Current divider:

$$\dot{I}_1 = \dot{I} \frac{Z_2}{Z_1 + Z_2} \qquad \dot{I}_2 = \dot{I} \frac{Z_1}{Z_1 + Z_2}$$

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Complex Impedance

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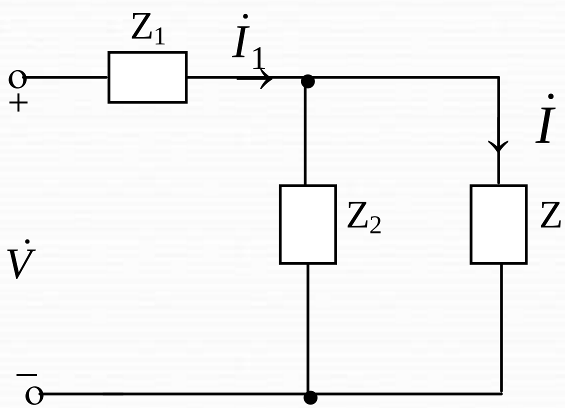

$$\dot{I} = \dot{I}_1 \frac{Z_2}{Z + Z_2}$$

$$\dot{I}_1 = \frac{\dot{V}}{Z_1 + \left(\frac{1}{Z_2} + \frac{1}{Z}\right)^{-1}} = \frac{\dot{V}(Z + Z_2)}{ZZ_1 + Z_2Z_1 + ZZ_2}$$

$$\dot{I} = \frac{\dot{V}Z_2}{ZZ_1 + Z_2Z_1 + ZZ_2}$$

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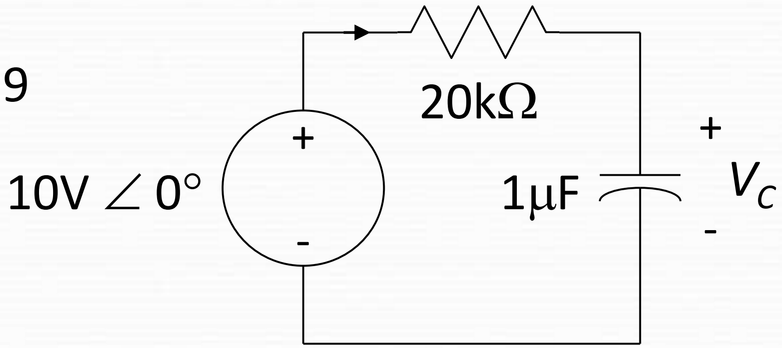
Complex Impedance

Phasors and complex impedance allow us to use Ohm's law with complex numbers to compute current from voltage and voltage from current

P4.9



$\omega = 377$

Find V_C

- How do we find V_C?
- First compute impedances for resistor and capacitor:

$$Z_R = 20k\Omega = 20k\Omega \angle 0^\circ$$

$$Z_C = 1/j\,(377 * 1\mu F) = 2.65k\Omega \angle -90^\circ$$

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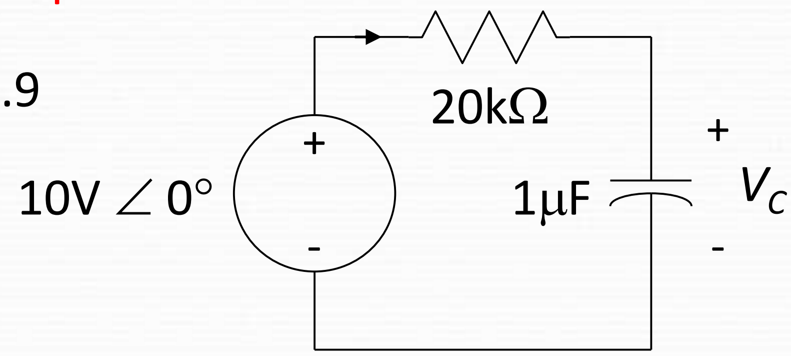
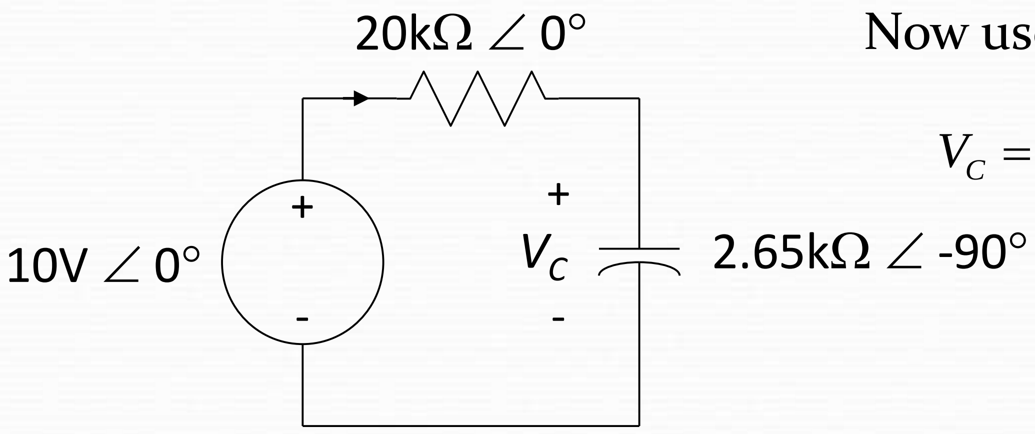
Complex Impedance

