# Magnetic Fields

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# Biot Savart's Law

- It states that the differential magnetic field intensity **dH** produced at a point P, by the differential current element Idl is proportional to the product Idl and the sine of the angle α between the element and the line joining the P to the element and is inversely proportional to the square of the distance R between P and the element,

That is,

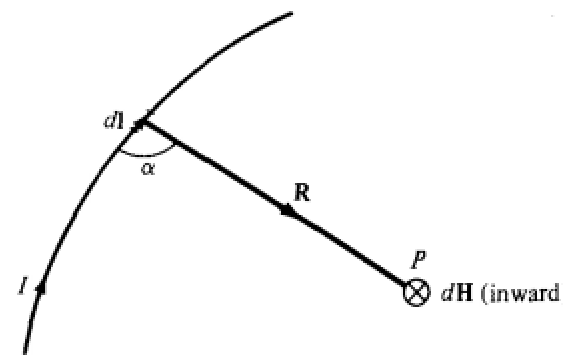
$$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

or

$$dH = \frac{kI\,dl \sin\alpha}{R^2}$$

where $k$ is the constant of proportionality. In SI units, $k = 1/4\pi$, so

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$



# Biot Savart's Law

- Vector form

$$\boxed{d\mathbf{H} = \frac{I\, d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\, d\mathbf{l} \times \mathbf{R}}{4\pi R^3}}$$

- For differential current distribution

$$\mathbf{H} = \int_L \frac{I\, d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(line current)}$$

$$\mathbf{H} = \int_S \frac{\mathbf{K}\, dS \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(surface current)}$$

$$\mathbf{H} = \int_v \frac{\mathbf{J}\, dv \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(volume current)}$$

# Field due to straight current carrying filamentary conductor of finite length

- We assume that the conductor is along z axis with its upper and lower ends respectively, subtending angle α1 and α2 at P, the pont at which **H** is to be determined.
- If we consider the contribution **dH** at P due to an element dl at (0,0,z)

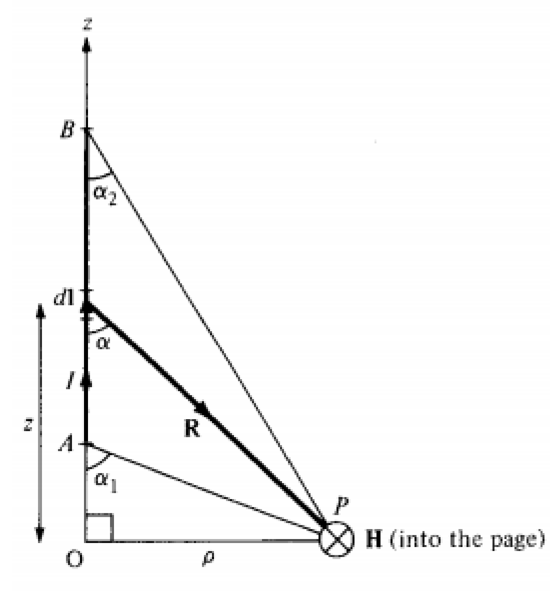
$$d\mathbf{H} = \frac{I\, d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

But $d\mathbf{l} = dz\, \mathbf{a}_z$ and $\mathbf{R} = \rho \mathbf{a}_\rho - z\mathbf{a}_z$, so

$$d\mathbf{l} \times \mathbf{R} = \rho\, dz\, \mathbf{a}_\phi$$

Hence,

$$\mathbf{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2 + z^2]^{3/2}}\, \mathbf{a}_\phi$$



# Field due to straight current carrying filamentary conductor of finite length

Letting $z = \rho \cot \alpha$, $dz = -\rho\, \mathrm{cosec}^2\, \alpha\, d\alpha$, and eq. (7.11) becomes

$$\mathbf{H} = -\frac{1}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2\, \mathrm{cosec}^2\, \alpha\, d\alpha}{\rho^3\, \mathrm{cosec}^3\, \alpha}\, \mathbf{a}_\phi$$

$$= -\frac{I}{4\pi\rho}\, \mathbf{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin \alpha\, d\alpha$$

or

$$\boxed{\mathbf{H} = \frac{I}{4\pi\rho} (\cos \alpha_2 - \cos \alpha_1)\mathbf{a}_\phi}$$

