Magnetism

Er.Somesh Kr.Malhotra
Assistant Professor
ECE Department,UIET,
CSJM University

Boundary conditions

- We define magnetic boundary conditions as the conditions that H (or B) field must satisfy at the boundary between two different media. We make use of Gauss's law for magnetic fields

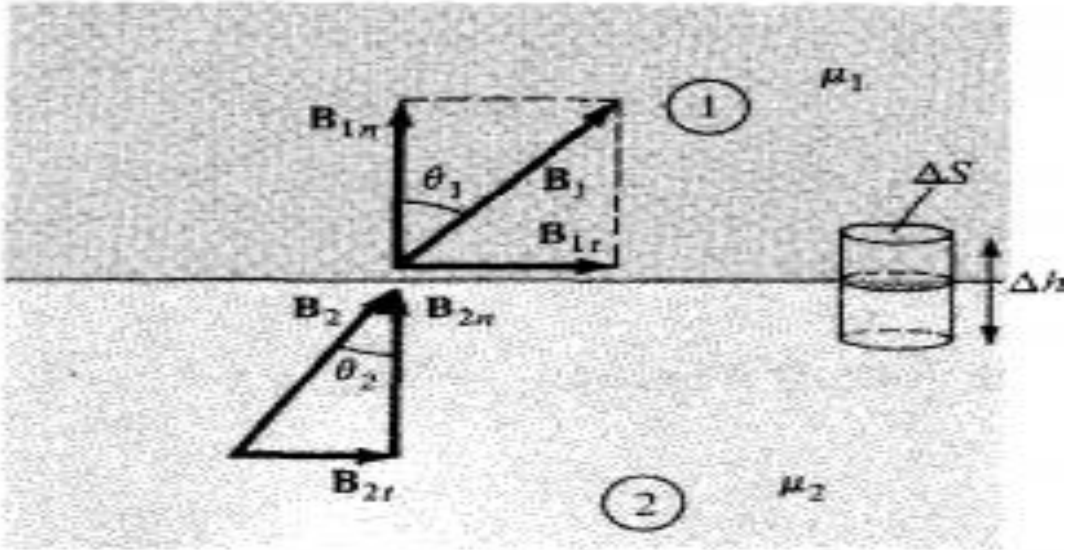
$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

and ampere’s circuital law

$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

Boundary conditions

- Consider the boundary between two magnetic media 1 and 2, characterized, respectively, by μ1 and μ2 as in Figure. Applying above equations to the pillbox (Gaussian surface) of Figure and allowing $\Delta h \to 0$, we obtain



(a)

BOUNDARY CONDITIONS

$$B_{1n}\,\Delta S - B_{2n}\,\Delta S = 0$$

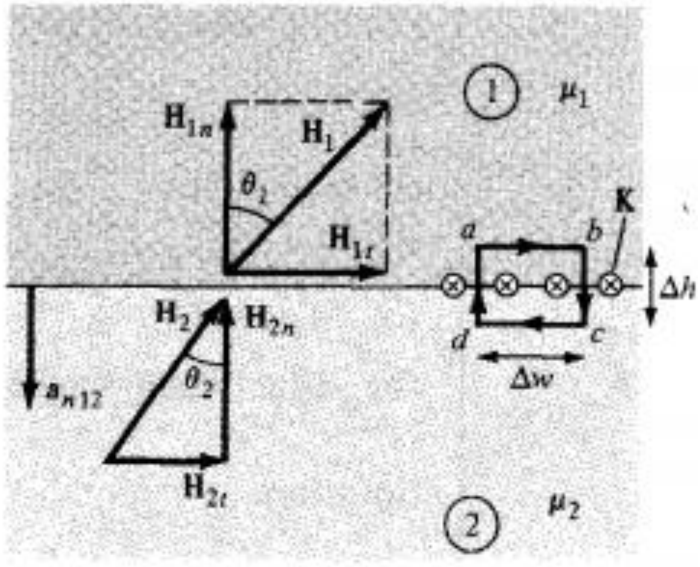
Thus

$$\boxed{\mathbf{B}_{1n} = \mathbf{B}_{2n}} \qquad \text{or} \qquad \mu_1 \mathbf{H}_{1n} = \mu_2 \mathbf{H}_{2n}$$

- since B = μH. Equation shows that the normal component of B is continuous at the boundary. It also shows that the normal component of H is discontinuous at the boundary; H undergoes some change at the interface.