P4.9



$\omega = 377$

Find V_C



Now use the voltage divider to find V_C:

$$V_C = 10V\angle 0^\circ \left(\frac{2.65k\Omega\angle -90^\circ}{2.65k\Omega\angle -90^\circ + 20k\Omega\angle 0^\circ}\right)$$

$$\mathbf{V_c} = 10\text{V}\ \angle 0^\circ \frac{2.65\angle -90^\circ}{20.17\angle -7.54^\circ}$$

$$= 1.31\text{V}\ \angle -82.46^\circ$$

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Complex Impedance

Impedance allows us to use the same solution techniques for AC steady state as we use for DC steady state.

- All the analysis techniques we have learned for the linear circuits are applicable to compute phasors
  - KCL & KVL
  - node analysis / loop analysis
  - superposition
  - Thevenin equivalents / Norton equivalents
  - source exchange
- The only difference is that now complex numbers are used.

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Kirchhoff's Laws

KCL and KVL hold as well in phasor domain.

KCL: $$\sum_{k=1}^{n} i_k = 0$$ i_k – Transient current of the $^{\#}k$ branch

$$\sum_{k=1}^{n} \dot{I}_k = 0$$

KVL: $$\sum_{k=1}^{n} v_k = 0$$ v_k – Transient voltage of the $^{\#}k$ branch

$$\sum_{k=1}^{n} \dot{V}_k = 0$$

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Admittance

- $\boldsymbol{I} = \boldsymbol{YV}$, $\boldsymbol{Y}$ is called admittance, the reciprocal of impedance, measured in siemens (S)
- Resistor:
  - The admittance is $1/R$
- Inductor:
  - The admittance is $1/j\omega L$
- Capacitor:
  - The admittance is $j \omega C$

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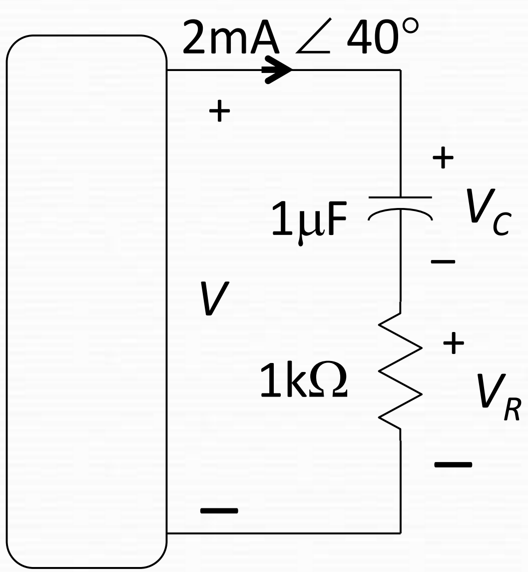
Phasor Diagrams

- A phasor diagram is just a graph of several phasors on the complex plane (using real and imaginary axes).
- A phasor diagram helps to visualize the relationships between currents and voltages.



$I = 2\text{mA} \angle 40°, \quad V_R = 2\text{V} \angle 40°$

$V_C = 5.31\text{V} \angle -50°, \quad V = 5.67\text{V} \angle -29.37°$