**Semiinfinite $\alpha 1=90$ and $\alpha 2=0$**

$$\mathbf{H} = \frac{I}{4\pi\rho}\, \mathbf{a}_\phi$$

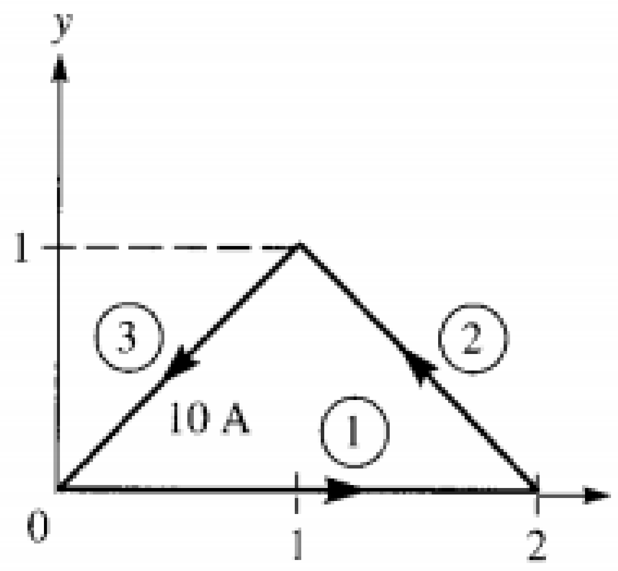
# Field due to straight current carrying filamentary conductor of finite length

**Infinite** $\alpha 1=180$ **and** $\alpha 2=0$

$$\boxed{\mathbf{H} = \frac{I}{2\pi\rho}\,\mathbf{a}_\phi}$$

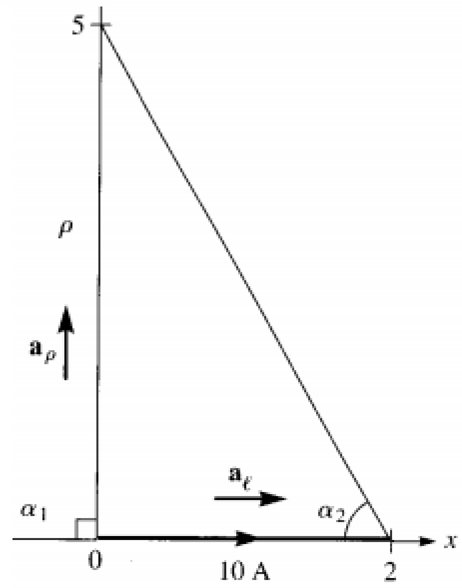
# Numerical 1

The conducting triangular loop in Figure carries a current of 10 A. Find **H** at (0, 0, 5) due to side 1 of the loop.



# Solution

$$\cos \alpha_1 = \cos 90^\circ = 0, \qquad \cos \alpha_2 = \frac{2}{\sqrt{29}}, \qquad \rho = 5$$