Boundary conditions

- Similarly, we apply ampere's law to the closed path *abcda* of Figure below where surface current *K* on the boundary is assumed normal to the path. We obtain



BOUNDARY CONDITIONS

$$K \cdot \Delta w = H_{1t} \cdot \Delta w + H_{1n} \cdot \frac{\Delta h}{2} + H_{2n} \cdot \frac{\Delta h}{2}$$
$$- H_{2t} \cdot \Delta w - H_{2n} \cdot \frac{\Delta h}{2} - H_{1n} \cdot \frac{\Delta h}{2}$$

As $\Delta h \to 0$, eq. (8.42) leads to

$$H_{1t} - H_{2t} = K$$

Boundary conditions

- This shows that the tangential component of H is also discontinuous. Equation may be written in terms of B as

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

Boundary conditions

In the general case, eq. becomes

$$\boxed{(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}}$$

where $\mathbf{a}_{n12}$ is a unit vector normal to the interface and is directed from medium 1 to medium 2. If the boundary is free of current or the media are not conductors (for K is free current density), $K = 0$ and eq. (8.43) becomes

$$\boxed{\mathbf{H}_{1t} = \mathbf{H}_{2t}} \quad \text{or} \quad \frac{\mathbf{B}_{1t}}{\mu_1} = \frac{\mathbf{B}_{2t}}{\mu_2}$$

Thus the tangential component of **H** is continuous while that of **B** is discontinuous at the boundary.

Boundary conditions

If the fields make an angle θ with the normal to the interface, eq. results in

$$B_1 \cos\theta_1 = B_{1n} = B_{2n} = B_2 \cos\theta_2$$

From the tangential condition

$$\frac{B_1}{\mu_1}\sin\theta_1 = H_{1t} = H_{2t} = \frac{B_2}{\mu_2}\sin\theta_2$$

Dividing above the equations

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

which is the law of refraction for magnetic flux lines at a boundary with no surface current.

NUMERICALS

Given that $\mathbf{H}_1 = -2\mathbf{a}_x + 6\mathbf{a}_y + 4\mathbf{a}_z$ A/m in region $y - x - 2 \leq 0$ where $\mu_1 = 5\mu_o$, calculate

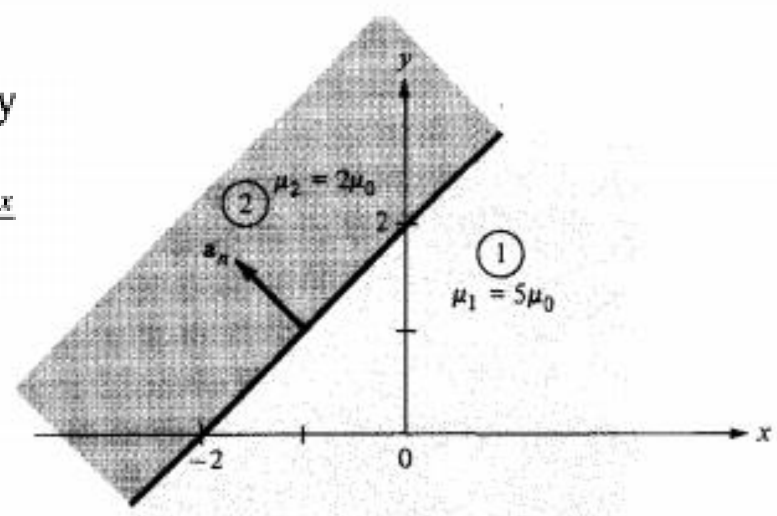
(a) $\mathbf{M}_1$ and $\mathbf{B}_1$

(b) $\mathbf{H}_2$ and $\mathbf{B}_2$ in region $y - x - 2 \geq 0$ where $\mu_2 = 2\mu_o$

Solution:

a unit vector normal to the plane is given by

$$\mathbf{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\mathbf{a}_y - \mathbf{a}_x}{\sqrt{2}}$$



NUMERICALS

(a)

$$\mathbf{M}_1 = \chi_{m1}\mathbf{H}_1 = (\mu_{r1} - 1)\,\mathbf{H}_1 = (5 - 1)(-2, 6, 4)$$
$$= -8\mathbf{a}_x + 24\mathbf{a}_y + 16\mathbf{a}_z \text{ A/m}$$

$$\mathbf{B}_1 = \mu_1\mathbf{H}_1 = \mu_o\mu_{r1}\mathbf{H}_1 = 4\pi \times 10^{-7}(5)(-2, 6, 4)$$
$$= -12.57\mathbf{a}_x + 37.7\mathbf{a}_y + 25.13\mathbf{a}_z\ \mu\text{Wb/m}^2$$

(b) $$\mathbf{H}_{1n} = (\mathbf{H}_1 \cdot \mathbf{a}_n)\mathbf{a}_n = \left[(-2, 6, 4) \cdot \frac{(-1, 1, 0)}{\sqrt{2}}\right]\frac{(-1, 1, 0)}{\sqrt{2}}$$
$$= -4\mathbf{a}_x + 4\mathbf{a}_y$$

