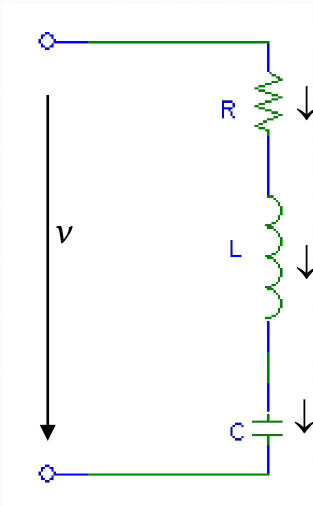
1.5 Series and Parallel Resonance

Series RLC Circuit (2nd Order RLC Circuit)



$$v = v_R + v_L + v_C$$

Phasor $\dot{V} = \dot{V}_R + \dot{V}_L + \dot{V}_C$

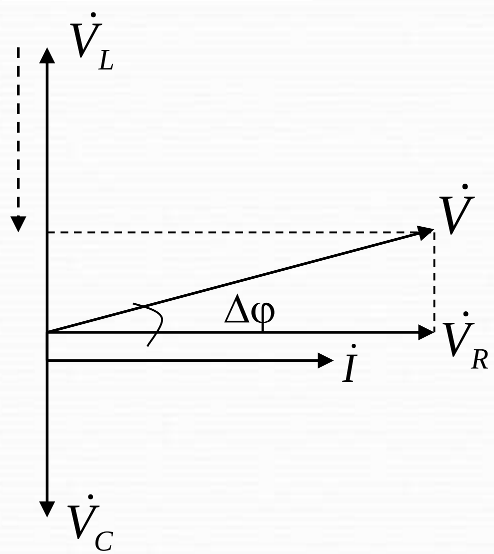
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$= I\sqrt{R^2 + X^2} \qquad (X = X_L - X_C)$$

$$= IZ$$

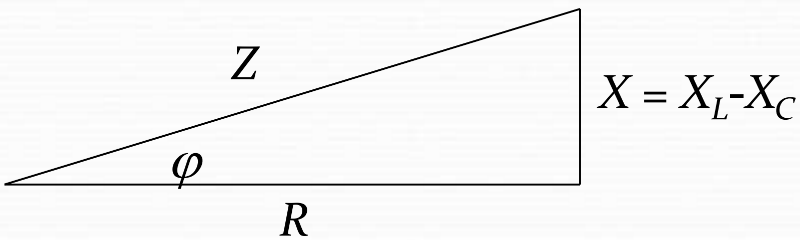


$$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}$$

1.5 Series and Parallel Resonance

Series RLC Circuit (2nd Order RLC Circuit)

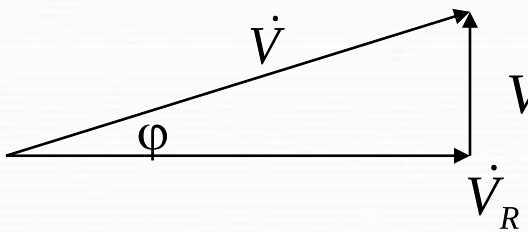
$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = IZ \qquad Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (\omega L - \frac{1}{\omega c})^2}$$



Phase difference:

$$= arctg\frac{V_L - V_C}{V_R}$$

$$= arctg\frac{X_L - X_C}{R}$$



$$\dot{V}_X = \dot{V}_L + \dot{V}_C$$

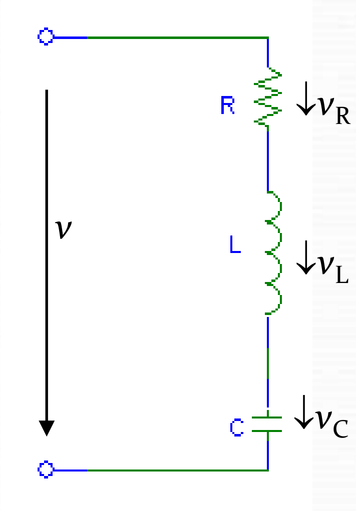
$X_L > X_C \rightarrow \varphi > 0$， v leads i by φ——Inductance Circuit

$X_L < X_C \rightarrow \varphi < 0$， v lags i by φ——Capacitance Circuit

$X_L = X_C \rightarrow \varphi = 0$， v and i in phase——Resistors Circuit

1.5 Series and Parallel Resonance

Series RLC Circuit (2nd Order RLC Circuit)



$$\dot{V} = \dot{V}_R + \dot{V}_L + \dot{V}_C = \dot{I}R + j\dot{I}X_L - j\dot{I}X_C$$