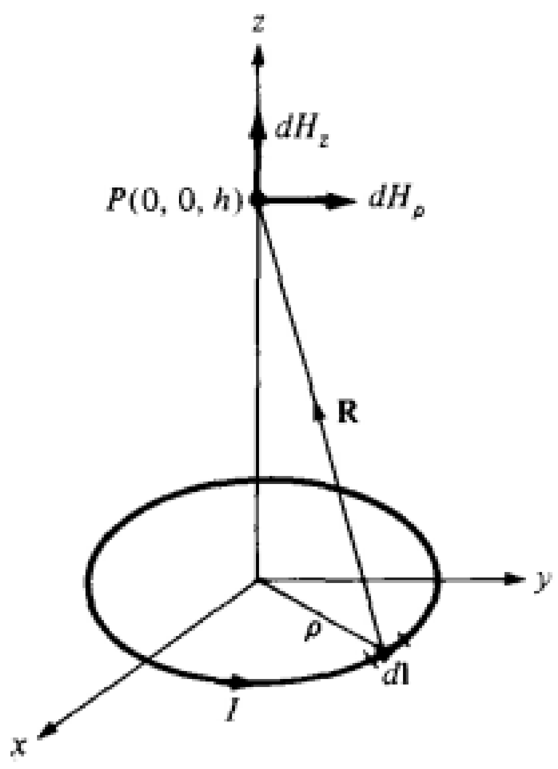

$$\mathbf{a}_\phi = \mathbf{a}_x \times \mathbf{a}_z = -\mathbf{a}_y$$

Hence,

$$\mathbf{H}_1 = \frac{I}{4\pi\rho}(\cos \alpha_2 - \cos \alpha_1)\mathbf{a}_\phi = \frac{10}{4\pi(5)}\left(\frac{2}{\sqrt{29}} - 0\right)(-\mathbf{a}_y)$$
$$= -59.1\mathbf{a}_y \text{ mA/m}$$

# Numerical 2

A circular loop located on $x^2 + y^2 = 9$, $z = 0$ carries a direct current of 10 A along $\mathbf{a}_\phi$. Determine $\mathbf{H}$ at (0, 0, 4) and (0, 0, −4).



# Solution

$$d\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

where $d\mathbf{l} = \rho\,d\phi\,\mathbf{a}_\phi$, $\mathbf{R} = (0, 0, h) - (x, y, 0) = -\rho\mathbf{a}_\rho + h\mathbf{a}_z$, and

$$d\mathbf{l} \times \mathbf{R} = \begin{vmatrix} \mathbf{a}_\rho & \mathbf{a}_\phi & \mathbf{a}_z \\ 0 & \rho\,d\phi & 0 \\ -\rho & 0 & h \end{vmatrix} = \rho h\,d\phi\,\mathbf{a}_\rho + \rho^2\,d\phi\,\mathbf{a}_z$$

Hence,

$$d\mathbf{H} = \frac{I}{4\pi[\rho^2 + h^2]^{3/2}}(\rho h\,d\phi\,\mathbf{a}_\rho + \rho^2\,d\phi\,\mathbf{a}_z) = dH_\rho\,\mathbf{a}_\rho + dH_z\,\mathbf{a}_z$$

# Solution

- Due to symmetry , radial components are cancelled out in pair, only z component are summed up

$$\mathbf{H} = \int dH_z\,\mathbf{a}_z = \int_0^{2\pi} \frac{I\rho^2\,d\phi\,\mathbf{a}_z}{4\pi[\rho^2 + h^2]^{3/2}} = \frac{I\rho^2 2\pi\mathbf{a}_z}{4\pi[\rho^2 + h^2]^{3/2}}$$

$$\mathbf{H} = \frac{I\rho^2\mathbf{a}_z}{2[\rho^2 + h^2]^{3/2}}$$

# Solution

(a) Substituting $I = 10\,A$, $\rho = 3$, $h = 4$ gives

$$\mathbf{H}(0, 0, 4) = \frac{10\,(3)^2\,\mathbf{a}_z}{2[9 + 16]^{3/2}} = 0.36\mathbf{a}_z \text{ A/m}$$