But

$$\mathbf{H}_1 = \mathbf{H}_{1n} + \mathbf{H}_{1t}$$

Hence,

$$\mathbf{H}_{1t} = \mathbf{H}_1 - \mathbf{H}_{1n} = (-2, 6, 4) - (-4, 4, 0)$$
$$= 2\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z$$

Numericals

Using the boundary conditions, we have

$$\mathbf{H}_{2t} = \mathbf{H}_{1t} = 2\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z$$

$$\mathbf{B}_{2n} = \mathbf{B}_{1n} \rightarrow \mu_2\mathbf{H}_{2n} = \mu_1\mathbf{H}_{1n}$$

or

$$\mathbf{H}_{2n} = \frac{\mu_1}{\mu_2}\mathbf{H}_{1n} = \frac{5}{2}(-4\mathbf{a}_x + 4\mathbf{a}_y) = -10\mathbf{a}_x + 10\mathbf{a}_y$$

Thus

$$\mathbf{H}_2 = \mathbf{H}_{2n} + \mathbf{H}_{2t} = -8\mathbf{a}_x + 12\mathbf{a}_y + 4\mathbf{a}_z \text{ A/m}$$

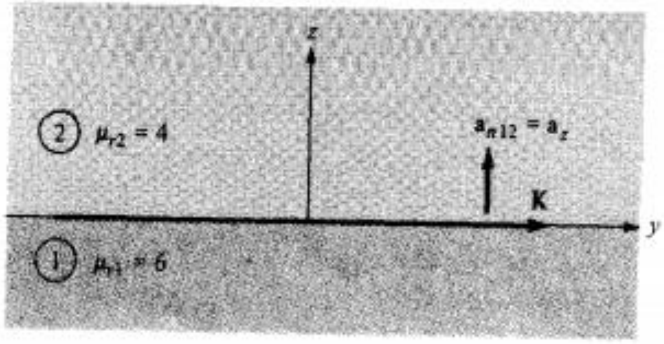
and

$$\begin{aligned}\mathbf{B}_2 = \mu_2\mathbf{H}_2 = \mu_o\mu_{r2}\mathbf{H}_2 &= (4\pi \times 10^{-7})(2)(-8, 12, 4)\\ &= -20.11\mathbf{a}_x + 30.16\mathbf{a}_y + 10.05\mathbf{a}_z \ \mu\text{Wb/m}^2\end{aligned}$$

Numericals

The xy-plane serves as the interface between two different media. Medium 1 ($z < 0$) is filled with a material whose $\mu_r = 6$, and medium 2 ($z > 0$) is filled with a material whose $\mu_r = 4$. If the interface carries current $(1/\mu_0)\ \mathbf{a}_y$ mA/m, and $\mathbf{B}_2 = 5\mathbf{a}_x + 8\mathbf{a}_z$ mWb/m^2, find $\mathbf{H}_1$ and $\mathbf{B}_1$.

Solution:



$$\mathbf{B}_{1n} = \mathbf{B}_{2n} = 8\mathbf{a}_z \rightarrow B_z = 8$$

But

$$\mathbf{H}_2 = \frac{\mathbf{B}_2}{\mu_2} = \frac{1}{4\mu_o}(5\mathbf{a}_x + 8\mathbf{a}_z) \text{ mA/m}$$

NUMERICALS

and

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_1} = \frac{1}{6\mu_o}(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z) \text{ mA/m}$$

Having found the normal components, we can find the tangential components using

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$$

or

$$\mathbf{H}_1 \times \mathbf{a}_{n12} = \mathbf{H}_2 \times \mathbf{a}_{n12} + \mathbf{K}$$

$$\frac{1}{6\mu_o}(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z) \times \mathbf{a}_z = \frac{1}{4\mu_o}(5\mathbf{a}_x + 8\mathbf{a}_z) \times \mathbf{a}_z + \frac{1}{\mu_o}\mathbf{a}_y$$

NUMERICALS

Equating components yields

$$B_y = 0, \qquad \frac{-B_x}{6} = \frac{-5}{4} + 1 \qquad \text{or} \qquad B_x = \frac{6}{4} = 1.5 \tag{8.8.5}$$

$$\mathbf{B}_1 = 1.5\mathbf{a}_x + 8\mathbf{a}_z \text{ mWb/m}^2$$

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_1} = \frac{1}{\mu_o}(0.25\mathbf{a}_x + 1.33\mathbf{a}_z) \text{ mA/m}$$

and

$$\mathbf{H}_2 = \frac{1}{\mu_o}(1.25\mathbf{a}_x + 2\mathbf{a}_z) \text{ mA/m}$$

Note that H_{1x} is $(1/\mu_o)$ mA/m less than H_{2x} due to the current sheet and also that $B_{1n} = B_{2n}$.