$$= \dot{I}(R + j(X_L - X_C)] = \dot{I}(R + jX) = \dot{I}Z$$

$$\longrightarrow Z = \frac{\dot{V}}{\dot{I}} = R + j(X_L - X_C)$$

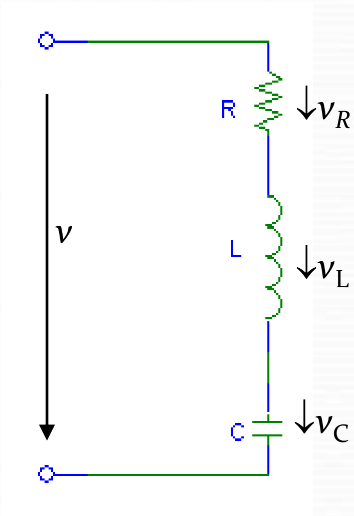
$$Z = R + jX = |Z|\angle\varphi \begin{cases} |Z| = \sqrt{R^2 + (X_L - X_C)^2} \\ \varphi = arctg\dfrac{X_L - X_C}{R} \end{cases}$$

$$\varphi = \varphi_v - \varphi_i$$

1.5 Series and Parallel Resonance

Series RLC Circuit (2nd Order RLC Circuit)

P4.9, R. L. C Series Circuit, $R = 30\Omega$, $L = 127\text{mH}$, $C = 40\mu\text{F}$, Source $v = 220\sqrt{2}\sin(314t + 20^o)$. Find 1) X_L、X_C、Z; 2) $\dot{I}$ and i; 3) $\dot{V}_R$ and v_R; $\dot{V}_L$ and v_L; $\dot{V}_C$ and v_C; 4) Phasor diagrams;



P4.10, Computing $\dot{I}$ by (complex numbers) Phasors

1.5 Series and Parallel Resonance

Series Resonance (2nd Order RLC Circuit)

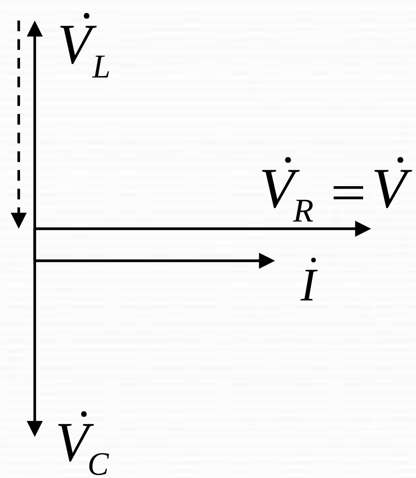
$$\dot{V} = \dot{V}_R + \dot{V}_L + \dot{V}_C = \dot{I}R + j\dot{I}X_L - j\dot{I}X_C \qquad \varphi = arctg\frac{V_L - V_C}{V_R} = arctg\frac{X_L - X_C}{R}$$

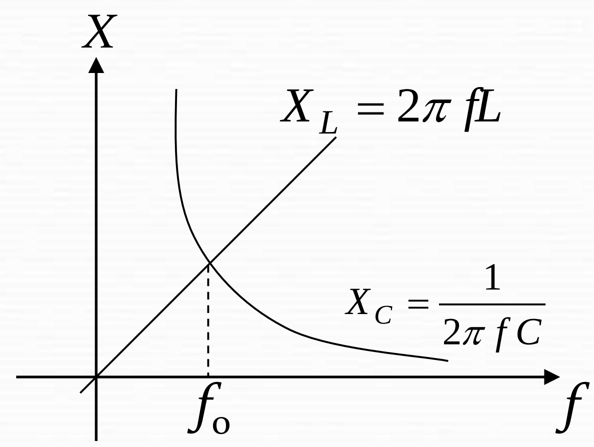
$$When\ X_L = X_C,\ \boxed{\frac{1}{\omega C} = \omega L} \rightarrow V_L = V_C \longrightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad or\ f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Resonance condition

Resonance frequency

$\longrightarrow$ $V_R = V$ and $\varphi = 0$ —— Series Resonance





1.5 Parallel and Series Resonance

Series Resonance (2nd Order RLC Circuit)

Resonance condition:

$$X_L = X_C \quad (\frac{1}{\omega C} = \omega L) \qquad \rightarrow V_L = V_C$$

- $$Z_0 = \sqrt{R^2 + (X_L - X_C)^2} = R \rightarrow \quad I_0 = \frac{V}{Z_0} = \frac{V}{R}$$

Z_{min}; when V=constant, $I=I_{max}=I_o$。

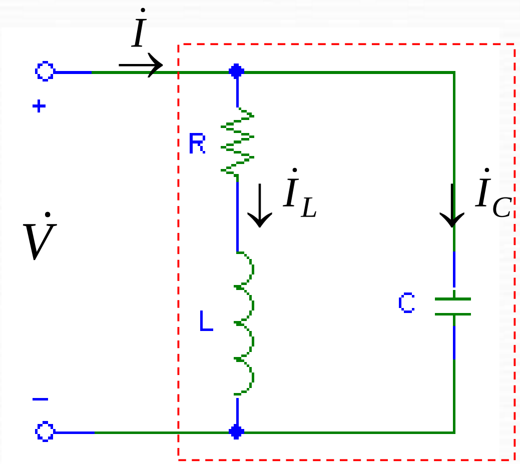
- When $X_L = X_C >> R \rightarrow \quad I_0 X_L = I_0 X_C >> I_0 R \longrightarrow V_L = V_C >> V$