(b) Notice from $d\mathbf{l} \times \mathbf{R}$ above that if $h$ is replaced by $-h$, the $z$-component of $d\mathbf{H}$ remains the same while the $\rho$-component still adds up to zero due to the axial symmetry of the loop. Hence

$$\mathbf{H}(0, 0, -4) = \mathbf{H}(0, 0, 4) = 0.36\mathbf{a}_z \text{ A/m}$$

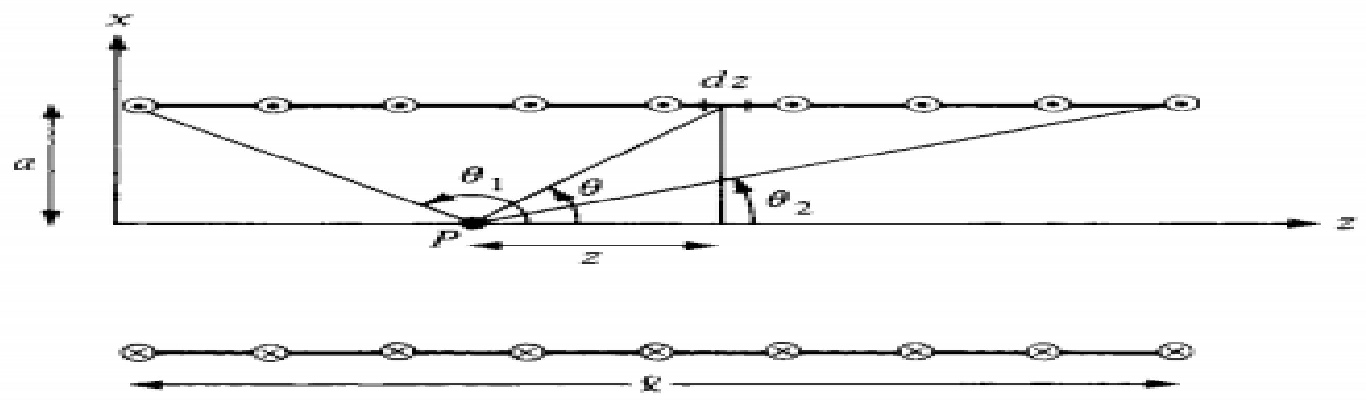
# Numerical 3

A solenoid of length $\ell$ and radius $a$ consists of $N$ turns of wire carrying current $I$. Show that at point $P$ along its axis,

$$\mathbf{H} = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)\mathbf{a}_z$$

where $n = N/\ell$, $\theta_1$ and $\theta_2$ are the angles subtended at $P$ by the end turns as illustrated in Figure Also show that if $\ell \gg a$, at the center of the solenoid,

$$\mathbf{H} = nI\mathbf{a}_z$$



# Solution

Consider the cross section of the solenoid in Fig. Since the solenoid consist of a circular loop.The contribution to the magnetic field H at P by an element of the of the solenoid of length dz is

$$dH_z = \frac{I\,dl\,a^2}{2[a^2 + z^2]^{3/2}} = \frac{Ia^2 n\,dz}{2[a^2 + z^2]^{3/2}}$$

# Solution

where $dl = n\,dz = (N/\ell)\,dz$. From Figure 7.9, $\tan\theta = a/z$; that is,

$$dz = -a\,\text{cosec}^2\,\theta\,d\theta = -\frac{[z^2 + a^2]^{3/2}}{a^2}\sin\theta\,d\theta$$

Hence,

$$dH_z = -\frac{nI}{2}\sin\theta\,d\theta$$

or

$$H_z = -\frac{nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta$$

Thus

$$\mathbf{H} = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)\,\mathbf{a}_z$$

# Solution

as required. Substituting $n = N/\ell$ gives

$$\mathbf{H} = \frac{NI}{2\ell}(\cos\theta_2 - \cos\theta_1)\,\mathbf{a}_z$$

At the center of the solenoid,

$$\cos\theta_2 = \frac{\ell/2}{[a^2 + \ell^2/4]^{1/2}} = -\cos\theta_1$$

# Solution

and

$$\mathbf{H} = \frac{In\ell}{2[a^2 + \ell^2/4]^{1/2}}\,\mathbf{a}_z$$

If $\ell \gg a$ or $\theta_2 \simeq 0°, \theta_1 \simeq 180°$,

$$\mathbf{H} = nI\mathbf{a}_z = \frac{NI}{\ell}\,\mathbf{a}_z$$