- Quality factor Q,

$$Q = \frac{V_L}{V} = \frac{V_C}{V} = \frac{X_L}{R} = \frac{X_C}{R}$$

1.5 Series and Parallel Resonance

Parallel RLC Circuit



$$Y = \frac{1}{R + j\omega L} + \frac{1}{-j/\omega C} = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{(R + j\omega L)(R - j\omega L)} + j\omega C$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2})$$

When $(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}) = 0$, $\quad Y_0 = \frac{R}{R^2 + \omega^2 L^2}$

$\dot{V}$ In phase with $\dot{I}$

$\longrightarrow$ Parallel Resonance

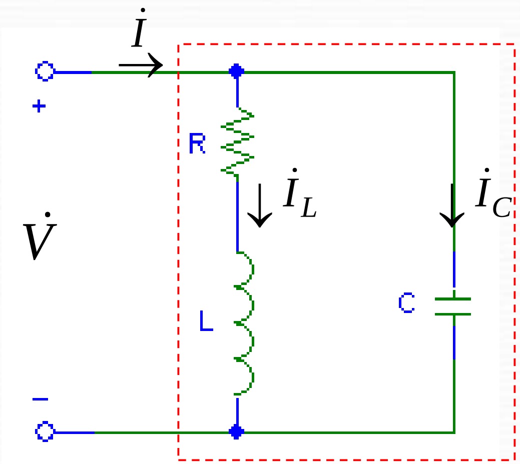
Parallel Resonance frequency $\quad \omega_0 = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{CR^2}{L}}$

In generally $R << X_L \longrightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad (f_0 = \frac{1}{2\pi\sqrt{LC}})$

$\longrightarrow$ Z_{max} I_{min}: $\quad I = I_0 = VY_0 = V\frac{R}{R^2 + \omega_0^2 L^2} = V\frac{R}{R^2 + \frac{1}{LC}L^2} = V\frac{R}{R^2 + \frac{L}{C}} \approx \frac{RC}{L}V$

1.5 Series and Parallel Resonance

Parallel RLC Circuit



$$\dot{I}_L = \dot{V}\frac{1}{R + j\omega_0 L} \approx -j\frac{\dot{V}}{\omega_0 L} = -j\sqrt{\frac{C}{L}}\dot{V}$$

$$\dot{I}_C = j\omega_0 C\dot{V} = j\sqrt{\frac{C}{L}}\dot{V}$$

$$|\dot{I}_L| = |\dot{I}_C| >> |\dot{I}_0| \approx 0 \longrightarrow Z \to \infty.$$

•Quality factor Q,

$$Q = \frac{I_C}{I_0} = \frac{I_L}{I_0} = \frac{Y_L}{Y_0} = \frac{Y_C}{Y_0}$$

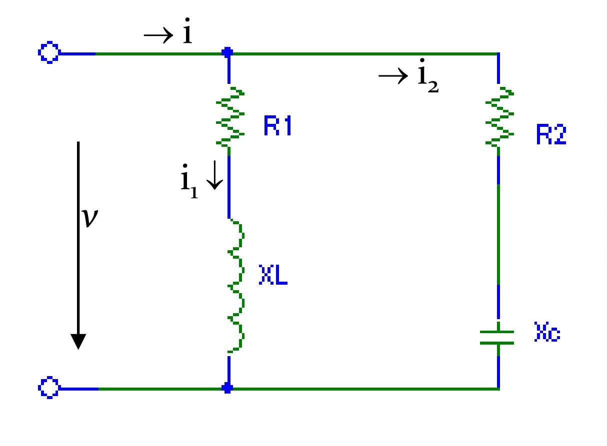
$$\dot{I}_L = -jQ\dot{I}_0 \qquad \dot{I}_C = jQ\dot{I}_0 \qquad Q \approx \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

1.5 Series and Parallel Resonance

Parallel RLC Circuit

P4.10, $R_1 = 3\Omega,\ X_L = 4\Omega,\ R_2 = 8\Omega,\ X_C = 6\Omega$ $v = 220\sqrt{2}\sin 314t$

Find i_1、i_2、i



1.5 Series and Parallel Resonance

Parallel RLC Circuit

Review

For sinusoidal circuit， Series : $v = v_1 + v_2$ $V \neq V_1 + V_2$

Parallel : $i = i_1 + i_2$ $I \neq I_1 + I_2$

?

Two Simple Methods:

Phasor Diagrams and Complex Numbers